



A Comprehensive study on the Evaluation of Swing Dynamics in Power System

Aditi Datta^{1*}, Kaberi Majumdar¹, Sanjoy Sarkar¹, Debasish Pal¹

¹Electrical Engineering Department, Tripura Institute of Technology, Tripura 799009

Abstract : Chaos is a fundamental aspect of a nonlinear dynamical system with feedback. As Chaos is extremely sensitive to the starting circumstances. A slight modification of the initial situation results in a huge amount of divergence in chaotic trajectories. Chaos is found in complex systems such as weather, astronomy and mostly in complex electrical power networks. The swing equation is essential for modeling and analyzing the power system dynamics, including transient stability also small-signal stability. The fact that it impacts the stability after being disturbed in a power system further supports the necessity of a nonlinear examination of the swing equation. Two types of stability i.e., voltage and power angle stability are considered to be vital in the study of power systems. Power angle stability is considered a matter of concern in this work. Power angle stability can be measured by analyzing the dynamics of the swing equation. The existence of chaos in the swing equation is a fact in the power system. Chaos can drive the system power angle into instability as chaos is a fundamental aspect of a nonlinear dynamical system. It is undesirable and it can also result in a total shutdown of the system. In this work a simulation-based model for swing equation is developed in MATLAB platform. Power angle(δ) vs time plot is generated at the scope connected at the output of the model for different fault clearing time. The presence of chaos i.e., instability in power angle is detected in the system with the help of entropy called Lyapunov Exponent. The entropy is calculated from time series data generated at the output of the Simulink based model of swing equation. An overview of a few potential uses of chaotic dynamics in power engineering is provided in this work. A review is given of recent research on chaos in electric power networks. The degree to which the observations can be explored is through the analysis of time series data. Additionally, use of chaos theory in power engineering is demonstrated.

IndexTerms - Swing equation, chaos, stability, Lyapunov Exponent.

I. INTRODUCTION

The swing equation [1] governs the power system's dynamics. The stability of a rotating synchronous machine inside a power system can be ascertained using the swing equation. One can calculate the power angle δ by figuring out the swing equation. Plotting δ vs time yields the swing curve. We can forecast the machine dynamics after a disturbance by examining the swing curve of a machine connected to the power supply. Since power systems are closely related to state quality upgrades, they have emerged as the most significant factor in the modern world. Maintaining the stability of the power system and securing the future depend heavily on the power system's regularity in reliability[2]. "STABILITY" refers to a power system's propensity to create restorative forces that are either greater than or equal to the disruptive forces to preserve the equilibrium. The study of the dynamics of the power system under disturbances is necessary for predicting power system stability. Two types of stability issues are there in the power system likely Voltage stability and power angle stability. Power angle stability is considered a matter of concern in this paper. Power angle stability can be measured by analyzing the dynamics of the swing equation. The existence of chaos in the swing equation is a fact in the power system. With the use of the mathematical formula known as the Swing Equation, we may better comprehend the dynamic interaction in a generator. This equation is particularly useful when analyzing the stability of a generator, especially in situations where there is a sudden change in the load on the machine. For example, depending on whether the rotor shaft is under or overloaded, the rotor will speed up or slow down to the stator field, which rotates synchronously. Entropy called the Lyapunov Exponent(LE) is explored here to measure the stability of the given generator after being disturbed by a sudden change of load. Swing equation is solved to plot power angle(δ) w.r.t time. The Simulink model's scopes have exported time series data. Utilizing a suitable procedure known as the Rosenstein algorithm, LE is calculated from the time series data. For varying fault clearance times following system disturbance, LE is computed. It has been discovered that the system verges toward instability as the critical clearing time increases, i.e., the positivity of the LE value increases. A nonlinear system's divergent attractors, which eventually lead to instability, are indicated by a positive LE value.

II. LITERATURE REVIEW

The swing equation with added delayed damping is discussed in the work [1]. Hopf bifurcations are examined in this setting using time delay as a system parameter. Hopf bifurcations occur repeatedly as time delays increase because of the emergence of limit cycles. After a certain value of the system parameter, i.e., the delay time, these limit cycles further cause the system to undergo a

period doubling cascade bifurcation, including the emergence of an endless number of attractor. The delay-free nonlinearity and time-delayed damping of second-order systems are computed. Bifurcation is the qualitative shift in system dynamics brought about by parameter changes.. Here, too, the numerical simulations demonstrate how adding time delay to a damping term in the delayed damping swing equation results in a diversity of performances.

Different types of chaos generated by various power system characteristics are proposed in this work [2]. Because electricity networks are the most complex systems, understanding chaos occurrences in them has become crucial. Various forms of disturbance-causing causes are detected, such as noise on an electromechanical system, generator excitation limitation circuit, fluctuation in linear and nonlinear loads, feedback system time delay, and turbine output variation impacting generator output. It is established that the two primary causes of chaos in the system are Ferro resonance and Sub synchronous resonance. The phenomenon of chaotic oscillation in electrical power networks is described in this review study. It has presented a brief summaries in the field of chaotic oscillation in nonlinear power network fields.

A transmission line with a single linear resistor and a single p-n diode at either end is examined in Paper [3]. Almost all electrical devices require diodes to protect the circuitry from hazardous high-voltage discharges. In this system, it is seen that changes in the diode's properties alter the chaotic dynamics when the diode is positioned in the network's primary circuit. The transmission line may sustain major damage as a result of these chaotic oscillations. Despite having infinite dimensions, the system is regarded as one-dimensional nonlinear in order to examine its chaotic behavior. It is now possible to stabilize this chaotic behavior by applying a sophisticated simulation model.. A promising way for managing common complicated spatiotemporal patterns seen in electrical circuits is explained by the chaos control method.

This study's main objective is to investigate the nonlinear dynamics in power systems in order to control chaos [4]. The author of this study has used a bifurcation diagram to observe the dynamics of the system over a range of parameter values. This can be achieved by comparing how the bifurcation diagram's parameter range changes with the system's dynamics. They have also shown that power systems are capable of many period-doubling bifurcations prior to complete collapse. In order to determine whether chaotic motion was present in the system, the LE was investigated in this paper. We performed numerical experiments to determine the effectiveness of these suggested control strategies.

The objective of reference paper[5] is to learn bifurcations of a power system with the variation of system parameter for observing the system dynamics. Bifurcations means qualitative change in system dynamics with the change in system parameters. Power systems work at a stable state point, and under typical circumstances, the system response varies gradually with changing system parameter, allowing detailed examination to be applied to the variation. Bifurcations identifies the range of parameter for stable operation of system. Points and transient stability both properties of those operating points. It makes bifurcation research valuable for stability analysis of power system. Both static and dynamic bifurcations have been studied here to address the issue of transient stability in power system.

