



Quantum Teleportation: A Step Toward Superluminal Communication

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Abstract

Numerous applications of quantum computing are thought to be better than those of its classical counterparts. "Quantum Teleportation" has been a well-known use since the dawn of the quantum computer era. It allows the transfer of quantum states between distant parties without the need to physically convey the particles, making it a fundamental component of quantum information technology. Noise, decoherence, and flaws in quantum entanglement make it difficult to achieve high-fidelity quantum teleportation in experimental situations, despite its theoretical elegance. In order to accomplish dependable and effective quantum teleportation, this paper suggests a unique method that combines quantum state tomography with the Peres-Horodecki Positive Partial Transpose (PPT) criterion. Our goal The proposed methodology leverages quantum tomography to reconstruct the density matrix of the shared entangled state, providing a comprehensive characterization of the system. Our goal is to develop a reliable technique that uses entangled particle pairs as the primary resource for teleporting quantum states over long distances. The results show that this integrated framework offers a scalable route for implementation in higher-dimensional quantum systems and improves the dependability of quantum teleportation protocols. The practical implementation of quantum communication technology has advanced significantly as a result of this effort.

1 Introduction

1.1 What is Quantum Teleportation

We are all familiar with the word "TELEPORTATION", which literally means the transfer of any object physically from one place to another. Well In quantum Computing, this term has a specific meaning. The name Telepor- tion is meant to be suggestive of the concept in science fiction where matter is transported from one location to another by some futuristic process, but it must be understood that matter is not teleported in quantum teleportation - what is actually teleported is "Quantum Information". In 1993 a paper described a quantum information protocol, termed as quantum teleportation, that shared several of the above properties. In this protocol, an Un- known quantum state of physical system is measured and sent to the remote location where it is subsequently reconstructed and reassembled. This process requires classical communication and excludes faster-than-light or superluminal communication. Most importantly, it requires quantum entanglement. Indeed, quantum teleportation can be seen as the protocol in quantum information that most clearly demonstrates the character of quantum entanglement as a resource. Without its presence, such a quantum state transfer would not be possible within the law of quantum mechan- ics.

1.2 Motivation

Despite its theoretical and experimental advancements, the conventional protocol faces challenges, such as it does not provide instantaneous communication, to complete this protocol classical com- munication is required, which is bound by the speed of light which is a barrier to long-distance communication. Addressing this issue is crucial for realizing practical implementation in long- distance communication.

1.3 Objective

This paper introduces a modified protocol for quantum teleportation that reduces the need to trans- fer classical information. Our approach uses the concept of quantum tomography where we recon- struct a quantum system from measurements taken on the identical system. It's used to characterize a quantum system state, including its density matrix, which is important for understanding entan- glement and coherence. The second main method we used in this protocol is the "PPT criterion" (Peres-Horodecki criterion) which stands for "Positive partial transpose criterion" which is a math- ematical test used to determine whether a quantum state is separable (meaning it can be described as a combination of independent smaller states) or entangled state. It acts as a key tool to identify the entanglement in the quantum system.

2 Literature review

2.1 Overview of Quantum Teleportation

Quantum teleportation is a cornerstone of quantum information theory, first proposed by Bennett et al. in 1993. The process enables the transfer of a quantum state from one location to another with- out physically transporting the particle itself. This is achieved through the utilization of quantum entanglement and classical communication

- **Entangled State Preparation:** A pair of particles (often photons, atoms, or qubits) is pre- pared in an entangled state.
- **Bell-State Measurement:** The sender (Alice) performs a Bell-state measurement on the particle to be teleported and her part of the entangled pair.

- **Classical Communication and Reconstruction:** The measurement results are sent to the receiver (Bob) via a classical channel, who uses this information to reconstruct the original quantum state through a unitary operation.

Key advancements in quantum teleportation include:

- **Experimental Demonstrations:** The first experimental realization by Bouwmeester et al. (1997) utilized photons entangled through polarization. Since then, teleportation has been demonstrated across larger distances and through diverse quantum systems, such as trapped ions and superconducting circuits.
- **Applications in Quantum Computing and Networking:** Quantum teleportation is a critical protocol for quantum repeaters, establishing long-distance quantum communication channels essential for quantum internet development.

2.2 Key Components of the Protocol

The standard quantum teleportation protocol involves three main components:

1. Entangled State Preparation

- Alice and Bob, share an entangled pair of qubits.
- The entangled state is typically represented as a Bell state:

$$\varphi^+ = \frac{1}{\sqrt{2}} (|00\rangle + |11\rangle)$$

- These qubits are prepared using the quantum gate such as the Hadamard gate followed by the CNOT gate.

