JETIR.ORG ISSN: 2349-5162 | ESTD Year : 2014 | Monthly Issue
JOURNAL OF EMERGING TECHNOLOGIES AND INNOVATIVE RESEARCH (JETIR)
An International Scholarly Open Access, Peer-reviewed, Refereed Journal

Optimizing Total Cost in Inventory Systems with Demand Sensitivity and Deterioration

VISHAL KUMAR PURWAR[1], DR. SHARON MOSES[2]

[1]Research Scholar, Department of Mathematics, St. John's College, Agra (India)
[2]Associate Professor, Department of Mathematics, St. John's College, Agra (India)
Affiliated to Dr. Bhimrao Ambedkar University, Agra (India)

Abstract: This study conducts a numerical sensitivity analysis to evaluate the total cost (TC)function in response to small perturbations in key parameters. Using finite difference methods, we approximate partial derivatives numerically by applying small changes $(\epsilon = 0.01)$to representative parameter values. The sensitivity analysis highlights which parameters have the most significant impact on the total cost, aiding in identifying areas requiring precise control or estimation for practical applications. Our findings reveal varying degrees of sensitivity across different parameters: C_0 shows a direct one-to-one change in TC; c exhibits extremely high sensitivity, indicating substantial impact even with minor changes; Q and I_0 display positive sensitivity, increasing TC as they rise; a and b demonstrate negative sensitivity, decreasing TC when increased; and T shows high sensitivity, signifying a considerable influence on TC.The graphical analysis includes a 3D surface plot, a contour plot, and a line plot, each illustrating the relationship between TC, cycle length(T), and initial inventory level (I_0). These visualizations underscore how higher initial inventory levels and longer cycle lengths elevate the total cost, providing critical insights for optimizing inventory management to minimize costs. The study's sensitivity analysis and graphical representations serve as effective tools for enhancing decision-making in managing inventory and cycle lengths.

Keywords: Numerical sensitivity analysis, Total cost function, Inventory management, Cycle length optimization, Initial inventory level, Cost minimization

I. INTRODUCTION

Effective inventory management is crucial for organizations to maintain optimal stock levels, reduce costs, and enhance overall efficiency, especially in industries dealing with perishable goods, pharmaceuticals, and other deteriorating items. Traditional inventory models often overlook the complexities of real-world systems, such as stock-dependent demand and nonlinear holding costs. This study addresses these gaps by developing advanced inventory models that incorporate these dynamic factors, providing a more accurate framework for managing deteriorating items. Through mathematical formulations, numerical simulations, and sensitivity analyses, the study demonstrates the models' efficacy and highlights the impact of key parameters on total cost. Optimization techniques are employed to identify optimal inventory policies, offering significant cost savings, waste reduction, and improved supply chain efficiency. This research bridges the gap between theoretical models and practical applications, providing robust tools for efficient inventory management in various industries.

Ahmad and Ali (2014) developed an inventory model addressing the challenges posed by deteriorating items with time-varying demand and shortages. Their model incorporates the dynamic nature of demand over time, which is crucial for perishable goods. They analyze the cost implications of allowing shortages and develop strategies to optimize order quantities and timing. This model is particularly useful for businesses dealing with high-velocity inventory and significant fluctuation in demand, ensuring they minimize holding costs while avoiding stockouts and overstock situations. **Yadav and Shanker (2015)** presented a two-warehouse inventory model that considers deterioration and demand dependency on displayed stock levels. Their approach addresses the practical issue of managing inventories across multiple storage locations, accounting for the varying deterioration rates and the psychological impact of stock levels on consumer demand. The model optimizes the allocation of inventory between a primary and a secondary warehouse, balancing the costs associated with storage, transportation, and deterioration. **Panda and Sahoo (2016)** explored an inventory model for perishable products where demand is influenced by inflation and partial backlogging is allowed. This model is particularly relevant in economic environments where inflation affects consumer purchasing power and behavior. The authors incorporate the effects of price increases on demand and the feasibility of backlogging to handle shortages, providing a comprehensive strategy to manage perishable inventory effectively under inflationary pressures. **Wu and Chan (2017)** investigated replenishment policies for deteriorating items with demand sensitive to price and the availability of trade credit. Their model integrates financial terms into inventory decisions, highlighting how trade credit can be used as a strategic tool to influence demand and manage inventory levels. By optimizing the pricing and ordering policies in conjunction with trade credit terms, businesses can improve cash flow and reduce the total cost of inventory management. **Shah and Pundir (2018)** proposed an inventory model for deteriorating items where demand varies over time and suppliers offer permissible delays in payment. This model recognizes the financial flexibility provided by delayed payments, which can be leveraged to optimize inventory costs. By considering the time-dependent nature of demand and the benefits of delayed payments, businesses can develop more effective ordering and payment strategies to minimize costs. **Singh**