The rising concentration of distributed generation units in the grid is challenging power system operation planning and management. Now a days polynomial chaos expansion (PCE) is explored for quantifying uncertainty in nonlinear dynamics. In this paper, author have proposed an efficient and accurate method i.e .modified PCE for measuring uncertainty in power flow networks. The method is capable of accurately handling the impact of uncertainty in a nonlinear description of the power grid[6].

Aim of this paper [7] is to analyze the chaotic event in Chua's circuit. Chua's circuit is also known as a typical chaotic circuit and has been explored by many researchers for producing chaos. This means that the output from this circuit becomes unpredictable and very sensitive to initial conditions for a certain values of system parameter. Chaotic systems are specifically used in electronics, random number generators communication systems. It has a wide range of uses, from weather prediction to encryption, in which a high 4Q degree of complex signal is required to prevent unnecessary decryption in the system.

In paper[8], authors used the Maximum Lyapunov Exponent(MLE) to analyse the after fault transient stability of the system. The MLE is calculated based on the measurement data of the system states, which can be measured by the Phasor Measurement Units (PMUs). Different types of faults with different time of durations and locations are simulated to analyse the stability of the system by computing MLE. Mainly transient stability is studied here as power system network experiences instability after large disturbances.

A hybrid Chebyshev-polynomial chaos (CPC) model is presented in this paper[9] to identify the existence of chaos in multiconductor transmission line (MTL) networks. Two different types of uncertainty i.e. uncertainty arising from inaccurate knowledge regarding the network parameters and uncertainty arising from the randomness in the power system network parameters are measured here using cpc model. Unfortunately, the model which is used to investigate such mixed problems, becomes exorbitantly complex. To mitigate this problem, a new dimension reduction-based algorithm has been developed in this paper for solving mixed problems.

Lyapunov Exponents can be computed from experimental data if the equations defining the system dynamics are unavailable. When the equations of any system are not well defined, this strategy is inappropriate. The only option left is to calculate the maximum Lyapunov Exponent (MLE) based on experimental findings at the output data. The method follows directly from the definition of LLE. For the purpose of calculating LLE from time series generated by dynamical systems, a number of algorithms are available. These include the Grassberger-Procaccia algorithm, and another type is Wolf's algorithm[2], also Sato algorithm, and the Rosenstein algorithm, etc. In this work[10], one of the most trustworthy techniques for calculating LLE from limited data sets is the Rosenstein Algorithm. The approach has been found to be quick, simple to use, and resistant to changes in the parameters given as encoding, size of the collected data, reconstruction delay, and noise level.

In the model and study of power system dynamics following disturbance, the swing equation is crucial. In paper[11], the incremental harmonic balance (IHB) method is used to obtain a rough solution to the limit cycle. It is discovered that by using higher order harmonics in the IHB approach, the issue of a more distorted limit cycle may be easily resolved. Here, the homoclinic bifurcation curve has been perfectly characterised by the authors. To examine a generalised swing equation with excitation voltage dynamics, the approach can be expanded.

The paper [12] addresses the critical review on the bifurcation analysis which discusses on the principle of bifurcation analysis. A well-known power system model and an appropriate load model that capture an element's physical behavior are selected for bifurcation analysis. The many analytic techniques aid in the resolution of power system issues. The use of FACTS to prevent crucial bifurcations is another topic covered by the writers.

The paper [13] addresses that the Bifurcation is the qualitative shift in dynamics that happens when a system parameter is altered. This work aims to analyse a perfect representation of the current-mode regulated power converter's bifurcation diagram. We may characterize the bifurcation diagram pattern—which gives rise to the complex pattern—using algebraic relations.

The paper [14] addresses about the synchronous motor's dynamic model, which resembles a Lienard type system in structure. The network's fluctuations in the equilibrium points that it must meet to have asymptotic or spiral stability are represented by the Hopf bifurcation.

The paper [15] addresses about the comprehensive power system model bifurcation study by examining how various control factors and constraints affect the bifurcation and ensuing system stability. Additionally, it focuses on the useful applications of bifurcation theory and how its stability affects realistic power systems demonstrations.

The paper [16] addresses about the analysis techniques, modelling strategies, and a list of typical power electronics bifurcation types. The paper [16] addresses about the voltage stability, which the bifurcation evaluation of the power system evaluation appears to be accurate. This work demonstrates a thorough bifurcation analysis of the power system, which is commonly expressed as an equation system in differential algebra. Additionally, it demonstrates how to translate its particular singularity into bordered matrix approaches and extending the techniques for differential algebraic equation systems.

The paper [17] addresses about a designed and created controller with UPFC for manage synchronous resonance split-offs in multi-machine power systems. UPFC is taken into consideration for managing the bus voltage as well as the flows of electricity in AC transmission lines, both reactive and active.

The paper [18] addresses about the impact of the controller controls the gain of a Thyristor-Controlled Series Capacitor (TCSC) on the bifurcation of sub synchronous resonance of an infinite bus power system for a single machine in two scenarios: one where the functioning of the generator's damper windings is taken into account, while one another where they are not.

The paper [19] addresses about extending a better model of fractional order of the dual alternatives to the classic integer-order paradigm for congestion control, resulting from a delayed inter-order model. We require the stability theorem by selecting the transmission latency in order to achieve stability and bifurcation.

The paper [20] addresses about a power system model displaying the static as well as dynamic behavior of bifurcation. It also demonstrates the unpredictable behavior that results from period doubling. Due to reactive power requirement reaching the system's steady-state operational limits, Voltage Collapse occurs. Additionally, it goes over some terminology related to bifurcations and offers numerous options for local dynamic phenomena.

The paper [21] addresses about the bifurcation in Sub synchronous Resonance is taken into consideration and a Hopf bifurcation and chaos are controlled using both linear and nonlinear controllers. Included are a power system stabilizer, automated voltage regulator, and damper winding.

III. SWING EQUATION IN POWER SYSTEM

This paper addresses the significance of rotor angle stability in power systems and proposes a method for evaluating rotor angle stability: calculating the power system's swing equation. The swing equation can be solved using non-linear techniques like Runge-Kutta and Euler, but in this paper, the stability of a single-machine infinite bus test system is investigated using the entropy, or LE, method. The results show that this method successfully locates the operating point of the system near its instability area, allowing the pointing out of the upper threshold level where the network's mechanical power input may grow suddenly. This work assesses the post-fault transient stability of a machine linked to an infinite bus using the Maximum Lyapunov Exponent (MLE) technique.