2. Quantum Measurement and Bell State Analysis

- Alice possesses the qubit to be teleported ($|\varphi^+\rangle$) and one qubit from the entangle pair.
- Alice performs a Bell state measurement on her two qubits, collapsing the state into one of four possible Bell states.
- The measurement outcomes are represented as classical bits (00, 01, 10, 11), which encode the transformations required to recover the original state.

3. Classical Communication and State Reconstruction

- Alice transmits the measurement result (two classical bits) to Bob via a classical channel.
- Bob, using his entangled qubit, applies a corresponding quantum operation (I, X, Z, XZ) to retrieve the original state $|\varphi\rangle$.

2.3 Mathematical Representation of the Protocol

The process can be mathematically expressed as follows:

1. The initial state:

$$|\Psi\rangle \otimes \varphi^+ = \frac{1}{\sqrt{2}}(a|0\rangle + b|1\rangle) \otimes \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

2. After Alice's measurement, the system collapses to one of four Bell states, and the result determines the transformation Bob needs to apply.

3. Bob's operations (unitary transformations) are defined as:

- 00 \rightarrow I (Identity)
- 01 \rightarrow X (Bit Flip)
- 10 \rightarrow Z (Phase Flip)
- 11 \rightarrow XZ (Bit and Phase Flip)

4. Bob's qubit then becomes $|\Psi\rangle$, completing the teleportation.

2.4 Density Matrix Representation

The density matrix formalism is an indispensable tool in quantum mechanics, particularly for describing mixed quantum states and subsystems of entangled systems. A quantum state can be represented by a density matrix ρ , which encapsulates all measurable information about the state.

The concept of the density matrix was introduced in the early 20th century by John von Neumann as part of the mathematical foundation of quantum mechanics. It provides a means to describe quantum systems in statistical ensembles, which is crucial for both pure and mixed states.

- (a) **Historical Context:** The density matrix was developed to address limitations in describing statistical mixtures of quantum states using wavefunctions alone. It has since become essential for the analysis of quantum systems where subsystems interact with their environment.
- (b) **Mathematical Foundation:** The density matrix formalism is rooted in the operator theory of quantum mechanics. The operator encodes probabilities and coherences of quantum states, enabling a comprehensive representation of quantum systems.
- (c) **Physical Interpretation:** The diagonal elements represent the probabilities of the system being in specific states, while the off-diagonal elements capture quantum coherences and phase relationships. This dual capacity makes the density matrix suitable for analyzing phenomena like entanglement and decoherence.

Key properties of the density matrix include:

- (a) **Hermiticity:** $\rho = \rho^\dagger$
- (b) **Trace One:** $\text{Tr}(\rho) = 1$, ensuring proper normalization.
- (c) **Positivity:** All eigenvalues of ρ are non-negative.

Intersection of Quantum teleportation and Density matrices The interplay between quantum teleportation and the density matrix formalism lies in the rigorous description of quantum states during teleportation. The density matrix is crucial for:

- **Analyzing Entangled States:** Quantifying entanglement resources and diagnosing experimental imperfections.
- **State Fidelity Evaluation:** Comparing the teleported state's fidelity to the original, essential for validating teleportation protocols.

2.5 Limitation of the Conventional Protocol

While revolutionary, the protocol has limitations:

- (a) **Requirement for Classical Communication:** The need for classical communication means quantum teleportation cannot be instantaneous, as the classical information is subject to the speed of light or other constraints of classical communication channels. This introduces a delay in the teleportation process.
- (b) **No Faster-Than-Light Communication:** This constraint means that quantum teleportation does not violate relativity, as it cannot be used for superluminal (faster-than-light) communication. Classical communication is always the bottleneck.
- (c) **Loss of Quantum Advantages for Classical Data:** Quantum teleportation is not optimized or advantageous for sending classical data. Classical channels remain more practical and efficient for this purpose.
- (d) **Limited Information Capacity:** The protocol does not enhance the amount of classical information that can be sent. Instead, it focuses on transferring quantum states, which encode information differently from classical bits.

3 Design of the Protocol

Our protocol begins with the fact that Alice needs to communicate with Bob (far from the earth). Alice and Bob share an e-bit (entangled qubits). To achieve our goal we will use teleportation protocol.

$$|e\rangle = \frac{1}{\sqrt{2}}(|\psi\rangle + |11\rangle)$$

This is the quantum circuit diagram



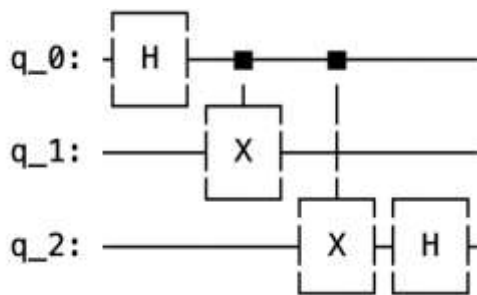


Figure 1: Quantum Circuit Diagram

Here q0 represents Alice's qubit, q1 represents Bob's qubit and q2 is the additional qubit entangled with Alice.