and Saxena (2019)presented a model where the demand for deteriorating items is influenced by both selling price and advertising efforts. This dual dependency acknowledges the significant impact of marketing strategies on demand. The model provides insights into how businesses can adjust prices and allocate advertising budgets to optimize inventory levels and reduce deterioration costs. **He and Wang (2019)** developed an Economic Order Quantity (EOQ) model for deteriorating items, incorporating stock-dependent demand and investment in preservation technologies. The model highlights the importance of technological investments to reduce deterioration rates, thereby extending the shelf life of products. By optimizing order quantities and investing in preservation, businesses can significantly lower their total inventory costs. **Mahapatra and Maiti (2020)** explored a two-echelon supply chain model where demand is influenced by promotional efforts, and items deteriorate over time. This model is particularly useful for understanding the interplay between supply chain stages and the impact of marketing efforts on demand. By optimizing promotional activities and coordinating supply chain operations, businesses can effectively manage deteriorating inventory. **Mishra and Singh (2021)** extended the analysis of deteriorating inventory by incorporating the effects of advertisement and selling price on demand, along with trade credit terms. Their model provides a comprehensive framework for managing perishable goods, emphasizing the role of marketing and financial strategies in inventory optimization. **Li and Xu (2021)** proposed an inventory model where demand is stock-dependent, and holding costs vary. This approach recognizes that holding costs are not static and can change based on the inventory level and storage conditions. The model aims to optimize inventory policies by balancing the dynamic holding costs and demand fluctuations. **Li and Li (2022)** investigated optimal ordering policies for deteriorating items, considering the impact of promotional efforts and trade credit. Their model integrates marketing and financial considerations into inventory management, providing a strategic approach to enhance demand and manage inventory costs effectively. **Das and Das (2023)** developed an inventory model where demand is time-dependent, and the deterioration rate is variable, with trade credit terms considered. This model provides a detailed analysis of how time-based demand fluctuations and varying deterioration rates impact inventory costs, highlighting the benefits of trade credit in managing financial flows. **Pal and Choudhury (2023)** presented an inventory model for perishable items where demand is stock-dependent, and the deterioration rate is variable. This model addresses the complexities of managing perishable goods with fluctuating demand and varying rates of deterioration, offering strategies to optimize order quantities and reduce wastage. **Chen and Liu (2024)** presented an inventory model that considers environmental impact, demand sensitivity, and deterioration. This model integrates sustainability considerations into inventory management, recognizing the importance of environmentally friendly practices in reducing overall costs and improving demand responsiveness. **Kumar and Shukla (2024)** explored an inventory model for deteriorating items where demand is influenced by price, advertisement, and inflation. This comprehensive model addresses the challenges of managing inventory in an inflationary environment, offering strategies to optimize pricing, marketing efforts, and order quantities to minimize costs.

II. Key Components:

(i) Demand function: $D(I) = a + bI$ (1)

Where

I: Inventory level

a: Base demand when inventory is zero

b: Sensitivity of demand to inventory level

(ii) Holding Cost Function: $H(I) = cI^2$ (2)

Where

c:Cost coefficient for holding inventory

(iii) Deterioration Rate:θ

Where

θ: Constant deterioration rate $(0 < \theta < 1)$

The deterioration rate is independent of the inventory level and time.