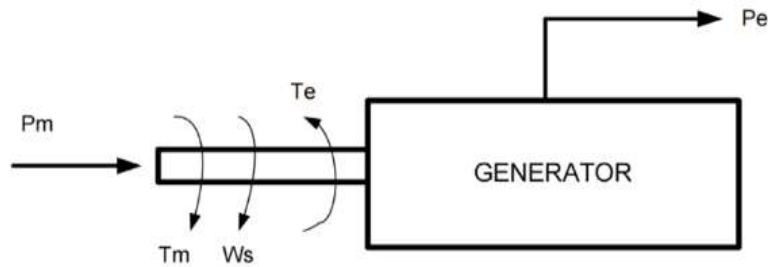


Figure 1 Synchronous machine operation

The classic swing equation is given below.

$$M \frac{d^2 \delta}{dt^2} = P_a = P_m - P_e \quad [1]$$

Where, $P_a = P_m - P_e$ is accelerating power,

P_e = Electrical power output in MW.

P_m = Mechanical Power input in MW.

Modifying this equation we can write,

$$\frac{d^2 \delta}{dt^2} = \frac{1}{M} (P_m - P_e)$$

$$\frac{d^2 \delta}{dt^2} = \frac{1}{M} (P_m - P_{\max} \sin \delta) \quad [2]$$

Where,

M = Angular Momentum, MJ - sec/elect. rad or in per unit system M can also be expressed as $M = \frac{GH}{\pi f}$, $M = \frac{H}{\pi f}$

H represents the inertia constant

$$P_e = P_{\max} \sin \delta$$

Where δ is the rotor angle and $P_{\max}$ represents maximum electrical power.

Swing Equation for Multi machine System:

In a multimachine system, selecting a common basis is important.

Let

$$G_{mach} = \text{machine base}$$

$$G_{system} = \text{system base}$$

Equation (2) can be written as

$$\frac{G_{mach}}{G_{system}} \left(\frac{H_{mach}}{f} \frac{d^2 \delta}{dt^2} \right) = (P_m - P_e) \frac{G_{mach}}{G_{system}}$$

Or,

$$\left(\frac{H_{system}}{\pi f} \frac{d^2 \delta}{dt^2} \right) = (P_m - P_e) \quad [3]$$

Where

$$H_{system} = H_{mach} \left(\frac{G_{mach}}{G_{system}} \right) = \text{machine inertia constant}$$

Swing Equation for Coherent System:

$$\delta_1 = \delta_2$$

$$\delta = \delta_1 = \delta_2$$

$$M_1 \frac{d^2 \delta_1}{dt^2} = P_{m1} - P_{e1} \quad [4]$$

$$M_2 \frac{d^2 \delta_2}{dt^2} = P_{m2} - P_{e2} \quad [5]$$

Adding both equation [4] and [5] we get

$$(M_1 + M_2) \frac{d^2 \delta}{dt^2} = (P_{m_1} + P_{m_2}) - (P_{e_1} + P_{e_2})$$

$$\begin{array}{ccc} \uparrow & \uparrow & \uparrow \\ M_{eq} & P_{m \text{ equ}} & P_{e \text{ equ}} \end{array}$$

$$M_{eq} = M_1 + M_2$$

$$\frac{G_{eq} H_{eq}}{\pi f} = \frac{G_1 H_1}{\pi f} + \frac{G_2 H_2}{\pi f}$$

$$H_{eq} = \frac{G_1 H_1 + G_2 H_2}{G_{equivalent}}$$

Swing Equation for Non-Coherent System

$$\delta_1 \neq \delta_2 \quad \delta_1 > \delta_2$$

$$M_2 \left(M_1 \frac{d^2 \delta_1}{dt^2} = P_{m_1} - P_{e_1} \right) \quad [6]$$

$$M_1 \left(M_2 \frac{d^2 \delta_2}{dt^2} = P_{m_2} - P_{e_2} \right) \quad [7]$$

$$M_1 M_2 \frac{d^2}{dt^2} (\delta_1 - \delta_2) = (P_{m_1} M_2 - P_{m_2} M_1) - (P_{e_1} M_2 - P_{e_2} M_1)$$

$$= (P_{m_1} M_2 - P_{m_2} M_1) - (P_{e_1} (M_1 + M_2))$$

$$\frac{M_1 M_2}{M_1 + M_2} \frac{d^2}{dt^2} (\delta_1 - \delta_2) = \frac{P_{m_1} M_2 - P_{m_2} M_1}{M_1 + M_2} - P_{e_1}$$

$$\begin{array}{ccc} \uparrow & & \uparrow \\ M_{eq} & & P_{m \text{ equ}} \end{array}$$

$$\frac{G_{eq} H_{eq}}{\pi f} = M_{eq}$$

$$H_{eq} = \frac{M_{eq} \cdot \pi f}{G}$$

Discretization of swing equation

Discretization is a general term used to describe the process of converting continuous functions or systems into discrete forms that can be analysed and solved numerically. The discretization process involves dividing the continuous function or system into smaller parts, such as time steps or spatial intervals. This allows us to represent the continuous function or system as a sequence of discrete values, which can then be analysed and manipulated using numerical methods[3,4]. There are several methods of discretization, incorporating the spectral approach, the finite element method, and the finite difference method. The best approach depends on the particular issue at hand because each strategy has advantages and disadvantages. One of the most popular ways of discretization has been the method of finite difference. It involves approximating the derivatives of a function using finite differences, which are the differences between the function values at neighbouring points in space or time. The resulting discretized equation can then be solved numerically using iterative methods. The discretized form of swing can be written by

$$\delta_n = \delta_{n-1} + \Delta \delta_n \quad [8]$$

$$\text{Where } \Delta \delta_n = \Delta \delta_{n-1} + \frac{(\Delta t)^2}{M} P_{a(n-1)} \quad [9]$$

By substituting [4] in equation [3] we get,

$$\Delta \delta_n = \delta_n - \delta_{n-1} = \Delta \delta_{n-1} + \frac{(\Delta t)^2}{M} P_{a(n-1)}$$

$$\delta_n = \delta_{n-1} + \Delta\delta_{n-1} + \frac{(\Delta t)^2}{M} P_{a(n-1)} \quad [10]$$

$$\text{Again we know} \quad \Delta\delta_{n-1} = \delta_{n-1} - \delta_{n-2} \quad [11]$$

Substituting equation [6] in equation [5] we can write,

$$\delta_n = \delta_{n-1} + \delta_{n-1} - \delta_{n-2} + \frac{(\Delta t)^2}{M} P_{a(n-1)}$$

$$\delta_n = 2\delta_{n-1} - \delta_{n-2} + \frac{(\Delta t)^2}{M} P_{a(n-1)} \quad [12]$$

