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Elaboration of Aryabhata's Kuttaka

By
Dr Ajay Saini

Abstract

Kuttaka Method, also known as the Kuttka-vieta Method, is a mathematical technique used to solve a certain type of indeterminate equation. It involves expressing variables in terms of parameters and then finding values for the perimeter that satisfy the condition of the equation. The method is particularly useful in Number theory problems and can yield integer solutions to the equations that might otherwise seem unsolvable. it relies on systematic manipulation of equation to uncover relationship between variation and parameters, ultimately leading to the solution.
indeterminate equation, Kuttka-vieta Method

Introduction

Indian writings from the period of Aryabhata (499 AD) contain systematic techniques for determining integer solutions of Diophantine equations. In his work Aryabhatiya, he provides the first explicit description of the general integral solution of the linear Diophantine problem ay - bx = c. In pure mathematics, this algorithm is regarded as Aryabhata's most significant contribution. Aryabhata used the method to provide integral solutions for first-degree simultaneous Diophantine equations, a subject with significant astronomical applications. Aryabhata did not call the algorithm Kuṭṭaka, and his explanation of the process was largely cryptic and difficult to understand. The algorithm was named Kuṭṭaka by Bhāskara-I (c. 600–c. 680), who published a full description of it in his Āryabhatiyabhāṣya, along with various instances from astronomy. The Kuṭṭaka algorithm is also known by various other names in literature, including Kuṭṭa, Kuṭṭakāra, and Kuṭṭikāra.
In just two stanzas of Aryabhatiya (verses 32 and 33 of the section Ganita), Aryabhata explains the algorithm. Bhaskara I (6th century AD) expounded upon his enigmatic verses in his commentary Aryabhatiyabhasya. Aryabhata's rule was demonstrated by Bhāskara-I using a number of examples, including 24 real-world astronomical issues. It would not have been easy to understand Aryabhata's lyrics without Bhaskara I's explanation (see [2] for details).
Appropriately, Bhāskara-I named the process 'kuttaka' (pulverisation); the algorithm would make the metaphorical meaning evident. A number of Indian mathematicians, including Brahmagupta (628 AD), Mahavira (850), Aryabhata II (950), Sripati (1039), Bhaskara II (1150), and Narayana (1350), subsequently discussed and refined the kuttaka. The concept of kuttaka was so significant to the Indians that it was first used to refer to the entire field of algebra as kuttaka-ganita, or simply kuttaka. Brahmagupta (628 AD) coined this name! It was long later that the modern Sanskrit term bijaganita arose.

Aryabhaṭa's formulation of the problem

Aryabhaṭa did not formulate the problem that can be solved by the Kuṭṭaka method as a linear Diophantine equation problem. Aryabhaṭa took into consideration the subsequent issues, all of which are comparable to the linear Diophantine equation problem:

Find an integer such that it leaves two given remainders when divided by two provided integers. There are two possible formulations for this problem:

Let N be the integer that has to be found, a and b be its divisors, and R1 and R2 be its remainders. The next challenge is to determine N such that Mod a, N≡R1, and mod b, N ≡ R2.
The task is to discover N such that there are numbers x and y such that
N = ax + R1 and N = by + R2.
Using N as the integer to be discovered, a, b, and R1 and R2 as the divisors and remainders, the goal is to find N such that x and y are integers such that
N = ax + R1 and N = by + R2.
This can also be expressed as -
ax − by = c, where c = R2 − R1.
Determine a number that, when multiplied by one given integer and either increased or lowered by another provided integer, divides by a third integer without leaving a residual. The task is to discover x such that (ax ± b)/c is an integer y, given the three integers, a, b, and c, and the integer to be determined, x. Finding two integers, x and y, such that (ax ± b)/c = y is the same as doing this. Consequently, this is comparable to the issue of determining integer solutions for ax ± by = ±c.
In Aryabhata's verses 32–33 of Ganitapada of Aryabhatiya, the algorithm for solving the linear Diophantine problem is

provided.[1] Bibhutibbhushan Datta has provided the translation of these verses, keeping in mind Bhāskara I's interpretation as well.