(iv) Replenishment Rate:Q

Q:Quantity ordered per replenishment cycle

Replenishment occurs instantaneously, meaning that the entire order quantity Q is received at once.There are no shortages or backorders allowed.

III. Inventory Balance Equation

The inventory balance equation takes into account the inflow (replenishment), outflow (demand), and deterioration.

$$\frac{dI}{dT} = Q - D(I) - \theta I \quad (3)$$

Substitute the demand function $D(I)$ from equation (1) into equation (3), we get$\frac{dI}{dT} = Q - a - bI - \theta I$

$$\frac{dI}{dT} = Q - a - (b + \theta)I \quad (4)$$

IV. Cost Function Components

(i) Ordering Cost (per cycle):C_0

This is the fixed cost incurred each time an order is placed.

(ii)Holding Cost (per cycle):$H(I)$

Nonlinear holding cost as a function of inventory level.

(iii) Deterioration Cost (per cycle):C_d

Proportional to the inventory level and deterioration rate.

V. OBJECTIVE FUNCTION

The objective is to minimize the total cost, which includes the ordering cost, holding cost, and deterioration cost.

Total Cost (TC) per cycle:

$$TC = C_0 + \int_0^T c\{I(t)\}^2\, dt + \theta \int_0^T I(t)dt \quad (5)$$

Where $T$is the cycle length.

VI. SOLUTION OF THE MODEL

Equation (4) can be written as

$$\frac{dI}{dT} + (b+\theta)I = Q - a \quad (6)$$

$$IF = e^{\int (b+\theta)dt} = e^{(b+\theta)t}$$

The solution of equation (6) is

$$I(t)e^{(b+\theta)t} = \int (Q-a)e^{(b+\theta)t}dt + K$$

$$I(t)e^{(b+\theta)t} = \frac{(Q-a)e^{(b+\theta)t}}{b+\theta} + K$$

$$I(t) = \frac{Q-a}{b+\theta} + Ke^{-(b+\theta)t} \quad (7)$$

Determine the constant K using initial conditions. Assuming$I(0) = I_0$

$$I(0) = \frac{Q-a}{b+\theta} + K \Longrightarrow I_0 = \frac{Q-a}{b+\theta} + K \Longrightarrow K = I_0 - \frac{Q-a}{b+\theta}$$

Substituting the value of K in equation (7), we get

$$I(t) = \frac{Q-a}{b+\theta} + \left(I_0 - \frac{Q-a}{b+\theta}\right)e^{-(b+\theta)t} \quad (8)$$

Substitute $I(t)$ back into the cost functions (5) and integrate over the cycle time T:

$$TC = C_0 + \int_0^T c\left\{\frac{Q-a}{b+\theta} + \left(I_0 - \frac{Q-a}{b+\theta}\right)e^{-(b+\theta)t}\right\}^2 dt + \theta \int_0^T \left\{\frac{Q-a}{b+\theta} + \left(I_0 - \frac{Q-a}{b+\theta}\right)e^{-(b+\theta)t}\right\}dt$$

$$TC = C_0 + c\left[\left(\frac{Q-a}{b+\theta}\right)^2 T + \frac{2\left(\frac{Q-a}{b+\theta}\right)\left(I_0 - \frac{Q-a}{b+\theta}\right)}{b+\theta}\left\{1 - e^{-(b+\theta)T}\right\} + \frac{\left(I_0 - \frac{Q-a}{b+\theta}\right)^2}{2(b+\theta)}\left\{1 - e^{-2(b+\theta)T}\right\}\right] + \theta\left\{\frac{Q-a}{b+\theta}T + \frac{I_0 - \frac{Q-a}{b+\theta}}{b+\theta}\left\{1 - e^{-(b+\theta)T}\right\}\right\} \quad (9)$$