- In the first step we will create a bell pair by first applying the Hadamard gate to Alice's qubit and then applying the Controlled-Not operation where Alice's qubit is the control bit and Bob's qubit is the targeted qubit.
- Then we will create the entanglement between Alice's qubit and an additional qubit C, by first applying the Control-Not operation where the additional qubit is the control qubit and Alice's qubit is the target qubit.
- Bob first calculates the density matrix and then applies the ppt criteria to check the entanglement between Alice's qubit and additional qubit. If entanglement is present, he will get 1; if not entangled, he gets 0.

This is the complete description of our quantum teleportation protocol. Here we seek to improve the existing quantum teleportation to reduce the need for classical communication with the help of density matrices and PPT criteria.

4 Quantum Teleportation Implementation

We will describe the steps of the quantum circuit below.

4.1 Setting up the circuit:

4.1.1 Imports and Authentication:

We begin with loading the Qiskit Simulator (IBM) and provide authentication information as below: We also use the additional and necessary imports as follows (We are going to use both **qiskit** and **numpy** imports).

```
1 from qiskit import QuantumCircuit, transpile
2 from qiskit_aer import AerSimulator
3 from qiskit.quantum_info import partial_trace, DensityMatrix
4 from qiskit_experiments.library import StateTomography
5 import numpy as np
6 from qiskit_experiments.framework import ExperimentData
7 from scipy.linalg import eigh
8 from scipy.linalg import sqrtm
```

and create a quantum circuit using **3 qubits**. We first have to create a bell pair. In Quantum circuit language, the way to create a bell pair between two qubits is first to transfer one of them to X basis using the **Hadamard gate**,

and then apply the **CNOT** gate on the other qubit controlled by the one in the X-basis.

```

1      # Create an entangled pair
2      qc = QuantumCircuit(3)
3      qc.h(0)
4      qc.cx(0, 1)
5      qc.cx(0, 2)
6      qc.h(2)
7      print(qc.draw())

```

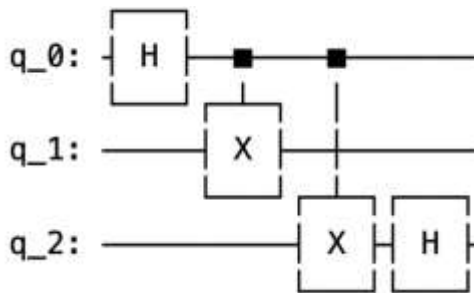


Figure 2: Bell pair

Let's assume that **Alice** has got A and **Bob** got B before they depart from each other. B applies a state tomography function that will reduce it to the density matrix, and will help Bob to know whether entanglement is present between A and C

```

1      def tomographyfunc(qc):
2          tomo = StateTomography(qc)
3          Aer = AerSimulator()
4          exp_data: ExperimentData = tomo.run(Aer).block_for_results()
5          rho = exp_data.analysis_results(0).value.data # Convert to numpy
6              array
7          reduced_rho_AB = partial_trace(rho, [2]).data # Trace out qubit C
8          reduced_rho_AC = partial_trace(rho, [1]).data # Trace out qubit B
9          reduced_rho_BC = partial_trace(rho, [0]).data # Trace out qubit A
10         is_entangled, pt_eigenvalues = check_ppt_entanglement(reduced_rho_AC)
11         # print("PPT Criterion:")
12         # print("Partially Transposed Eigenvalues:", pt_eigenvalues)
13         for item in pt_eigenvalues:
14             if item < 0:
15                 return 1
16         else:
17             return 0

```

To check the entanglement between A and C, Bob uses the **Positive Partial Transpose criteria**, which determines whether a given quantum is separable (not entangled) or entangled. The PPT function will take the density matrix and compute the partial transpose of a system with respect to the second system. Then it examines the eigenvalues, if all eigenvalues are **non-negative**, the state satisfies the PPT criteria and is separable. If the subsystem has negative eigenvalues, the state is **entangled**.


```
1 def check_ppt_entanglement(rho):
2     """
3     Check entanglement using the ppt (positive Partial Transpose)
4     Args:
5         rho: The density matrix.
6     Return :
7         True if entangled, False otherwise.
8     """
9
10    if rho.shape[0] != rho.shape[1]:
11        raise ValueError("Density matrix must be square.")
12    pt_rho = partial_transpose(rho, [2,2], 1) # Parial transpose with
13    respect to the second system
14
15    if pt_rho.shape[0] != pt_rho.shape[1]:
16        raise ValueError("Partial transpose resulted in a non square
17    matrix.")
18
19    # Compute eigenvalue of the partially transpose matrix
20    eigenvalues = np.linalg.eigvalsh(pt_rho)
21    return np.any(eigenvalues < 0), eigenvalues
```