The accelerating power at the completion of (n-1)th interval,

$$P_{a(n-1)} = P_m - P_{\max} \sin \delta_{n-1} \quad [13]$$

Putting the expression for $P_{a(n-1)}$ in equation [7] we get,

$$\delta_n = 2\delta_{n-1} - \delta_{n-2} + \frac{(\Delta t)^2}{M} (P_m - P_{\max} \sin \delta_{n-1}) \quad [14]$$

The equation [14] represents the discretized form of the swing equation and describes the behaviour of a system over time. The equation is recursive, meaning that to calculate the value of δ_n , we need to know the values of δ_{n-1} and δ_{n-2} , etc. Starting from some initial values of $\delta(0)$ and $\delta(1)$, we can use the equation to iteratively calculate the values of $\delta(n)$ for each subsequent time step. The behaviour of the system is determined by the values of $P_{\max}$, P_m and M . By varying these values and initial conditions, we can study the behaviour of the system under different conditions and make predictions regarding the dynamics of the system. For the convenience of analysis time step is considered to be unity in the equation [14].

IV. SIMULATION OF SWING EQUATION IN MATLAB & OUTPUTS

In today's society, electric power loads are anticipated to be consistently met. A power system is the infrastructure used to produce and deliver electricity to final users. The power system needs to be resilient enough to endure unexpected component losses or electric short circuits, among other abrupt disruptions. It should be highlighted that the majority of disruptions, including component failure, do not result in an event. In the event of a disruption, a governor controls a machine's speed to modify a generator's output power based on the state of the network. For this reason, the transient stability evaluation is required to determine whether the power system is stable following a disruption. When taken as a whole, power systems are made up of mechanical and electrical components that adhere to Kirchhoff's rules and energy conservation. These components are combined to form the so-called swing equation.

The swing equation is a multi-variable, heterogeneous, nonlinear second-order differential equation. For the most part, numerical methods are used to solve the swing equation. This method yields the best results when assessing the power system's stability.

This section has covered the MATLAB-based Simulink platform's software simulation of the swing equation, as stated in equation (1). The values of maximum electrical power $P_{\max}$, mechanical power input P_m , and angular momentum M , were constantly varied through out the numerical experiment. The difference between P_m and P_e has been obtained through a summing block and the signal is passed through a gain block. Then the signal after the gain block is passed through cascaded integrator. Four types of blocks namely Constant Parameter, Summing, Gain, Integrator, Transfer Function and Scope blocks are used in this simulation platform in accordance with the equation's specifications in order to construct the swing equation model in Simulink. It has been illustrated in Fig-2.

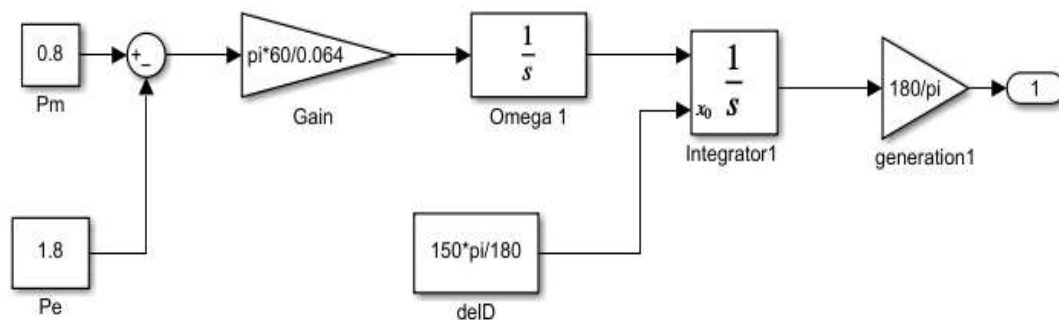


Fig 2: - Swing Equation Model

The output Graphs given below are simulated using Simulink model of swing equation with $H=50$ MJ. They all represent single machine systems.

Case:1**For System with $P_m < P_e$**

The figures attached below represents a stable system. The system deaccelerates due to presence of retardation power i.e. $P_m < P_e$. Values of P_m and P_e are kept as follows $P_m = 0.8$ while $P_e = 1.8$.

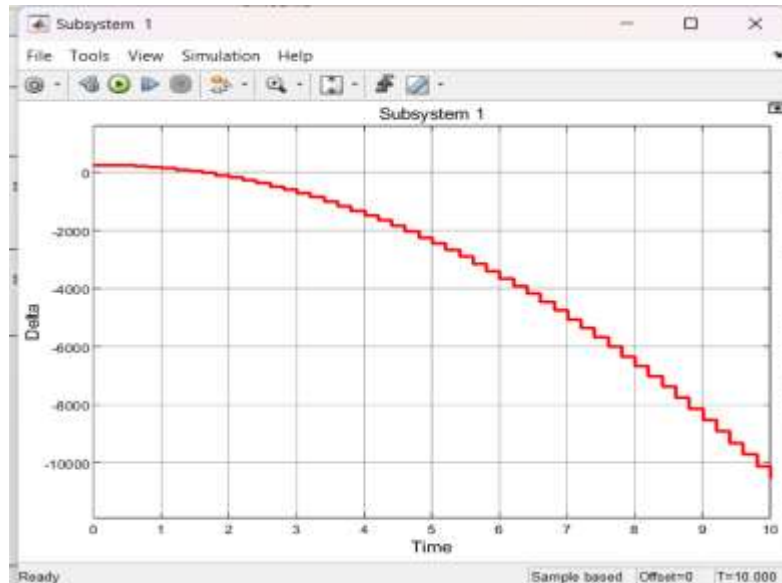


Fig-3 (a) Stable Swing Curve for $P_m < P_e$.

Case:2**For system 2 with $P_m > P_e$**

For system 2 in the single machine system, the simulation time for 10 seconds is completed. Simulation has been done for $P_m = 1.8$ while $P_e = 0.8$. The system accelerates and diverges towards instability due to presence of accelerating force i.e. $P_m > P_e$. Swing curve with simulation time is given below. The system become instable due to accelerating power.