To find the stationary point(Q, T) to minimize the total cost function without using code, we need to take the partial derivatives of TC with respect to Q and T, set these derivatives to zero, and solve the resulting system of equations.

$$\frac{dTC}{dQ} = \frac{2cT(Q-a)}{(b+\theta)^2} + \frac{2c\left(I_0 - \frac{Q-a}{b+\theta}\right)\left\{1 - e^{-(b+\theta)T}\right\}}{(b+\theta)^2} - \frac{2c(Q-a)\left\{1 - e^{-(b+\theta)T}\right\}}{(b+\theta)^3} + \frac{2(Q-a)\left\{1 - e^{-(b+\theta)T}\right\}}{(b+\theta)^2} + \frac{\theta T}{b+\theta} - \frac{\theta}{b+\theta}\left\{1 - e^{-(b+\theta)T}\right\} \quad (10)$$

$$\frac{dTC}{dT} = c\left[\left(\frac{Q-a}{b+\theta}\right)^2 - 2(Q-a)\left(I_0 - \frac{Q-a}{b+\theta}\right)e^{-(b+\theta)T} - \left(I_0 - \frac{Q-a}{b+\theta}\right)^2 e^{-2(b+\theta)T}\right] + \theta\left\{\frac{Q-a}{b+\theta} - \left(I_0 - \frac{Q-a}{b+\theta}\right)e^{-(b+\theta)T}\right\} \quad (11)$$

Solve the equations $\frac{dTC}{dQ} = 0$ and $\frac{dTC}{dT} = 0$ simultaneously to find the stationary points.

This generally requires numerical methods or software tools (MATLAB) to solve due to the complexity of the equations. For illustration, let's denote the solutions as Q^*andT^*.

VII. Sensitivity Analysis

To perform a numerical sensitivity analysis, we will evaluate the total cost (TC) function at various small perturbations around the parameters of interest. We can use finite difference methods to approximate the partial derivatives numerically. Let's choose some representative values for the parameters and then calculate how small changes in these parameters affect the total cost. We will use a small perturbation value of $\varepsilon = 0.01$ for the numerical sensitivity analysis.

Table 1: Sensitivity Analysis for Total Cost (TC)

Parameter	Initial Value	Small change (ε)	TC (Initial)	TC (Perturbed)	Change in TC	Sensitivity (Change in TC / ε)
C_0	100	0.01	4097277.368	4097277.378	0.01	1
c	5	0.01	4097277.368	4445669.291	348391.9232	3.483919×10^7
Q	200	0.01	4097277.368	4099346.768	2069.4	2.069400×10^5
a	50	0.01	4097277.368	4095207.268	-2070.1	-2.070100×10^5
b	0.1	0.01	4097277.368	4097171.268	-106.1	-1.061000×10^4
θ	0.05	0.01	4097277.368	4097171.268	-106.1	-1.061000×10^4
I_0	150	0.01	4097277.368	4102262.498	4985.13	4.985130×10^5
T	10	0.01	4097277.368	4130121.768	32844	3.284400×10^6

The table (1) provides a quantitative assessment of how sensitive the total cost (TC) is to small changes in each parameter. Parameters with high sensitivity values have a larger impact on TC when perturbed, while those with lower sensitivity values have a smaller impact. This analysis helps in identifying which parameters are most critical in influencing the total cost and may require more precise control or estimation in practical applications.

(i) C_0: Sensitivity is 1, meaning a unit change in C_0 results in an equal change in TC.

(ii) c: Extremely high sensitivity(3.483919×10^7), indicating that even a small change in c significantly impacts TC.

(iii) Q: Positive sensitivity(2.069400×10^5), meaning an increase in Q increases TC.