अधिकाग्रभागहारं छिन्द्यादूनाग्रभागहारेण ।
शेष परस्परभक्तं मतिगुणमग्रान्तरे क्षिप्तं ॥
अधउपरिगुणितमन्त्ययुगूनाग्रच्छेदभाजिते शेष ।
मधिकाग्रच्छेदगुणं द्विच्छेदाग्रमधिकाग्रयुतम् ॥

Kuttaka's description as provided by Aryabhata in Aryabhatiya

"Divide by the divisor that corresponds to the smaller residual to get the divisor for the greater remainder.The last quotient should be multiplied by an optional integer and then added(if the number of quotients of the mutual division is even) or s ubtracs (if the number of quotients is odd) by the difference of the remainders. The residue and the divisor corresponding to the smaller remainder should be mutually divided until the remainder becom es zero. (put the other mutual division quotients in a column, one below the other; place the Recently acquired result and the optinal integer underneath it.) Any number below (the penultimate) is added by the number below it and multiplied by the number immediately above it. After multiplying the residue by the divisor corresponding to the greater remaining and adding the greater remainder, divide the last number (obtained by doing so repeatedly) by the divisor corresponding to the smaller remainder. The number that corresponds to the two divisors will be the outcome."
A few remarks are necessary-
The algorithm finds the lowest positive integer that, when divided by the given values,
returns the desired remainder. By converting the procedure into contemporary mathematical notation, the validity of the technique can be demonstrated.
Several iterations of this technique have been devised and discussed by later Indian mathematicians such as Brahmagupta (628 AD), Mahavira (850), Aryabhata II (950), Sripati (1039), Bhāskara II (1150), and Narayana (1350).[1]

Elaboration of Aryabhatta's Kuttaka

Ab.2.32. adhikāgrabhāgahāraṃ chindyād ūnāgrabhāgahāreṇa|
śeṣaparasparabhaktaṃmatiguṇam agrāntare kṣiptam||
Ab.2.33ab. adhopariguṇitam antyayugūnāgracchedabhājite śeṣam
aṣṭau kenābhyastāḥ ṣaḍrūpayutā hṛtāḥ trayodaśabhiḥ |
dadyuḥ śuddhaṃ bhāga ko guṇakāraḥ kimāptam ca
karaṇam – bhājyabhāgahārarāśī rūpeṇāpavartitau

One should reduce a large number (adhikāgrabhāgahāra) by a small number (ūnāgrabhāgahāra) by division method. The mutual division of the remainders having a clever for multiplier and added in the inside of a number (agrāntara)

(the quotient apta, labdha) (the pulveriser, the multiplier)

$$y = \frac{ax + c}{b}$$

“Ab.2.32. To simplify, divide a large number (adhikāgrabhāgahāra) by a little number (ūnāgrabhāgahāra). The remaining is constantly divided mutually. The residue with a clear multiplier added inside a number (agrāntara) is divided by the final divisor.

Ab.2.33ab. The one above is multiplied by the one below, which is then increased by the last one. When the remaining higher quantity is divided by the divisor, which is a little number, the pulverizer remains. The quotient is created when the lowest remaining number is divided by the dividend.”
Here we will discuss two case of Diophantine equations ax+c=by
Case I a>b
(A). if quotient is odd in number
(B). if quotient is odd in number
Case II a< b
(A). if quotient is odd in number
(B). if quotient is odd in number

Case I a>b

If in given Diophantine equations ax+c=by coefficient of x is greater than coefficient of y i.e. a>b then

Table 1

x) y (Q ↓ y_1) x (q_1 ↓ x_1) y_1 (q_2 ↓ y_2) x_1 (q_3 ↓ x_2) y_2 (q_4 ↓ y_3) x2 (q_5 ↓ x_3) y3 (q_6 ↓ Y_4) x3 (q_7 ↓ ----- x_{n-1}) y_{n-1}(q_{2n-2}. ↓ y_n) x_{n-1}(q_{2n-1} ↓ x_n) y_n(q_{2n} ↓ y_{n+1}	b) a (Q ↓ r_1) b (q_1 ↓ r_2) r_1 (q_2 ↓ **r_3) r_2 (q_3** ↓ r_4) r_3 (q_4 ↓ r_5) r_4 (q_5 ↓ r_6) r_5 (q_6 ↓ r_7) r_6 (q_7 ↓ ----- r_i) r_{i-1} (q_i ↓ r_{i+1}) r_i (q_{i+1} ↓ r_{i+2} -------------------