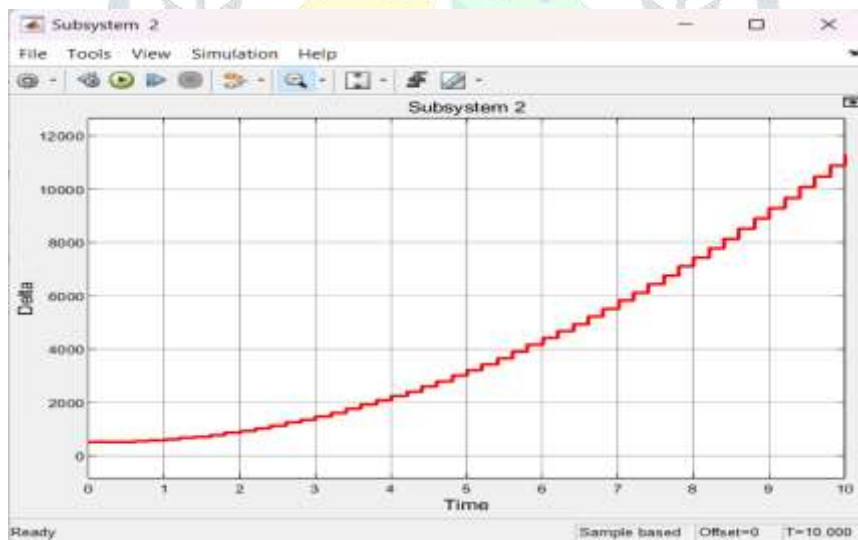


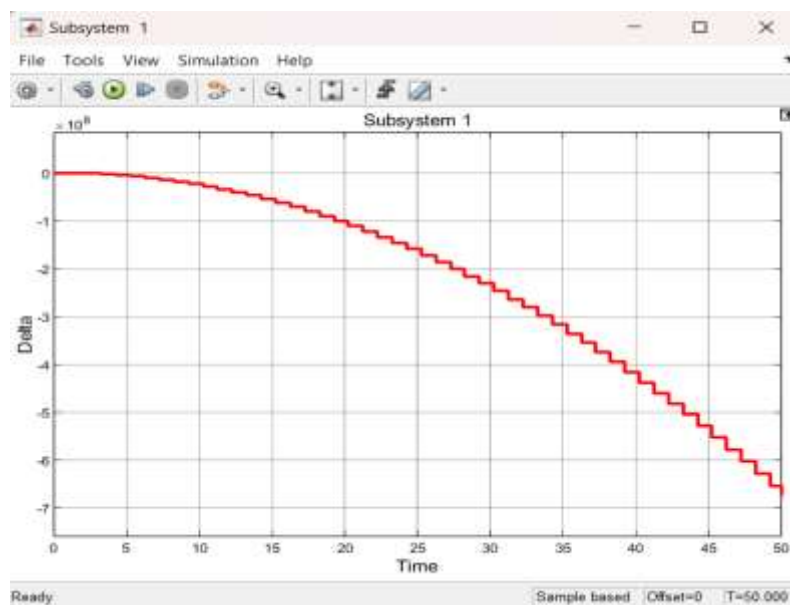
Fig. 3 (b) Unstable Swing Curve for $P_m > P_e$

Simulation of Multi-Machine System with $H_{eq} = 0.02$ pu

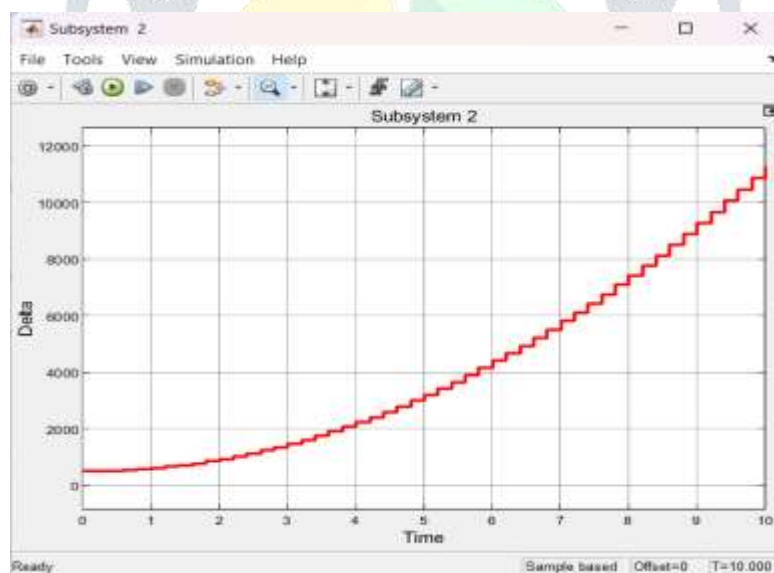
Now all the sub system are treated as coherent multi-machine system and numerical experiment has been carried out with the following value of equivalent H_{eq} i.e. per unit inertia constant $H_{eq} = 0.02$ pu.

Case: 1**For System with $P_m < P_e$**

For multi-machine system 1, the simulation time is considered 50 seconds with $P_m < P_e$ and $P_m = 0.8$, $P_e = 1.8$.

Fig 4 (a) Stable Swing Curve for for $P_m < P_e$ **Case:2****For system 2 with $P_m > P_e$**

For multi-machine Subsystem2, simulation is completed in 10 seconds with $P_m > P_e$ i.e. $P_m = 1.8$ and $P_e = 0.8$. Outputs become unstable just like the single machine system due to presence of accelerating force.

Fig-4 (b) Unstable Swing Curve For $P_m > P_e$

The model of swing equation in MATLAB platform(Fig-2) has been modified by incorporating a fault clearing block as given in fig-5. The following values are taken into account for the simulation: maximum electrical power $P_{max} = 1.71$ pu, mechanical power input $P_m = 0.8$ pu, and angular momentum $M = 0.00057$ pu, with a crucial clearance time ranging from 0.01 to 0.1 sec. A constant block was added first and its value was set to the necessary value in accordance with the equation's specifications in order to construct the swing equation model in Simulink[1]. The output from the constant block is then fed into the positive terminal of the Subtractor block, which has been installed. The product block's output is extended to the subtractor block's negative terminal. A gain block connects the subtractor's output to the first integrator. After going via a Sin block, the signal from integrator 2 is multiplied by the switching block's output at the product block's input. To achieve the model, product block's output is feed to the subtractor's negative terminal. After converting the angle from radians to degrees using one gain block, a

scope is connected so that the output, that is, the time series produced at the swing equation's output—can be seen. This model has been run for various clearing time values. The output's LE has been computed using various fault clearance times. A switch is used to determine the fault clearance time and consequent system stability. In this case, the Rosenstein technique is explored for calculating entropy, or LE, from the experimental data.

