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Volumetric Strain Energy Theory Of Failure

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Abstract

The Volumetric Strain Energy Theory (VSET) of Failure provides a theoretical basis to predict material failure under complex stress conditions. This theory proposes that failure occurs when the volumetric strain energy within the material reaches a specific threshold, identified by the energy stored in a standard tensile test sample at failure. By concentrating on volumetric energyalterations rather than stresses or strains alone, VSET offers an enriched perspective on materialfailure.

***Keywords**:* Volumetric Strain Energy

1. Introduction

This research paper presents a comprehensive analysis of the Volumetric Strain EnergyTheory (VSET)of failure, an advanced theoretical framework for predicting material failure under complex loading conditions. The VSET is introduced as a robust alternative, emphasizing its foundation on the volumetric strain energy density, which accounts for hydrostatic component ofthe stress state.

The theoretical development of VSET is detailed, including the derivation of failure criteria and the formulation of corresponding constitutive models. The paper elaborates on the mathematical representation of volumetric strain energy and its correlation with material failure.

Furthermore, the paper explores the practical implications of VSET in engineering design and material selection, highlighting its potential to enhance the reliability and performance of structural components.

This study concludes with recommendations for future research, emphasizing the necessity for experimental validation and the advancement of computational tools to promote VSET application in industry.

Theorem 1. *"Failure of a component subjected to a complex state of stress occurs when the Volumetric Strain Energy stored in it becomes equal to the Volumetric Strain Energystored in asimple Tensile Test Specimen"*

Failure criteria are defined as σ1 + σ2 + σ3 = ± (σy/FOS), where the primary stresses reacha yield threshold divided by the safety factor. This ensures the design maintains a robust margin from the failure point under operational loads.

$$\sigma_1 + \sigma_2 + \sigma_3 < \pm (\sigma_y/FOS)$$ (This is design criteria)

Where:
σ_1 , σ_2 , σ_3 are the principle stresses
σ_y is the failure stress in simple tensile testing specimen

2. General Definition:

2.1 Stress Tensor (in 3 dimensions)

It is a three dimensional matrix whose diagonal elements represents the normal stresses(σxx ; σyy ; σzz) acting in all three directions (x, y, z) and remaining elements represents the shear stresses(σxy ; σyz ; σzx ; σyx ; σzy ; σxz)on all the three planes (xy, yz,zx) as shown below:

$$\begin{matrix} \sigma xx & \sigma xy & \sigma xz \\ \sigma yx & \sigma yy & \sigma yz \\ \sigma zx & \sigma zy & \sigma zz \end{matrix}$$

2.2 Principal Stresses[2]

σ_1 , σ_2 , σ_3 are the principle stresses (sum of hydrostatic stress and distortional stresses) inthe directions x , y and z respectively these stresses are combination of both the hydrostatic and distortional stresses as shown in the figure.These can be calculated as theroots of the following cubic equation:

$S^3 - (\sigma_{xx} + \sigma_{yy} + \sigma_{zz})S^2 + (\sigma_{xx}\sigma_{yy} + \sigma_{yy}\sigma_{zz} + \sigma_{zz}\sigma_{xx} - \sigma_{yz}^2 - \sigma_{zx}^2 - \sigma_{xy}^2)S - (\sigma_{xx}\sigma_{yy}\sigma_{zz} + 2\sigma_{yz}\sigma_{zx}\sigma_{xy} - \sigma_{xx}\sigma_{yz}^2 - \sigma_{yy}\sigma_{xz}^2 - \sigma_{zz}\sigma_{xy}^2) = 0$ [1]

2.3 Hydrostatic Stress

The hydrostatic stress is responsible for “CHANGE IN SIZE” and this results in volumetricstrain energy which forms

the base of this theory.

2.4 Distortional Stress

The distortional stresses are responsible for “CHANGE IN SHAPE” and results in distortionenergy.

2.5 Total Strain Energy

Both the volumetric strain energy and distortion energy gets stored in the body as total strainenergy.

2.6 Theories Of Failure[5]

Failure theories are essential concepts in the field of strength of materials, focusing on understanding and predicting how engineering materials fail under various loading conditions. These theories offer engineers and designers crucial insights into the safety and reliability of structures and components, helping them make well-informed decisions during the design and analysis stages.