Table 2

$y = Qx+y_1$ $x = q_1y_1 + x_1$ $y_1 = q_2x_1 + y_2$ $x_1 = q_3y_2 + x_2$ $y_2 = q_4x_2 + y_3$ $x_2 = q_5y_3 + x_3$ $y_3 = q_6x_3 + y_4$ $x_3 = q_7y_4 + \ldots x_{n-1}$ $y_{n-2} = q_{2n-4}\, x_{n-2} + y_{n-1}$ $x_{n-2} = q_{2n-3}\, y_{n-1} + x_{n-1}$ $y_{n-1} = q_{2n-2}\, x_{n-1} + y_n$ $x_{n-1} = q_{2n-1}\, y_n + x_n$ $y_n = q_{2n}x_n + y_{n+1}$ $x_n = q_{2n+1}y_{n+1} + x_{n+1}$	$a = Q\, b + r_1$ $b = q_1\, r_1 + r_2$ $r_1 = q_2\, r_2 + r_3$ $r_2 = q_3\, r_3 + r_4$ $r_3 = q_4\, r_4 + r_5$ $r_4 = q_5\, r_5 + r_6$ $r_5 = q_6\, r_6 + r_7$ ------------------------------ $r_{i-2} = q_{i-1}\, r_{i-1} + r_i$ $r_{i-1} = q_i\, r_n + r_{i+1}$ $r_i = q_{i+1}\, r_{i+1} + r_{i+2}$ say i=2n $r_{2n-2} = q_{2n-2}\, r_{2n-2} + r_{2n}$ $r_{2n-2} = q_{2n-1}\, r_{2n-1} + r_{2n}$ $r_{2n-1} = q_{2n}\, r_{2n} + r_{2n+1}$ $r_{2n} = q_{2n+1}\, r_{2n+1} + r_{2n+2}$

Now we will analyse this given table data and try to find out the value of unknown variable i.e x and y. first we will go through analysation then we will synthesis this data.

ax +c = by is given equation. We have to find out value unknown variable x and y.