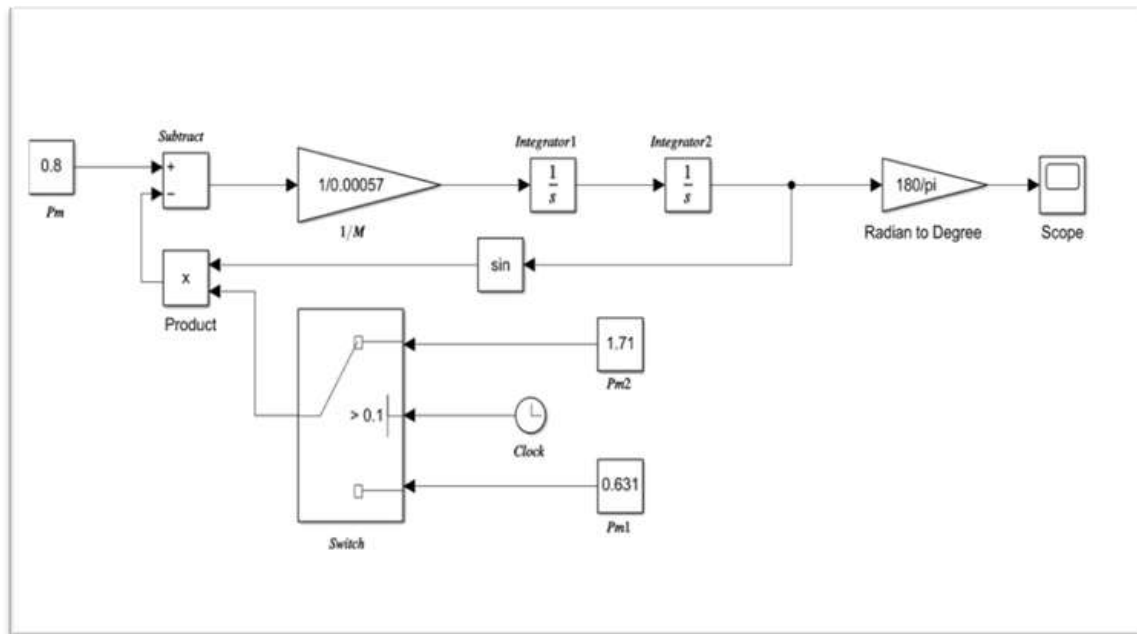


Fig-5 Simulation of swing equation in Simulink using fault clearing block.

While a negative LE value guarantees a stable system, a positive LE value identifies disorder or instability in the system. The system maintains stability for a critical clearing time of less than 0.06 seconds with $P_m=0.631$ pu. For that specific value of PM, the system becomes unstable if the crucial clearing time surpasses that threshold. Additionally, we have used MATLAB to simulate the swing equation by visualizing the power angle vs discrete time using Discrete Equation No. [15]. A few of the plots are also included in the results and discussion section, as shown in Figures 9 and 10. As stated in the figures, both positive and negative values of LE were discovered for two distinct sets of system parameters.

In this study, the stability of power systems is analysed by building a Simulink model of the swing equation using MATLAB[1,8]. We changed the critical clearing time, or the switching parameter, of the nonlinear system and looked at the results for various switching time values. Throughout the numerical experiment, the critical clearing time is changed. Below are some simulation results for various crucial clearance periods along with the appropriate figure captions.

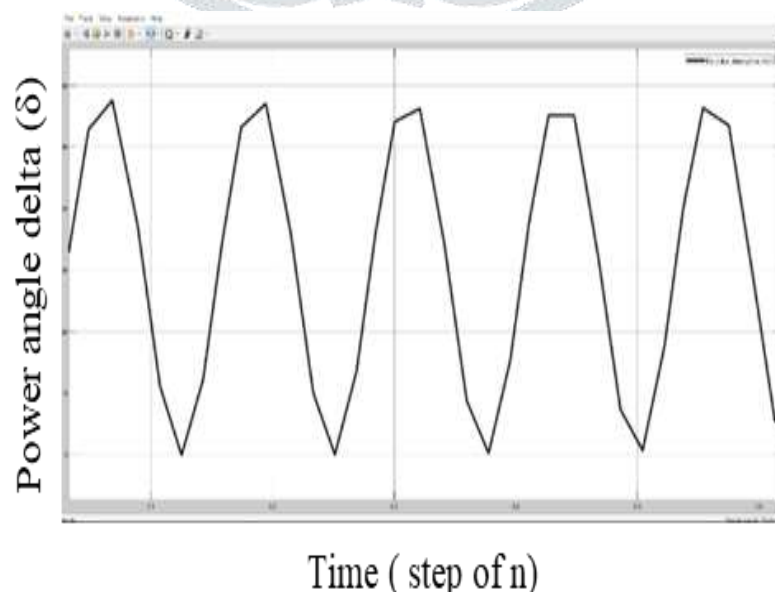


Figure 6 Simulation output for critical clearing time > 0.01 s (LE= - 0.0087)

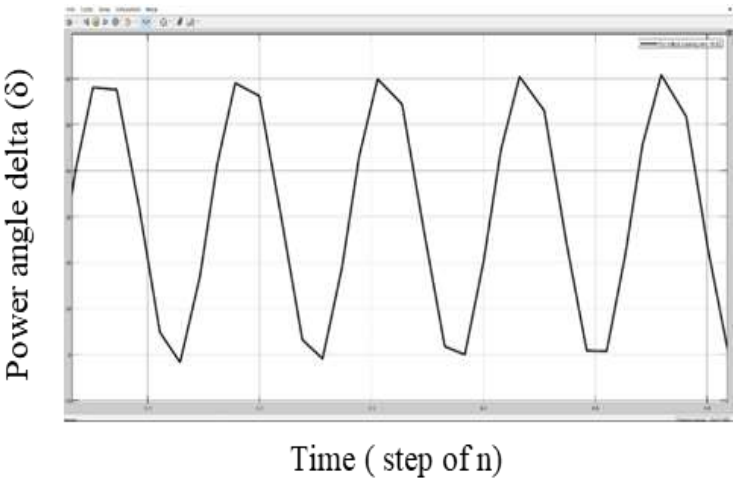


Figure 7 Simulation output for critical clearing time > 0.02s(LE = -0.0053)

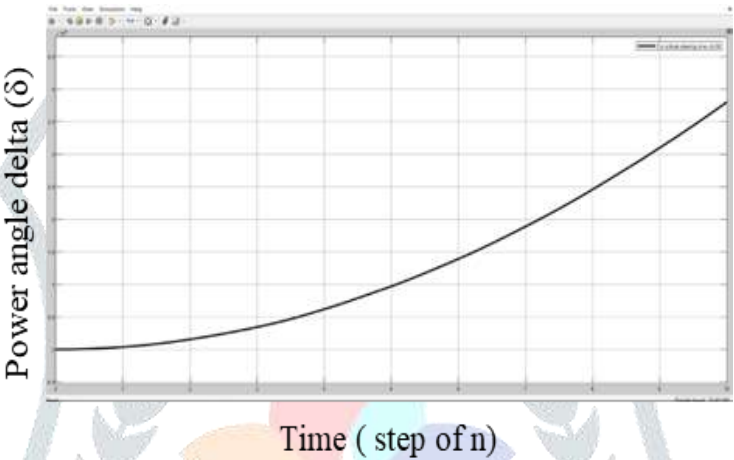


Figure 8 Simulation output for critical clearing time > 0.06 sec (LE = 0.005)