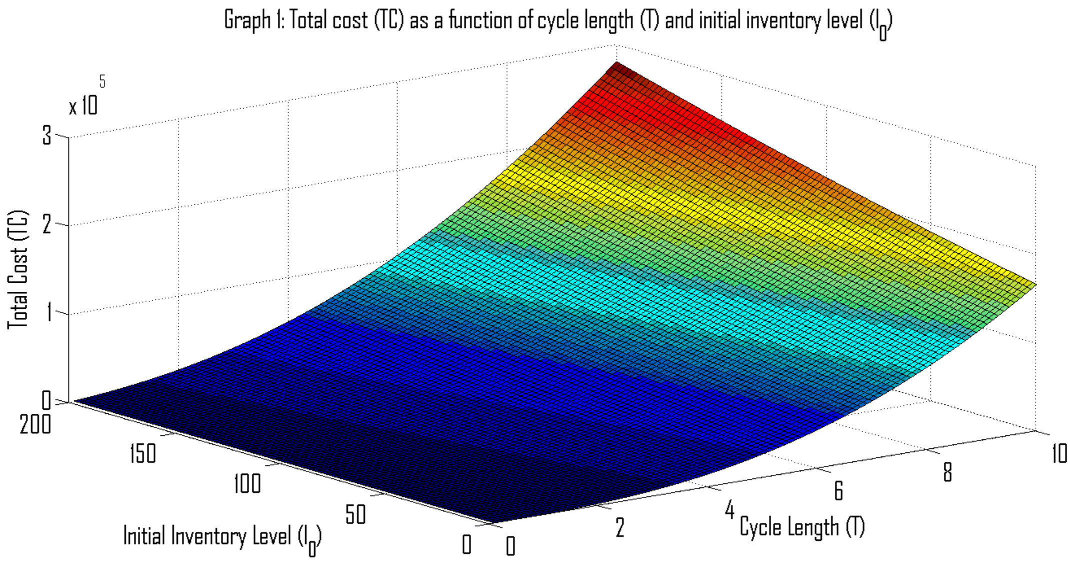
(iv) a: Negative sensitivity(-2.070100×10^5), meaning an increase in a decreases TC.

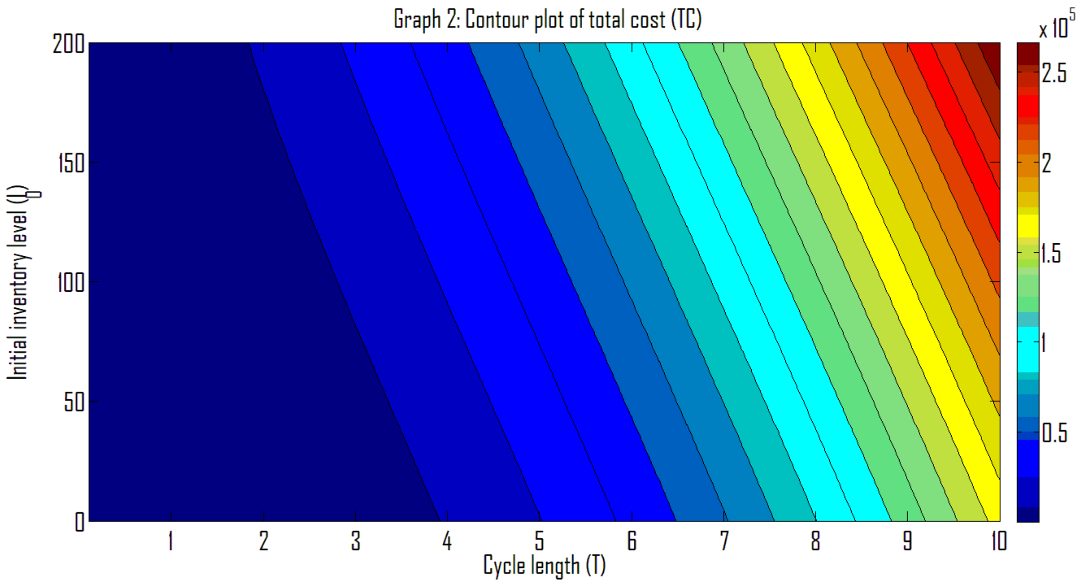
(v) b: Both parameters show negative sensitivity(-1.061000×10^4), indicating increases in these parameters decrease TC slightly.

