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Domination of Graph Based on the Vizing's Conjecture

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Abstract: In graph idea, a dominating set for a graph S = (V, E) is a subset D of V. The domination number γ(S) is the range of vertices in a smallest dominating set for S. The Cartesian product L of graphs S such that, the vertex set of S and L is the Cartesian product and two vertices and are adjoining in S, L if and simplest if both and is adjoining to in L. Vizing's conjecture from 1968 proves that the domination variety of the Cartesian fabricated from graphs is at least as big because the made of their domination numbers. On this paper, the vizing's conjecture states that for each pair of graphs S, L.

Keywords: Minus total domination; signed total domination; k-partite graph; Triangle free graph; Hypergraph

1. INTRODUCTION

Graph theory is the study of the graphs used to model pairwise relationships between objects. A graph is composed of vertices connected by edges. Graph is distinguish between undirected graphs that the edges are connected symmetrically to two vertices and the direct graphs that the edges are connected two vertices asymmetrically. An edge is one of the two basic units of which graphs are constructed. Edges are be in directed or undirected. The undirected edges referred as lines. In the undirected graphs, an edge is represented as the set of its vertices. A directed edges are referred as arcs or arrows. In a directed simple graph it is represented as an ordered pair of its vertices. Vertices of graphs are often considered as atomic objects with no internal structure. The graph is one of the main objects of study of discrete mathematics. The Prime symbols is used to modify notation for graph invariants so that it applies to the line graph. A set D of vertices of a simple and non-oriented graph $S = (V, E)$ is dominant if every vertex in VD is adjacent to a vertex in D. The other dominant factor requires at least vertices $\gamma(S)\gamma(L)$. It is also known that the Vizing hypothesis applies to cycles and graphs with the domain number two.

1.1 Graph theory Applications

Graph theory concept used in different areas among various applications. It includes a study of atoms and molecules, construction of bonds in chemistry. It is also used in biological applications where a vertex represents the region in which certain species exist and the borders represent the migration path or the movement between the regions. It is used for tracking the spread of disease. Network activity is used to solve a large number of combinatorial problems. It is also used in the modeling of transport networks, activity networks and game theory. For example, the problem of the street vendor, the shortest tree in a weighted graph, obtaining an optimal correspondence between jobs and men and identifying the shortest path between two vertices in a graph.

Snipping tIn a graph S, a set of edges g is said to cover S if every vertex in S is incident on at least one edge in g. The set of edges covering a graph S is said to be a covering of the edges or a cover sub-graph or simply a cover of S. Ex. A spanning tree in a linked graph is a cover. The example computer network with the relative minimum vertex coverage is shown below.

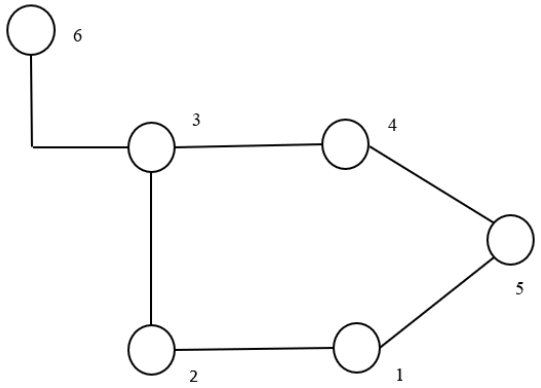

Figure 1: The vertex Set g = {1,3,4} covers all vertices in S

1.3 Coloring of the graph

Graph coloring is the most vital standard in graph theory and is used in many computer programs. Numerous coloring methods can be used as a basis for the requirement. The correct coloring of a graph is defined as the coloring of vertices and edges with a minimum variety of colors that does not offer a color equal to 2 vertices. The minimum range of colors is known as the color quantity and the chart is also referred to as a colored chart.

1.4 Domination in graphs

Let S = (V, E) be a graph with the set of vertices V and the set of boundaries E. A set is a dominant set of S if each vertex is adjacent to the vertex in s. The domain number of S, indicated by γ (S) which is the minimum cardinality of a dominant set of S.

The supremacy in diagrams was used in the investigated branch of the diagram concept. The graph principle is one of the branches of contemporary mathematics and computer programs. Growing the principle of combining problems with optimizing problems, combinatorial problems and classical problems. It has a variety of applications that are used in many areas such as engineering, body sciences, social and life sciences, and so on. The dominance was the core of the research interest in the graph concept these days. This is basically a summary of new parameters that can be developed from the primary definitions of dominance. The NP - completeness of basic problems of domination and their relation to other NP completeness of research and development of activity in the principle of domination.

1.5 Applications of Domination in Graph

The applications of Domination in the graph have been used in numerous fields. The dominion arises in the problems of the places of structures, in which the great variety of centers (eg Hospitals, hearth stations) is constant and space must be reduced to a minimum. Standards of domination that is, to minimize the amount of places a surveyor should commit to taking peak measurements for an entire area.