$$y = \frac{ax+c}{b}$$

put value of y in equation $y = Qx + y_1$

$$\frac{ax+c}{b} - Qx = y_1$$

$$\frac{x(a-Qb)+c}{b} = y_1$$

$$\frac{x(r_1)+c}{b} = y_1$$

$$y_1 = \frac{x(r_1)+c}{b}$$

$$x = \frac{by_1-c}{r_1}$$

put value of x equation in $x = q_1y_1 + x_1$

$\frac{by_1-c}{r_1} - q_1y_1 = x_1$

$\frac{y_1(b-q_1r_1)-c}{r_1} = x_1$

$\frac{y_1(r_2)-c}{r_1} = x_1$

$x_1 = \frac{y_1(r_2)-c}{r_1}$ ------------ ODD

$y_1 = \frac{x_1(r_1)+c}{r_2}$

put value of y_1 in $y_1 = q_2x_1 + y_2$

$\frac{x_1(r_1)+c}{r_2} - q_2x_1 = y_2$

$\frac{x_1(r_1-q_2r_2)+c}{r_2} = y_2$

$\frac{x_1(r_3)+c}{r_2} = y_2$

$y_2 = \frac{x_1(r_3)+c}{r_2}$ --------------- EVEN

$x_1 = \frac{y_2(r_2)-c}{r_3}$

put value of x_1 in $x_1 = q_3y_2 + x_2$

$\frac{y_2(r_2)-c}{r_3} - q_3y_2 = x_2$

$\frac{y_2(r_2-q_3r_3)-c}{r_3} = x_2$

$\frac{y_2(r_2-q_3r_3)-c}{r_2} = x_2$

$\frac{y_2(r_4)-c}{r_3} = x_2$ ------------ ODD

$x_2 = \frac{y_2(r_4)-c}{r_3}$

$y_2 = \frac{x_2(r_3)+c}{r_4}$

put value of y_2 in $y_2 = q_4x_2 + y_3$

$\frac{x_2(r_3)+c}{r_4} - q_4x_2 = y_3$

$\frac{x_2(r_3-q_4r_4)-c}{r_4} = y_3$

$\frac{x_2(r_5)+c}{r_4} = y_3$

$y_3 = \frac{x_2(r_5)+c}{r_4}$ ----------------- EVEN

$x_2 = \frac{y_3(r_4)-c}{r_5}$

put value of x_2 in $x_2 = q_5y_3 + x_3$

$\frac{y_3(r_4)-c}{r_5}$ - $q_5y_3 = x_3$

$\frac{y_3(r_4-q_5r_5)-c}{r_5} = x_3$

$\frac{y_3(r_6)-c}{r_5} = x_3$

$x_3 = \frac{y_3(r_6)-c}{r_5}$

$y_3 = \frac{x_3(r_5)+c}{r_6}$

In general,

$x_{n-1} = \frac{y_{n-1}(r_{2n-2})-c}{r_{2n-3}}$ **and** $y_{n-1} = \frac{x_{n-1}(r_{2n-3})+c}{(r_{2n-2})}$

put value of y_{n-1} in $y_{n-1} = q_{2n-2}x_{n-1} + y_n$

$\frac{x_{n-1}(r_{2n-3})+c}{(r_{2n-2})} - q_{2n-2}x_{n-1} = y_n$

$\frac{x_{n-1}(r_{2n-3})-q_{2n-2}r_{2n-2})+c}{r_n} = y_n$

$\frac{x_{n-1}(r_{2n-1})+c}{r_{2n-2}} = y_n$

$y_n = \frac{x_{n-1}(r_{2n-1})+c}{r_{2n-2}}$ ------------- EVEN

As we are noticing that number of quotients is in even number after leaving first quotient i.e q_{2n-2}= even where n= N^+

$x_{n-1} = \frac{y_n(r_{2n-2})-c}{r_{2n-1}}$

put value of x_{n-1} in $x_{n-1} = q_{2n-1}y_n + x_n$

$\frac{y_n(r_{2n-2})-c}{r_{2n-1}} - q_{2n-1}y_n = x_n$

$\frac{y_n(r_{2n-2}-q_{2n-1}r_{2n-1})-c}{r_{n+1}} = x_n$

$\frac{y_n(r_{2n})-c}{r_{2n-1}} = x_n$

$\mathbf{x_n} = \frac{y_n(r_{2n})-c}{r_{2n-1}}$ ---------------ODD

As we are noticing that number of quotients is in odd number after leaving first quotient i.e. q_{2n-1}= odd where n= N^+

$\mathbf{y_n} = \frac{x_{n-1}(r_{2n-1})+c}{r_{2n-2}}$ ------------------------EVEN

$\mathbf{x_n} = \frac{y_n(r_{2n})-c}{r_{2n-1}}$ --------------------ODD

As we reached on result means we have found out value of unknown variable with specific condition i.e. even or odd concept.

Now we will synthesis this data. Actually, synthetic methods in mathematics are about leveraging foundational principles and abstract reasoning to solve problems and prove theorems. They often involve a more direct approach to understanding and proving mathematical truths, without breaking problems down into more specific or computationally intensive parts. We can say that synthesis simply means -to place thing together or to join separate parts. It is the process of relating known bits of data to a point where the unknown become true. Now we will go through given below table. First we will see if quotient is odd. Then we will see if if quotient is even.