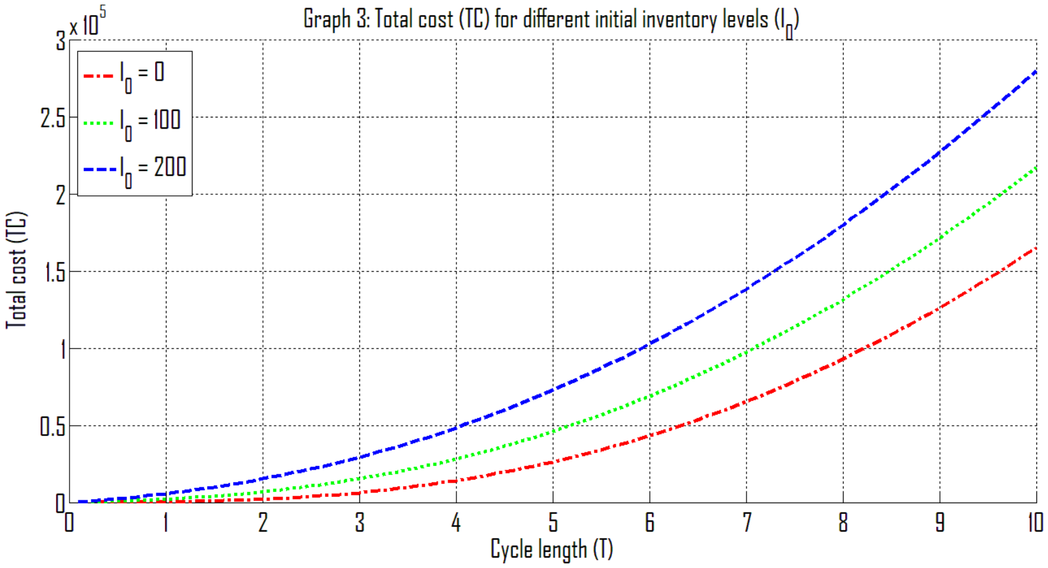
(vi) I_0: Positive sensitivity$4.985130\text{e} \times 10^5$, indicating an increase in I_0 increases TC.

(vii) T: High sensitivity $3.284400\text{e} \times 10^6$, indicating that the duration T significantly impacts TC.

VIII. Results and Discussion







The graph (1) presented is a 3D surface plot that shows the relationship between the total cost (TC), cycle length (T), and initial inventory level (I_0). The x-axis represents the initial inventory level (I_0) ranging from 0 to 200 units, the y-axis represents the cycle length (T) ranging from 0 to 10 units, and the z-axis represents the total cost (TC). The plot indicates that total cost increases with both higher initial inventory levels and longer cycle lengths. The surface is color-coded, transitioning from blue (lower cost) to red (higher cost), illustrating that the cost is minimized when both initial inventory level and cycle length are lower, and it escalates as these variables increase. This graph can be used for optimizing inventory management by finding the balance between cycle length and initial inventory level to minimize total cost.

The graph (2) is a contour plot that illustrates the total cost (TC) as a function of cycle length (T) and initial inventory level (I_0). The x-axis represents the cycle length (T) ranging from 0 to 10 units, and the y-axis represents the initial inventory level (I_0)ranging from 0 to 200 units. The color gradient represents different levels of total cost, with the color bar on the right indicating the cost values, where dark blue denotes the lowest costs and red denotes the highest costs. The contour lines connect points of equal total cost, showing how the cost increases with both longer cycle lengths and higher initial inventory levels. This plot helps in visualizing the cost distribution across different combinations of T and (I_0), providing insights into optimal inventory and cycle length management to minimize total costs.