2. LITERATURE SURVEY REVIEW

Elisabeth et.al. Proposes conjecture of vizing as a polynomial pair so the polynomial is not negative. This method is able to produce positive certificates of low negativity of non-negativity for certain classes of graphs using an innovative collection of techniques ranging from semi-defined programming to computer algebra. For example, the conjecture of Vizing with parameters $K_S = n_S$ and $k_L = n_L - 1$ has a Positivstellensatz with 1 sum of squares and with parameters $k_S = n_S$ and $k_L = n_L - 2$ has a Positivstellensatz with 2 sum of squares.

Michael et.al. approaches the essential upper limits on the total domination number of a graphic method. In this method, the S graph is demonstrated without isolated vertices and isolated edges that have the form of unified ones. The decision problem is to determine the total domination number of a graph. The result of the graphs is extended with the minimum degree δ.

Davila et.al. Proposes the zero forcing number with lower bounds. The zero force number of S is the minimum cardinality of a zero forcing set in S and is indicated by Z (S). This method is introduced by the AIM Minimum Rank - Special Graphs work group as a tool to limit the minimum rank of matrices associated with the S graph.

Borzooei et.al. demonstrates the domination in graph with application. The domain in the graphs has been used for the branch of the theory of graphs sought. Graph theory is one of the branches of modern mathematics and computer applications. In a graph S = (V, E), it is a dominant set of S if every vertex is in I or joined by an edge to a vertex in I.

Michael A Henning et.al. demonstrate the problem concerning the domination in the graphs that states that the number of domination of the Cartesian product of any two graphs is at least as large as the product of their domination numbers. It also proves that the product of the total domination numbers of any non-trivial tree and any graph without isolated vertices is at most twice the total domination number of their Cartesian product.

Bert L. Hartnell et.al proposes the complete graph of order two. Vizing's conjecture is sharp due to the development of graphs with number of infinite classes of graphs.

Bert L. Hartnell et.al establishes dominant limits of the Cartesian product of any two graphs. This method establishes a new lower limit for the domain number and also improves an upper limit of the evaluation method. To yield a good lower bound, at least one of the two graphs must have a 2-packing that is almost as large as its domination number.

Noga Alon et.al. demonstrates the coloring of graphs using acyclic edge concept. If no incident edge pairs are colored, the edge coloring diagram is correct. Then the correct coloring of the edge is a color cycle in S. The chromatic number of the acyclic edge is the least number of colors in an acyclic edge coloring of S.

Robert et al. suggests the maximum total number of domination. The total power $\gamma_t(s)$ of the graph $S=(V,E)$ is the cardinality of a smallest subset of V such that every vertex of V has a neighbor in S. It is known that if S is a contiguous graph with n vertices $\gamma_t(s) \le \left\lfloor 2n/3 \right\rfloor$. This limit is valid if the graph is connected and has at least two vertices.

Favaron et.al. proposes minimum degree three of domination graphs. The minimum degree among the vertices of S is denoted by $\delta(S)$ and the maximum degree by $\Delta(S)$. The decision problem for determining the total number of counts of a graph is known to be NP-complete. Therefore, it is to determine the upper limits for the total number of counts of a graph.

Paul Andrew et al. proposes the applications and variations of graph theory. In this Roman concept of domination is introduced. A function f: V → {0,1,2} such that every node v for which f (v) = 0 has a neighbor w with f (w) = 2. This function is called Roman dominating function. Roman dominating functions is used for finding minimum weight for trees and lattices.

Edwin clark et.al. approaches an inequality sequence of domination. Based on this approach, the domain of the Cartesian product of two graphs without isolated vertices is limited above a constant multiple of the total domain of their Cartesian product.

Carsten Thomassen et.al. establishes the reflection of graphs. An S graph can be broken down into edge-disjoint cycles in pairs. If and only if, for each partition of the set of vertices of S in two sets A and B, there is an even or infinite number of edges between A and B.

Tverberg et.al. proposes the complete decomposing graph on n-vertices in n-2 that are uncomfortable. In this method, a coupled dominant set, designated PDS, of a graph S is a set A of vertices of S such that each vertex is adjacent to a vertex in A and the subgraph induced by A contains a perfect match.

Cockayne et.al. demonstrates the description of total dominating sets in graph. The cardinality of a smaller dominant set in a graph S is called the total domain number of S and is indicated by γ_t (S).The independent domain number, i (S), is the smallest cardinality of an independent maximum set of vertices of C.

V.G. Vizing et.al deals with unsolved problems relating to non-oriented graphs only. A multigraph M = (X, U) is defined by giving two finite sets X and U and a one-to-one mapping of X into the set of unordered pairs of different elements of U. The smallest length of a simple cycle in a multigraph is called the compass of the multigraph. A connected multigraph without cycles is a tree. An invariant of a graph of no small importance is the number of all nonisomorphic subgraphs.