If quotient is odd (2n-3=5) a>b

Table 3

Q						$Qx+y_1=$ **y**
q_1				q_1	$q_1y_1 + x_1=$ **x**	
q_2			q_2	$Q_2x_1 + y_2 = y_1$		
q_3		q_3	$q_3y_2(y_{n-2})$ $+\ldots x_{n-2} = x_1$	x_1		
q_{2n-4}		$q_{2n-3}\ x_{n-2} + y_{n-1}$ $= y_{n-2}$	$y_{n-2} = y_2$			

q_{2n-3}	$q_{2n-3}\ y_{n-1}$ + $x_{n-1}= x_{n-2}$	x_{n-2}				
y_{n-1}	y_{n-1}					
x_{n-1}						

Where $x_{n-1} = \frac{y_{n-1}(r_{2n-2})-c}{r_{2n-3}}$ **&** $\boldsymbol{r_{2n-2} = 1}$

As we have seen in table, we from last row and after calculation we reached on first row and find value of unknown variable.

if quotient is even (2n-2=6) a>b **Table 4**

Q						$Qx+y_1= y$
q_1					q_1	$q_1y_1 + x_1= x$
q_2				q_2	$q_2x_1 +y_2= y_1$	y_1
q_3			q_3	$q_3y_{n-2}+\ldots x_{n-1}= x_1$	x_1	
q_4		q_4	$q_4x_{n-2} +y_{n-1}= y_{n-2}$	y_{n-2}		
q_{2n-3}	q_{2n-3}	$q_{2n-3}\ y_{n-1} +x_{n-1}= x_{n-}$	x_{n-2}			
q_{2n-2}	$q_{2n-2}\ x_{n-1} +y_n= y_{n-}$	y_{n-1}				
x_{n-1}	x_{n-1}					
y_n						

Where $y_n = \frac{x_{n-1}(r_{2n-1})+c}{r_{2n-2}}$ & $r_{2n-1} = 1$

If ax+c=by where a<b **Table 5**

y) x (Q ↓ x_1) y(q_1 ↓ y_1) x_1 (q_2 ↓ x_2) y_1 (q_3 ↓ y_2) x_2 (q_4 ↓ x_3) y_2 (q_5 ↓ y_3) x3 (q_6 ↓ **x_4) y_3 (q_7** ↓ y_{n-1}) x_{n-1}(q_{2n-2}. ↓ x_n) y_{n-1}(q_{2n-1} ↓ y_n) x_n(q_{2n} ↓ x_{n+1}	a) b (Q ↓ r_1) a (q_1 ↓ r_2) r_1 (q_2 ↓ **r_3) r_2 (q_3** ↓ r_4) r_3 (q_4 ↓ r_5) r_4 (q_5 ↓ r_6) r_5 (q_6 ↓ r_7) r_6 (q_7 ↓ ----- r_i) r_{i-1} (q_i ↓ r_{i+1}) r_i (q_{i+1} ↓ r_{i+2}

Table 6

$x = Qy+x_1$ $y = q_1x_1 + y_1$ $x_1 = q_2y_1 + x_2$ $y_1 = q_3x_2 + y_2$ $x_2 = q_4y_2 + x_3$ $y_2 = q_5x_3 + y_3$ $x_3 = q_6y_3 + x_4$ $y_3 = q_7x_4 + \ldots y_{n-1}$ ----------------------------------- $x_{n-2} = q_{2n-4}\, y_{n-2} + x_{n-1}$ $y_{n-2} = q_{2n-3}\, x_{n-1} + y_{n-1}$ $x_{n-1} = q_{2n-2}\, y_{n-1} + x_n$ $y_{n-1} = q_{2n-1}\, x_n + y_n$ $x_n = q_{2n}y_n + x_{n+1}$ $y_n = q_{2n+1}y_{n+1} + y_{n+1}$	$b = Q\, a + r_1$ $a = q_1\, r_1 + r_2$ $r_1 = q_2\, r_2 + r_3$ $r_2 = q_3\, r_3 + r_4$ $r_3 = q_4\, r_4 + r_5$ $r_4 = q_5\, r_5 + r_6$ $r_5 = q_6\, r_6 + r_7$ -------------------------------- $r_{i-2} = q_{i-1}\, r_{i-1} + r_i$ $r_{i-1} = q_i\, r_n + r_{i+1}$ $r_i = q_{i+1}\, r_{i+1} + r_{i+2}$ say i=2n $r_{2n-3} = q_{2n-2}\, r_{2n-2} + r_{2n-1}$ $r_{2n-2} = q_{2n-1}\, r_{2n-1} + r_{2n}$ $r_{2n-1} = q_{2n}\, r_{2n} + r_{2n+1}$ $r_{2n} = q_{2n+1}\, r_{2n+1} + r_{2n+2}$