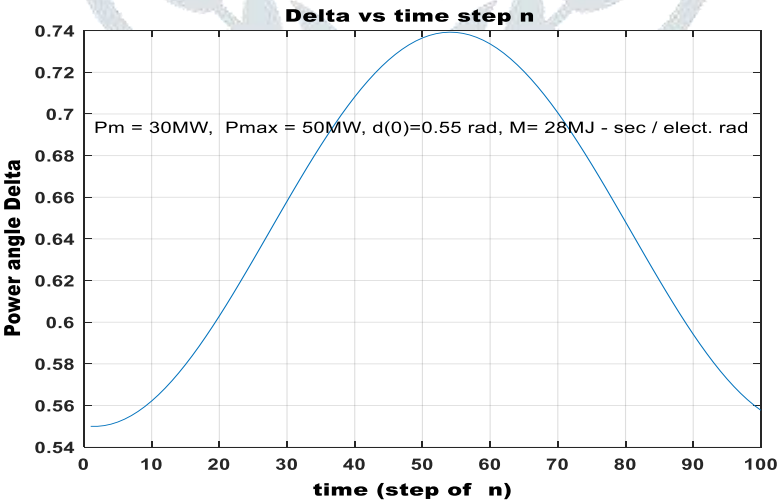


Figure 9 Power Angle Delta vs Time step (n) curve (LE = -0.0101325)

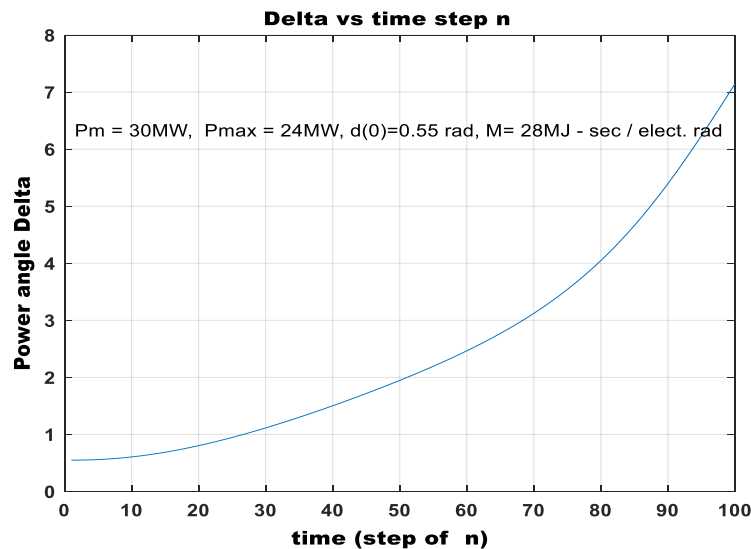


Figure 10 Power Angle Delta vs Time step (n) curve (LE = 0.01067)

It has been depicted from the figures that the system verges toward instability beyond a critical time greater than 0.06s. Hence critical clearing time should be kept at less than 0.06s. Along with the Simulink model, we have also simulated the swing equation in discrete mode in MATLAB. In MATLAB, a program has been created and simulated for different values of P_m , P_{max} , and M , and the initial conditions of δ . Two such values are given below. Values of LE have also been computed to predict the stability of the system. From the nature of the delta vs time plots, it is evident that fig-10 ensures the stability of the system, whereas fig-11 represents an unstable system. By selecting proper values for P_m , P_{max} , M , and the initial condition δ we can restrict the system within a stable zone. The present study has investigated the stability of power systems by utilizing a simulation-based model for swing equations in the MATLAB platform. To evaluate the stability of the system, we employed a measurement tool known as the entropy called LE. The LE provides insight into the system's stability, with a negative value indicating stability and a positive value indicating instability. Our findings reveal that the clearing time is a critical factor that determines the stability of the system, and there is a specific threshold value beyond which the system becomes unstable.

V. DISCUSSIONS

For instance, when we set the clearing time at 0.01 seconds, we discovered that the value of LE was negative, implying that the system is stable. Furthermore, as we increased the clearing time to 0.03 seconds, the Lyapunov Exponent remained negative, suggesting a stable system. However, as we raised the clearing time to 0.06 seconds, the Lyapunov Exponent became positive, indicating that the system verges towards instability. As the clearing time was further increased, the Lyapunov Exponent continued to become more positive, signifying that the system was progressively becoming unstable. This very fact is depicted in Table 1. These findings are crucial to power system engineers as they assist them in determining crucial clearing times and formulating effective stability measures to prevent power system malfunctions.

The correct choice of the Rosenstein algorithm's parameters is one of the difficult tasks we examine here. At the final stage of the Rosenstein algorithm, LLE is computed using the curve fitting method. The divergence of nearest trajectories is plotted as the system evolves with time. The linear range of the curve is determined to compute the slope which is the measure of the value of the LLE of the system. This curve-fitting technique is another sensitive step for the proper calculation of LE.

VI. CONCLUSION

In this study, we looked at how chaos affects power systems and their stability. We used the Rosenstein algorithm to calculate entropy from time series functions and examined system stability at different critical clearing times [9,10,14]. By keeping some factors constant, such as P_m , M , and P_{max} , we have also determined the entropy δ as a function of the time step from the discretized swing equation. Both stable and unstable conditions were detected depending on the values of P_m , M , and P_{max} . Research indicates that entropy is a useful tool for assessing power systems' stability and identifying potential sources of chaos. By calculating entropy for different clearing times, we were able to measure the system's stability and locate critical points where instability may occur and we were also able to identify a range for the values of P_m , M , P_{max} and critical clearing time within which the system always remains stable. Expanding upon the outcomes of this study, we sought to deepen our understanding of chaos in power systems by employing the swing equation to compute entropy and constructing bifurcation diagrams. By undertaking bifurcation analysis, we were able to identify distinct regions of chaos corresponding to different entropy values, providing a compelling visual representation of the system's behavior. This comprehensive investigation allowed for a more accurate analysis of power system dynamics and shed light on the profound influence of various parameters on system stability. The findings presented in this study provide a solid foundation for future research endeavors aimed at unraveling the intricate link between chaos and power system stability and reliability. By conducting further investigations in this area, we can enhance our ability to anticipate and prevent power system breakdowns, ultimately fortifying the reliability and resilience of power grids.