3. VIZING'S CONJECTURE

The Vizing conjectures refer to the dimensions of minimal dominant sets in the S and L graphs with the dimensions of a minimal dominant set in the Cartesian product graph. In principle, the Vizing theorem states that any simple non-oriented graph can be multi-colored with the use of some colors.

3.1 Classification of graphs

A few graphs are classify as being of sophistication one or class two. If the vertices of the major diploma Δ in a graph S form an impartial whole or more generally if the precipitated subgraph for this set of vertices is a forest, then S should be sophisticated.

3.1.1 Planar graphs

A planar graph is of refinement if its maximum degree is at least eight. There exist a planar graphs of sophistication if any most diploma in the variety from 2 to 5. For degree 3, 4 and 5, planar graphs are composed of platonic solids using the change of an unmarried part to a path of two adjacent edges. The planar graph conjecture states that all simple planar graphs with a maximum degree of six or seven are class one. Planar refinement graphs constructed through the use of subdivision of Platonic solids are not ordinary because they have vertexes of degree in addition to high school diploma vertices. The four-color theorem on vertexing planar graphs is equivalent to the announcement that any planar graph with three bridges is class one.

3.1.2 Non-Planar Graphs

A multifaceted embedding is a graphic embedding so that each face of the embedding is topologically a disk and such that the double embedding plot is simple without auto-loops or multiple adjacencies.

Let S = [V (S), E (S)] be a finite. For subsets of vertices, P dominates K if K ⊆ N [P] i.e, if every vertex of K is in P or is adjacent to a vertex of P. When K and P are disjoint and P dominates K, then P dominates externally K. The domain number of S is the smallest denoted cardinal γ (S) dominating V (S). If D dominates V (S), then D dominates the graph S and that D is a dominant set of S.

Any dominant set of S must intersect each closed quarter in S. Therefore, the domain number of S is at least equal to the cardinality of any set $X \subseteq V(S)$ having the property that for distinct x_1 and x_2 in X , $N[x_1] \cap N[x_2] = \theta$. Such a set X is called 2-packing and the maximum cardinality of a 2-packing in S is denoted by σ (S) and is called the 2-packing number of S. The smallest cardinality of a dominant set which is also independent is denoted i (S) and the independence number of the vertex of S is the maximum cardinality α (S) of an independent set of vertices in S.

If S is not a complete graph, then for each pair of vertices v_1 and v_2 they are not adjacent to S, it is clear that $\gamma(S)-1 \le \gamma(S+v_1v_2) \le \gamma(S)$. If S has the property that $\gamma(S)-1=\gamma(S+v_1v_2)$ for each of these pairs of non-adjacent vertices, then S is critical with respect to the domain (or critical for brevity).

Let S be a graph with the domain number u. If the set of vertices of S can be covered by all the subgraphs, then S is called as a separable graph.

Theorem 1.1. For each pair of finite graphs S and L

$$\gamma(S \times L) \ge \gamma(S)\gamma(L) \quad (1)$$

Where, γ stands for the domain number and $S \times L$ stands for Cartesian product of the graphs S and L.

Theorem 1.2. Let S be a graph that satisfies the vizing's conjecture and $S^{'}$ be a spanning subgraph. Both a spanning subgraph of S such that $\gamma(S^{'}) = \gamma(S)$. So it also satisfies the Vizing's conjecture.

If L is a graph, then

$$\gamma(S^{'})\gamma(L) = \gamma(S)\gamma(L) \leq \gamma(S \times L) \leq \gamma(S^{'} \times L) \tag{2}$$

The last inequality holds because $S^{'} \times L$ is a spanning subgraph of $S \times L$.

Theorem 1.3 If a graph S is a spanning subgraph of a decomposable graph $S^{'}$, such that $\gamma(S) = \gamma(S^{'})$, then L holds for each graph L,

$$\gamma(S \times L) = \gamma(S)\gamma(L) \tag{3}$$

All graphs are considered as undirected, simple, coherent and finite. In particular, let S be a graph with the vertex set V = V (S) and the edge set E = E (S). Two vertices m, n ∈ V are neighbors, or if mn ∈ E. The open neighborhood of m ∈ V is the set of neighbors of m, denoted by N_S(m), while the closed neighborhood N_S [m] = N_S (m) ∪ {m}. The open neighborhood of D ⊆ V is the set of all neighbors of vertices in D, denoted by N_S (D), while the closed neighborhood of D is N_S [D] = N_S (D) ∪ D. If S is removed from the context, it may be written as N (D) and N [D] instead of N_S (D) and N_S [D] respectively. The distance between two vertices m, n ∈ V is the length of a shortest (m, n) path in S and is denoted by d_S(m,n). The Cartesian product of two graphs S(V_1, E_1) and L(V_2, E_2) denoted by $S \times L$, is a vertex-set graph $V_1 \times V_2$ and edge set $E(S \times L) = \{((u_1, v_1), (u_2, v_2)) : v_1 = v_2 and (u_1, u_2) \in E_1, or u_1 = u_2 and (v_1, v_2) \in E_2\}$.