$ax + c = by$

$x = \frac{by-c}{a}$

put value of y in equation $x = Qy + x_1$

$\frac{by-c}{a} - Qx = x_1$

$\frac{y(b-Qa)-c}{a} = x_1$

$\frac{x(r_1)-c}{a} = x_1$

$x_1 = \frac{x(r_1)-c}{a}$

$y = \frac{ax_1+c}{r_1}$

put value of x in equation $y = q_1x_1 + y_1$

$\frac{ax_1+c}{r_1} - q_1x_1 = y_1$

$\frac{x_1(a-q_1r_1)+c}{r_1} = y_1$

$\frac{x_1(r_2)+c}{r_1} = y_1$

$y_1 = \frac{x_1(r_2)+c}{r_1}$

$x_1 = \frac{y_i(r_1)-c}{r_2}$

put value of x_1 in equation $x_1 = q_2y_1 + x_2$

$\frac{y_i(r_1)-c}{r_2} - q_2y_1 = x_2$

$\frac{y_1(r_1-q_2r_2)-c}{r_2} = x_2$ --------quotient even

$\frac{y_1(r_3)-c}{r_2} = x_2$

$x_2 = \frac{y_1(r_3)-c}{r_2}$ -----------even

$y_1 = \frac{x_2(r_2)+c}{r_3}$

put value of y_1 in equation $y_1 = q_3x_2 + y_2$

$\frac{x_2(r_2)+c}{r_3}$ - $q_3x_2 = y_2$

$\frac{x_2(r_2-q_3\,r_3)+c}{r_3} = y_2$ quotient odd

$\frac{x_2(r_4)+c}{r_3} = y_2$

$y_2 = \frac{x_2(r_4)+c}{r_3}$ odd

$x_2 = \frac{y_3(r_3)-c}{r_4}$

in general

$y_{n-1} = \frac{x_{n-1}(r_{2n-2})+c}{r_{2n-3}}$

$x_{n-1} = \frac{y_{n-1}(r_{2n-3})-c}{r_{2n-2}}$

put value of x_{n-1} in equation $x_{n-1} = q_{2n-2}\, y_{n-1} + x_n$

$\frac{y_{n-1}(r_{2n-3})-c}{r_{2n-2}}$ - $q_{2n-2}\, y_{n-1} = x_n$

$\frac{y_{n-1}(r_{2n-3}-q_{2n-2}r_{2n-2})-c}{r_{2n-2}} = x_n$ ------- quotient even

$x_n = \frac{y_{n-1}(r_{2n-1})-c}{r_{2n-2}}$ EVEN

$y_{n-1} = \frac{x_n(r_{2n-2})+c}{r_{2n-1}}$

put value of y_{n-1} in equation $y_{n-1} = q_{2n-1}\, x_n + y_n$

$\frac{x_n(r_{2n-2})+c}{r_{2n}}$ - $q_{2n-1}\, x_n = y_n$

$\frac{x_n(r_{2n-2}-q_{2n-1}\,r_{2n-1})+c}{r_{2n-1}} = y_n$ -----quotient odd

$\frac{x_n(r_{2n})+c}{r_{2n-1}} = y_n$ ODD

if quotient is odd(2n-3=5) ax+c=by where a<b

Q					Q	Qy+x1= **x**
q_1				q_1	$q_1x_1 + y_1$ = **y**	
q_2			q_2	$q_2y_1 + x_2 = x_1$		
q_3		q_3	$q_3x_2(x_{n-2})+\ldots y_{n-2} = y_1$			
q_{2n-4}	q_{2n-4}	$q_{2n-4}\, y_{n-2} + x_{n-1} = x_{n-2}$	$x_{n-2} = x_1$			
q_{2n-3}	$q_{2n-3}\, x_{n-1} + y_{n-1} = y_{n-2}$	y_{n-2}				
x_{n-1}						
y_{n-1}						