3. Equation Derivation

In The Case Of Body Subjected To Complex State Of Stress:

Volumetric strain energy $E_v = (1/2)^* \bar{\sigma}^* \varepsilon_v \varepsilon_v = \varepsilon_z + \varepsilon_y + \varepsilon_z$

Where;

$\varepsilon_x = (\sigma_1/E) - (\mu^*(\sigma_2 + \sigma_3)/E)$ (Strain along x-direction)ε_y

$= (\sigma_2/E) - (\mu^*(\sigma_3 + \sigma_1)/E)$ (Strain along y-direction)

$\varepsilon_z = (\sigma_3/E) - (\mu^*(\sigma_1 + \sigma_2)/E)$ (Strain along z-direction)ε_v

: Volumetric strain in the body

$\bar{\sigma}$: Hydrostatic or Volumetric stress (As seen in Fig. 2)

As we know that the volumetric strain energy due to distortional stresses is equal to zeroInthe case of distortional stresses that is

$\sigma_{d1} = \sigma_1 - \bar{\sigma}$ (as we can see from the Fig. 2 is the distortional stress in the x direction) $\sigma_{d2} = \sigma_2 - \bar{\sigma}$ (as we can see from the Fig. 2 is the distortional stress in the y direction)$\sigma_{d3} = \sigma_3 - \bar{\sigma}$ (as we can see from the Fig. 2 is the distortional stress in the z direction)

μ : Poisson’s ratio

E : Young’s modulus

$\varepsilon_{d1} = (\sigma_{d1}/E) - ((\mu/E)^*(\sigma_{d2} + \sigma_{d3}))$ (This is the strain along x direction due to the distortional stresses)

$\varepsilon_{d2} = (\sigma_{d2}/E) - ((\mu/E)^*(\sigma_{d3} + \sigma_{d2}))$ (This is the strain along y direction due to the distortional stresses)$\varepsilon_{d3} = (\sigma_{d3}/E) - ((\mu/E)^*(\sigma_{d1} + \sigma_{d2}))$ (This is the strain along z direction due to the distortional stresses) $\varepsilon_{dv} = \varepsilon_{d1} + \varepsilon_{d2} + \varepsilon_{d3}$ (Volumetric strain due to distortional stresses)

But as we know that the Volumetric strain due to distortional stresses must be equal to zero asthe distortional stresses won’t cause any change in the size of the body.

As a result of which we get:

$\varepsilon dv = (\sigma d1/E) - ((\mu/E)^*(\sigma d2 + \sigma d3)) + (\sigma d2/E) - ((\mu/E)^*(\sigma d3 + \sigma d2)) + (\sigma d3/E) - ((\mu/E)^*(\sigma d1 + \sigma d2)) = 0$

$(\sigma_{d1} + \sigma_{d2} + \sigma_{d3})^*((1-(2^*\mu))/E) = 0 (\sigma_{d1} + \sigma_{d2} + \sigma_{d3}) = 0$

$(\sigma_1 + \sigma_2 + \sigma_3) - (3^*\bar{\sigma}) = 0$

Therefore, $\bar{\sigma} = (\sigma_1 + \sigma_2 + \sigma_3)/3$

$\varepsilon_v = (\sigma_1/E) - (\mu*(\sigma_2 + \sigma_3)/E) + (\sigma_2/E) - (\mu*(\sigma_3 + \sigma_1)/E) + (\sigma_3/E) - (\mu*(\sigma_1 + \sigma_2)/E) \varepsilon_v = (\sigma_1 + \sigma_2 + \sigma_3)*((1-(2*\mu))/E)$

$$E_v = (1/2)*(\sigma_1 + \sigma_2 + \sigma_3)^2*((1-(2*\mu))/(3*E)) = (1/(6*E))*(1-(2*\mu))*(\sigma_1 + \sigma_2 + \sigma_3)^2 \quad \text{(Eq.1)}$$

In Case Of Body Subjected To Simple Tension At The Time Of Failure:

At the time of failure we know that the body will be subjected to a stress equal to the yieldstress andthe principle stresses in this case will be as follows:

$\sigma_1 = \sigma_y \sigma_2 = 0$

$\sigma_3 = 0$

$\varepsilon_v = \varepsilon_x + \varepsilon_y + \varepsilon_z$

$\varepsilon_x = (\sigma_1/E) - (\mu*(\sigma_2 + \sigma_3)/E) = (\sigma_y/E)$ (Strain along x-direction)

$\varepsilon_y = (\sigma_2/E) - (\mu*(\sigma_3 + \sigma_1)/E) = -(\mu*(\sigma_y)/E)$ (Strain along y-direction)$\varepsilon_z = (\sigma_3/E) - (\mu*(\sigma_1 + \sigma_2)/E)$