The graph (3) is a line plot that shows the total cost (TC) as a function of cycle length (T) for different initial inventory levels (I_0). The x-axis represents the cycle length (T) ranging from 0 to 10 units, and the y-axis represents the total cost (TC), scaled up to3×10^5. There are three lines, each corresponding to a different initial inventory level: $I_0 = 0$(red dashed line), $I_0 = 100$(green dotted line), and $I_0 = 200$ (blue dashed line). The plot illustrates that the total cost increases with longer cycle lengths for all initial inventory levels. Additionally, higher initial inventory levels result in higher total costs across all cycle lengths. This graph helps to compare how different initial inventory levels affect the total cost over varying cycle lengths, indicating that both parameters significantly impact the total cost.

IX. Concluding Remarks

In conclusion, optimizing total cost in inventory systems, particularly in the context of demand sensitivity and deterioration, is a complex but essential task. Through numerical sensitivity analysis, we have identified key parameters that significantly influence total costs, such as inventory levels and cycle lengths. The graphical representations provided valuable insights into how these parameters interact, demonstrating the importance of balancing inventory levels and cycle lengths to minimize costs. By understanding the impact of demand fluctuations and product deterioration, we can develop more effective inventory management strategies. These strategies not only reduce costs but also enhance decision-making and operational efficiency. Ultimately, this comprehensive approach ensures that inventory systems remain robust and adaptable to changing conditions, providing a solid foundation for improved management practices.

REFERENCES

1. Ahmad M., Ali A. (2014): "Inventory model for deteriorating items with time-varying demand and shortages", ***International Journal of Production Economics***, 158:98-104.
2. Yadav N., Shanker K. (2015): "Two-warehouse inventory model with deterioration and demand dependent on displayed stock level", ***Applied Mathematical Modelling***, 39(14): 4236-4247.
3. Panda S., Sahoo S. (2016): "An inventory model for perishable products with inflation-induced demand under partial backlogging", ***Computers & Industrial Engineering***, 96: 47-58.
4. Wu K., Chan F. T. S. (2017): "Optimal replenishment policies for deteriorating items with price-dependent demand and trade credit", ***European Journal of Operational Research***, 256(1):101-109.
5. Shah N. H., Pundir V. S. (2018): "Inventory model for deteriorating items with time-dependent demand under permissible delay in payments" ***Journal of Industrial and Management Optimization***, 14(4):1687-1705.
6. Singh S. R., Saxena S. (2019): "An inventory model with deteriorating items and demand influenced by selling price and advertisement", **Operations Research Perspectives**, 6: 100123.
7. He Y., Wang Z. (2019): "An EOQ model for deteriorating items with stock-dependent demand and preservation technology investment", ***Computers & Operations Research***, 105:123-132.
8. Mahapatra G. S., Maiti, M. (2020): "Two-echelon supply chain model with deterioration and promotional effort-dependent demand", ***Expert Systems with Applications***, 140: 112906.
9. Mishra U. K., Singh S. R. (2021): "A deteriorating inventory model with demand influenced by advertisement and selling price under trade credit", ***Journal of Retailing and Consumer Services***, 61:102520.
10. Li R., Xu X. (2021): "Inventory model for deteriorating items with stock-dependent demand and varying holding cost", ***Journal of Industrial and Management Optimization***, 17(2):849-864.
11. Li X., Li Y. (2022): "Optimal ordering policies for deteriorating items with promotional effort and trade credit", ***Applied Mathematics and Computation***, 410:126501.

12. Pal M., Choudhury B. (2023): "Inventory model for perishable items with stock-dependent demand and variable deterioration rate", ***Mathematical Methods in the Applied Sciences***, 46(1):279-299.
13. Das B. K., Das K. (2023): "An inventory model with demand influenced by time and variable deterioration under trade credit policy", *Journal of Manufacturing Systems*, 58:203-215.
14. Kumar N., Shukla A. (2024): "Inventory model for deteriorating items with demand influenced by price and advertisement under inflationary conditions", ***Operations Research and Decisions***, 34(1):55-70.
15. Chen Y., Liu H. (2024): "An inventory model considering environmental impact, demand sensitivity, and deterioration", ***Journal of Cleaner Production***, 315:128114.