REFERENCES

- [1]. Scholl, T. H., Gröll, L., &Hagenmeyer, V. (2019). Time delay in the swing equation: A variety of bifurcations. *Chaos*, 29(12). <https://doi.org/10.1063/1.5122784>
- [2]. Al-Hussein, A. B. A. (2021). Chaos Phenomenon in Power Systems: A Review. *Iraqi Journal for Electrical & Electronic Engineering*, 17(2)
- [3] Triandaf, I. (2010, December). Chaos control in a transmission line model. In 2010 17th IEEE International Conference on Electronics, Circuits and Systems (pp. 1006-1008). IEEE.
- [4] Chang, S. C. (2020). Stability, chaos detection, and quenching chaos in the swing equation system. *Mathematical Problems in Engineering*, 2020:27–30. Pp.
- [5] Tan, C. W., Varghese, M., Varaiya, P., & Wu, F. F. (1995). Bifurcation, chaos, and voltage collapse in power systems. *Proceedings of the IEEE*, 83(11), 1484-1496.
- [6] Métivier, D., Vuffray, M., & Misra, S. (2020). Efficient polynomial chaos expansion for uncertainty quantification in power systems. *Electric Power Systems Research*, 189, 106791.
- [7] Kawata, J., Nishio, Y., &Ushida, A. (1996, May). Analysis of Chua's circuit with transmission line. In 1996 IEEE International Symposium on Circuits and Systems. Circuits and Systems Connecting the World. ISCAS 96 (Vol. 3, pp. 201-204). IEEE.
- [8] Banerjee, P., Srivastava, S. C., & Srivastava, K. N. (2014, December). A Lyapunov exponent based method for online transient stability assessment. In 2014 Eighteenth National Power Systems Conference (NPSC) (pp. 1-6). IEEE.
- [9]Prasad, A. K., & Roy, S. (2018). Reduced dimensional Chebyshev-polynomial chaos approach for fast mixed epistemic-aleatory uncertainty quantification of transmission line networks. *IEEE Transactions on Components, Packaging and Manufacturing Technology*, 9(6), 1119-1132.
- [10] Rosenstein, M. T., Collins, J. J., & Luca, C. J. De. (1993). A practical method for calculating largest Lyapunov exponents from small data sets. *Physica D: Nonlinear Phenomena*, 65(1-2), 117–134.
- [11]Qiu, Q., Ma, R., Kurths, J., & Zhan, M. (2020). Swing equation in power systems: Approximate analytical solution and bifurcation curve estimate. *Chaos: An Interdisciplinary Journal of Nonlinear Science*, 30, 13110. <https://doi.org/10.1063/1.5115527>
- [12]. Michael Fette, Ingo Winzenik, "Bifurcation analysis of Power System load characteristics", *Mathematica and Computer Modelling of Dynamical Systems*, vol. 11, issue 4, 425-445, 2005.
- [13]. Mohsen Ruzbehani, Luowei Zhou, Mingyu Wang, "Bifurcation diagram feature of a dc-dc converter under current-mode control. *Chaos, Solitons & Fractals*", 2006, vol 28, pp.205-212.
- [14]. Fernando Mesa, German Correa, J. Barba-Ortega, "Hopf Bifurcation in the Study of synchronous Motor Stability," *Ciencia en Desarrollo* 13 (1), 1-7, 2022.
- [15].Andre Arthur Perleberg Lerm, Claudio A Canizares, Aguinaldo Silveira e Silva, Multiparameter Bifurcation Analysis of south Brazilion Power System, *IEEE Transactions on Power Systems* 18 (2), 737-746, 2003.
- [16]. Mario di Bernardo and chi k. Tse, Chaos in Power Electronics: An Overview, *Chaos in circuits and systems*, 317-340, 2002.
- [17]. Stijn cole, Kailash Srivastava, Muhamad Reza and Ronnie Belmans, New numerical techniques for bifurcation analysis of power systems with application to grid connected voltage source converter, *European Transactions on Electrical Power* 22 (5), 704-720, 2012.
- [18]. Majdi M. Alomari, Mohammad S. Widyan, Hopf Bifurcation Control of Sub synchronous Resonance Utilizing UPFC, *Engineering, Technology & Applied Science Research* 7 (3), 1588-1594, 2017.
- [19]. M.S. Widyan & A.M. Harb, ON the Effect of TCSC and TCSC-Controller Gain on Bifurcation of Sub synchronous Resonance in Power Systems, *International Journal of Modelling and Simulation* 30 (2), 252-262, 2010.
- [20]. Min Xiao, Guoping Jiang, Jinde Cao, Weixing Zheng, Local Bifurcation Analysis of a Delayed Fractional-order Dynamic Model of Dual Congestion Control Algorithm, *IEEE/CAA Journal of Automatica Sinica* 4 (2), 361-369, 2016
- [21]. Chin-Woo Tan, Matthew Varghese, Pravin Varaiya and Felix Wu, Bifurcation and chaos in Power Systems, *Sadhana* 18, 761-786, 1993.
- [22]. Majdi M. Alomari and Benedykt S. Rodanski, 2011, Control of Hopf Bifurcation and Chaos as Applied to Multimachine System. *Nonlinear science and complexity*, pp. 37-48.
- [23]. Ajarapu, V. and Lee, B., 1992, Bifurcation theory and its application to nonlinear dynamical phenomena in an electrical power system. *IEEE Traiisticlions on Power Systems*. 7. 424 431
- [24]. Budd. C.J. and Wilson, J.P-. 2002. Bogdanov-Takens bifurcation points and Sil'nikov homoclinicity in a simple power system model of voltage collapse. *IEEE Transactions on Circuits and Svstems I*. 49, 575 -590.
- [25]. Chiang. H.D.. Liu. C.W.. Varaiya. P.P . Wu. F.F. and Lauhy, M.G.. 1993. Chaos in a simple power system. *IEEE Transactions on Power Systems*. 8. 1407- 1417.
- [26]. Mithulaiianthan, N., Cafiizares, C.A., Reeve. J. and Rogers, G.J., 2003, Comparison of PSS, SVC. And STATCOM controllers for dumping power system oscillations. *IEEE Transactions on Power Systems*.18, 786 792.
- [27]. Perleberg Lerm, A.A., Cafiizares, C.A. and e Silva, A.S., 2003, Multiparameter bifurcation analysis of the south Brazilian power system. *IEEE Transactions on Power Systems*, 18, 737-746.