$= -(\mu*(\sigma_y)/E)$ (Strain along z-direction)$\varepsilon_v = (\sigma_y/E)*(1-(2*\mu))$

$\sigma_v = (\sigma_1 + \sigma_2 + \sigma_3)/3 = (\sigma_y/3)$

Therefore,

$$E_v = (1/2)*(\sigma_y)^2*((1-(2*\mu))/(3*E)) = (1/(6*E))*(1-(2*\mu))*(\sigma_y)^2 \quad \text{(Eq.2)}$$

Upon equating the equations 1 and 2 we get:

$(1/(6*E))*(1-(2*\mu))*(\sigma_1 + \sigma_2 + \sigma_3)^2 = (1/(6*E))*(1-(2*\mu))*(\sigma_y)^2$

Final Expression for this theory is as follows:

$$\sigma_1 + \sigma_2 + \sigma_3 = \pm \sigma_y$$

In case of planar stress $\sigma_3 = 0$Therefore, $\sigma_1 + \sigma_2 = \pm \sigma_y$

If factor of safety (FOS) is considered then we get: ("+" for tension and "-" for compression) Failure criteria are defined as $\sigma 1 + \sigma 2 + \sigma 3 = \pm (\sigma y/FOS)$, where the primary stresses reacha yield threshold divided by the safety factor. This ensures the design maintains a robust margin from the failure point under operational loads.

$\sigma_1 + \sigma_2 + \sigma_3 < \pm (\sigma_y/FOS)$ (This is design criteria

4. Figures

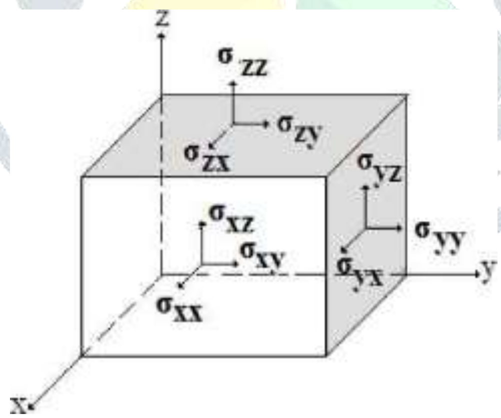

Fig. 1 Three Dimensional Stress Element

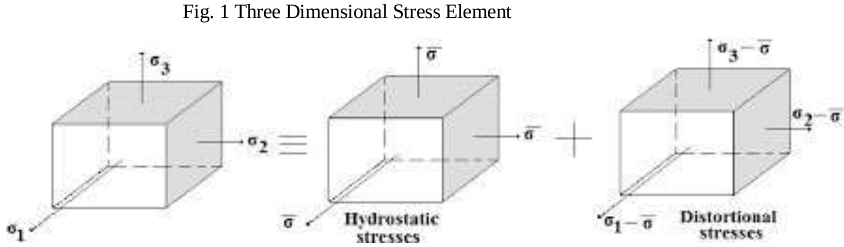

Fig.2 Stress Component

5. Conclusion

Statement : “Failure of a component subjected to a complex state of stress occurs when the Volumetric Strain Energy stored in it becomes equal to the Volumetric Strain Energy stored in asimple Tensile Test Specimen”

Application : As this theory considers normal stresses this theory is applicable for BRITTLE MATERIALS as we know that they are weak in TENSION as well as this theoryconsiders only strain energy due to change in size so it is applicable for HYDROSTATICSTATE OF STRESS as it involves only size change.

Future Scope : The study concludes with a discussion on the experimental investigations and thedevelopment of computational tools to facilitate the widespread adoption of VSETin industry.

6. References

[1] ANSEL C. UGURAL , SAUL K. FENSTER, Advanced Mechanics of Materials and Applied Elasticity, 6th Edition, Pg.No. 38-41
[2] J. E. Shigley, C. R. Mischke, R. G. Budynas, Mechanical Engineering Design, 8th ed., McGraw-Hill, New York, 2004,Pg. No. 75-79
[3] Arthur P. Boresi and Richard J. Schmidt, Advanced Mechanics of Materials, 6th ed., John Wiley & Sons, New York, 2003
[4] Joseph Marin, Engineering Materials, Prentice-Hall, Englewood Cliffs, N.J., 1952.
[5] PSG Design Data Handbook, Faculty Of Mechanical Engineering, PSG College of Technology, Coimbatore

Competing Interests
The authors, Kumaran T and Dr. K. MalarMohan, declare that they have no competinginterests.