(4)

A subset of vertices D ⊆ V (S) is called a semi-totally dominant set, if N [D] = V (S), and for each vertex u ∈ D a vertex v ∈ D exists, such that d (u, v) ≤ 2. A vertex set D half dominantly dominates a vertex set T when D is a semi-total dominant set in the induced subgraph D ∪ T of S. The half-sum dominance number of S, written as γ_{t2}(S), is the size of a minimum half-sum dominating set of S. A 2-pack is a subset of vertices T of S such that each pair of vertices in T is at least 3 apart. The size of a maximum 2-pack of S is called the 2-pack number written with ρ (S).

Theorem 1.4 For all isolate-free graphs S and L

$$\gamma_{t2}(S \times L) \geq \rho(S)\gamma_{t2}(L) \tag{5}$$

Proof:

Assume without restriction of generality that ρ (S) = γ (S) and let {v_1,..., v_ρ(S)}be a maximum 2-packing of S. Since every vertex of our packing is at least 3 away from other vertex. Note that for i = 1,..., ρ (S) the closed neighborhoods $N_S[v_i]$ are pairwise disjoint. Let {V_1,..., V_ρ(S)}be a partition of V (S) such that $N_S[v_i]$ ⊆ V_i,for 1 ≤ i ≤ ρ (S). Let B be an $\gamma_{t2}(S \times L)$ S-set. For i = 1,..., ρ (S), let $B_i = B \cap (V_i \times V(L))$, and let . Further, let C_i be a minimum set of vertices $S \times L$ that completely dominate L_i and contain as many vertices as possible in L_i. Then C_i ⊆ V_i×V(L). Next, he assumed that C_i contains a vertex x such that x is not in L_i. Then x is the unique vertex that completely dominates x 'for some x' ∈ L_i. Since x 'has neighbors that belongs to L_i , all of which are dominated by vertices in C_i, C_i is still a semisumerally dominant set when x is replaced by x' in C_i. Thus, a set of vertices that half-fully dominate L_i and contain more vertices in L_i than, which is C_i, is a contradiction. Therefore, C_i ⊆ L_i are subsets, and therefore C_i is a half-total dominance of L in $S \times L$ induced by the set L_i.Since B_i half-total dominance {v_i}×V(L), |B_i|≥|C_i|. So $\gamma_{t2}(S \times L) \geq \rho(S)\sum i = 1|C_i| \geq \rho(S)\sum i = 1\gamma_{t2}(S \times L) = \rho(S)\gamma_{t2}(L)$.

Next, the result of a Vizing type based only on the subtotal of the domination number must be proved. By dividing the minimum half-sum dominating sets into parts that completely dominate and do not completely dominate. Note that for each graph S, when U = {u_1,..., u_k} is a minimum semisurve dominant set of S, U can be separated into two sets X and Y, where X is the set of vertices of U that are adjacent to at least one other vertex of U, and Y = U \ X. Such sets are called X, allied and Y, free. For each graph S, consider the set of minimum semi-sums dominating sets of vertices {U_1,..., U_k}, and let 1≤i≤k, X_i and Y_i be partitions of U_i in allied and free sets, respectively. Therefore, it is called U_i, so that | X_i | is of maximum size for 1 ≤ i ≤ k ,a maximum allied half sum dominant set of S, the partition { X_i, Y_i} a maximum allied partition of S, the set X_i a maximum allied set of S, and the Set Y_i a minimum free set of S. For every maximal allied partition of S{X, Y} let x (S) = | X | and y (S) = | Y |.

4. CONCLUSION

The main aim of this research is to present the importance of graph theoretical ideas in various areas of Science & Engineering for researches that they can use Domination in graph theoretical concepts for the research. An overview is presented especially to project the idea of graph theory. So, the graph theory section of each paper is given importance than to the other sections. Researches may get some information related to graph theory and its applications in various field and can get some ideas related to their field of research.

As Vizing's conjecture is not yet proved for all graphs, several researchers have studied Vizing like conjectures for other graph products and other types of domination. And also the graph theory stated a few conjectures which remain open problems and it contribute to efforts to prove Vizing's conjecture. Vizing type based only on the subtotal of the domination number are proved for isolate free graphs. The graph theory modeled Vizing's conjecture as an ideal/polynomial pair such that the polynomial was nonnegative on the variety of a particularly constructed ideal if and only if Vizing's conjecture was true

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