Where $y_{n-1} = \frac{x_n(r_{2n-2})+c}{r_{2n-1}}$ & $r_{2n-2} = 1$

If quotient is odd(2n-2=6) a<b

Q							Qy+x1= **x**
q_1					q_1	$q_1x_1 + y_1$ = **y**	
q_2				q_2	$q_2y_1 + x_2 = x_1$		
q_3			q_3	$q_3x_2 + y_2 = y_1$	y_1		
q_4		q_4	$q_4y_2(y_{n-2}) + x_3 = x_2$	x_2			
q_{2n-3}	q_{2n-3}	$q_{2n-3}\, x_{n-1} + y_{n-1} = y_{n-2}$	$y_{n-2} = y_2$				
q_{2n-2}	$q_{2n-2}\, y_{n-1} + x_n = x_{n-1}$	$x_{n-1} = x_3$					
y_{n-1}							

x_n							

Where $x_n = \frac{y_{n-1}(r_{2n-1})-c}{r_{2n-2}}$ & $r_{2n-1}=1$

Example

Let us solve linear equation 137x+10=60y

Here it is in form of ax+c=by and a>b (a=137, b=60, c=10)

Divide a large number (adhikāgrabhāgahāra) by a little number (ūnāgrabhāgahāra). Use division method to do following recursive action-

137=2.60+17
60=3.17+9
17=1.9+8
9=1.8+1

Construct table (as Quotient is Odd so 2n-3=3 then n=3)

Q	2			2	**297 =y**
q_1	3		3	**130 =x**	
q_2	1	1	**37**	37	
q_3	1	**19**	19		
$y_{n-1}=y_2$	**18**	18			
$x_{n-1}=x_2$	1				

$x_{n-1} = \frac{y_{n-1}(r_{2n-2})-c}{r_{2n-3}}$ $x_2 = \frac{y_2(r_4)-10}{r_3}$ $x_2 = \frac{y_2(1)-10}{8}$

if $x_2=1$ then $y_2=18$
so as per table-
x=130 and y=297

Example

Let us solve linear equation 129x+10=40y

Here it is in form of ax+c=by and a>b (a=129, b=40, c=10)

Divide a large number (adhikāgrabhāgahāra) by a little number (ūnāgrabhāgahāra). Use division method to do following recursive action

129=3*40+9 40=4* 9+4 9= 2*4+1

Construct table (as Quotient is even in number so 2n-2=2 then n=2)

Q	3		3	97 = y
q_1	4	4	30 = x	
q_2	2	7	**7**	
$x_{n-1}=x_1$	2	**2**		
$y_{n-}=y_2$	**3**			

$y_n = \frac{x_{n-1}(r_{2n-1})+c}{r_{2n-2}}$, $y_2 = \frac{x_1(r_3)+c}{r_2}$ $y_2 = \frac{x_1(1)+10}{4}$

if $y_2=3$ then $x_1=2$
so as per table-
x=30 and y=97

Conclusion

The knowledge of Mathematical Sciences, the finest mathematical facts, figures and theories have been achieved by the continuous practices and researches of hundreds of mathematicians for the centuries. Many people had contributed to the development of Mathematical sciences which mathematicians, scientists and social scientists benefit today. The contribution of Indian mathematicians is immense and remarkable in this sense. It is the need of the today's era to promote ahead the heritage of mathematicians so as to encourage and cherish the magnificent traditional roots of the country in mathematics.

Aryabhata was a great Indian mathematician and Astronomer of the ancient world whose contribution in solving indeterminate equations had enormous influence around the globe. kuttaka methodology is the great thinking of great Indian mathematician The Aryabhata. The simplicity of kuttaka method lies in the fact that it divides the large time computing operations into several modular arithmetic with smaller number in each of the iterations which may be utilized by the computer scientists in developing crypto algorithm.

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