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ESTIMATION OF BINOMIAL SAMPLING BETA AND GAMMA DISTRIBUTION

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ABSTRACT

In this paper we consider the Pascal sampling equal to Binomial sampling in some sense. The Binomial sampling and the sample size is fixed at n in advance. We show that for some researches, the testing costs dependsome more on the number of failures s than on the number of trials n: This is particularly true if failures cost significantly more than survivors.

Keywords : *Fuzzy, Binomial, Pascal, Beta and Gamma etc.*

Introduction :

Let us assume that the survivors are not reported exactly under some surprising situation in case, the item is not be completely failed during the test, or some unsuccessful items are not exactly recorded owing to human errors. Hence, the survival probability cannot be sure of as an exact real number. Therefore, the survival probability is regarded as a fuzzy real number under this situation. we estimate the (fuzzy) existence probability of an item based on the Binomial sampling and Pascal sampling.

Let us suppose that there are n items all item *i*might be represented as a Bernoulli random variable Y_i with survival probability p. Then the number of survivors $X = \sum_{i=1}^{n} Y_i$ is recorded. The probability distribution of the number of survivors is the Binomial distribution given by

$$f(x|p) = \Pr\{x \text{ survivors will ovvur in n trials}|p\} = \frac{n!}{(n-x)!\,x!} p^x (1-p)^{n-x} x = 0{,}1 \; and \; 0 < p < 1$$

We find the Bayes point estimator of the survival probability p; by taking survival probability p as a random variable P with pdf π(p), where π(p) is termed as a prior distribution of the survival probability P. Then the posterior distribution of P is then given by

$$\pi(p|x) = \frac{f(x|p).\pi(p)}{\int_0^1 f(x|y).\pi(y)dy} = \frac{p^x(1-p)^{n-x} x\,\pi(p)}{\int_0^1 y^x(1-y)^{n-x}\pi(y)dy} \quad for\; 0 < p < 1$$

where x is the observed number of toughies in n trials.The prior distribution for P is the Beta distribution $B(\theta_1, \theta_2)$ with pdf given by

$$\pi(p;\theta_1,\theta_2) = \frac{\Gamma(\theta_1+\theta_2)}{\Gamma(\theta_1)\Gamma(\theta_2)} = p^{\theta_1-1}(1-p)^{\theta_2-1} \qquad \text{... (1.1)}$$

where $0 \le p \le 1$ and $\theta_1, \theta_2 > 0$, since the conjugate prior distribution of the Binomial distribution is Beta delivery. Therefore equation the posterior distribution of P given x is also a Beta distribution $B(x + \theta_1, n + \theta_2 - x)$ with pdf given by

$$\pi(p|x;\; \theta_1,\theta_2) = \frac{\Gamma(n+\;\theta_1+\theta_2)}{\Gamma(x+\;\theta_1)\Gamma(n+\;\theta_2+x)} = p^{x+\theta_1-1}(1-p)^{n+\theta_2-x-1}$$

The Bayes point estimator $\hat{p}$ for the survival probability p is the mean of the posterior distribution as in Eq. (1.1) under the formed error loss function, and it is given by :

$$\hat{p} = \frac{X+\;\theta_1}{n+\;\theta_1+\theta_2} \qquad \text{... (1.2)}$$

Let $n_0 = \theta_1 + \theta_2$ and $x_0 = \theta_1$. We reparameterize the Beta distribution, denoted as $B_1(x_0, n_0)$ to give its as follows.

$$\pi(p; x_0, n_0) = \frac{\Gamma(n_0)}{\Gamma(x_0)\Gamma(n_0 - x_0)} = p^{x_0-1}(1-p)^{n_0-x_0-1} \quad \ldots(1.3)$$

Therefore, the Bayes point estimate of p can be rewritten as

$$\hat{p} = \frac{x+x_0}{n+n_0} \quad \ldots(1.4)$$

But the expectation of Beta distribution B(θ$_1$, θ$_2$) is θ$_1$/θ$_1$, +θ$_2$), is the expectation of Beta distribution B_1(x_0 , n_0) is then x_0/n_0. Therefore, the numbers n_0 and x_0 may interpreted as 'pseudo' number of sample size and 'pseudo' number of survivors. The 'pseudo' number of survivors may be taken from the past experience, e.g. the average of survivors from the previous tests. Let us assume x_0 to be an integer and consider the prior Beta distribution B_1(x_0 , n_0).

The survival probability is not sure of an exact real number. For Bayesian approach, the (fuzzy) survival probability is scheduled as a fuzzy random variable $\tilde{P}$ with the assumptions that n_0 integer representing the 'pseudo' number of sample size and $\tilde{x}_0$ is a known fuzzy real number representing the 'pseudo' number of survivors $\tilde{x}_0$ is assumed as a fuzzy real number. Then, from Eq. (1.2), the Bayes point estimates of $\tilde{p}_\alpha^L$ and $\tilde{p}_\alpha^U$ are given by

$$\widehat{\tilde{p}_\alpha^L} = \frac{x + (\hat{x}_0)_\alpha^L}{n + n_0} \; and \; \widehat{\tilde{p}_\alpha^U} = \frac{x + (\hat{x}_0)_\alpha^U}{n + n_0} \quad \ldots(1.5)$$

respectively, for all α = [0, 1]. Let us

$$A_\alpha = \left[\min\left\{\min_{\alpha \le \beta \le 1} \widehat{\tilde{P}_\beta^L}, \min_{\alpha \le \beta \le 1} \widehat{\tilde{P}_\beta^U}\right\}, \max\left\{\max_{\alpha \le \beta \le 1} \widehat{\tilde{P}_\beta^L}, \max_{\alpha \le \beta \le 1} \widehat{\tilde{P}_\beta^U}\right\}\right] \quad \ldots(1.6)$$

Then this interval contain all of the Bayes point estimates for each $p \in [\tilde{p}_\alpha^L, \tilde{p}_\alpha^U]$. Since $(\tilde{x}_0)_\alpha^L \le (\tilde{x}_0)_\alpha^L$. It is obvious that $\widehat{\tilde{P}_\alpha^L} \le \widehat{\tilde{P}_\alpha^U}, \widehat{\tilde{P}_\alpha^L} \le \widehat{\tilde{P}_\beta^U}$ *and* $\widehat{\tilde{P}_\alpha^U} \le \widehat{\tilde{P}_\beta^U}$ *for* $\alpha < \beta, i.e. \widehat{\tilde{P}_\alpha^L} \le \widehat{\tilde{P}_1^L} \le \widehat{\tilde{P}_1^U} \le \widehat{\tilde{P}_\alpha^U}$. Thus, $A_\alpha$may be written as

$$A_\alpha = \widehat{\tilde{P}_\alpha^L}, \widehat{\tilde{P}_\alpha^U}$$

But the membership function of the fuzzy Bayes point estimate of $\tilde{p}$, denoted as $\hat{\tilde{p}}$, is defined by

$$\xi_{\tilde{p}}(p) = \sup_{0 \le \alpha \le 1} \alpha . 1_{A_\alpha}(P)$$

As deductible via the form of Resolution identity.

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