4.2 Testing the protocol in the Qiskit Simulator:

In this test, we will use the histocount function that will take the quantum circuit as an argument. histocount function apply the tomography function to our circuit, and run this 100 times. This function returns us a histogram that will show the frequency of entanglement and separability.

```

1  def histocount(qc):
2      results = [tomographyfunc(qc) for _ in range(100)]
3      plt.hist(results, bins=2, align="mid", rwidth=0.5)
4      plt.xticks([0,1], ["Not Entangled", "Entangled"])
5      plt.xlabel("Entanglement Status")
6      plt.ylabel("Frequency")
7      plt.title("Entanglement detection in 100 runs")
8      plt.show()
9  histocount(qc)

```

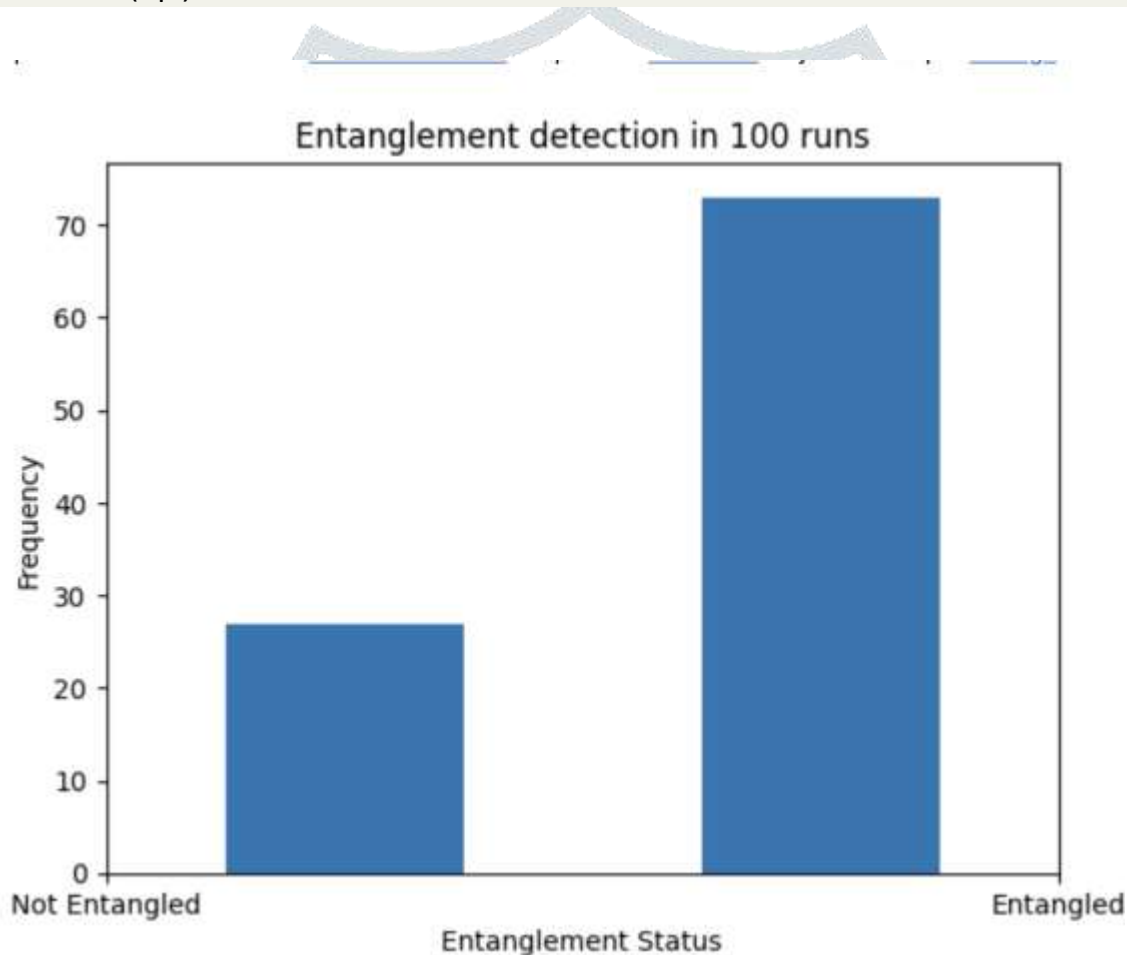


Figure 3: histogram results

As we see here, there are a few results in which we measure 0 instead of 1. This arises due to the Precision issues that are in near-threshold cases (e.g., very small negative eigenvalues close to

zero), numerical precision may impact the result. Secondly, The computational methods to test PPT rely on numerical approximations for the eigenvalues of the partial transpose. Small errors or tolerance of the computation might lead to misclassification.

5 Future Scope and Conclusion

As we see it's possible to transform information without transforming the actual information. This is the first step toward a completely different future. We can use an 8 qubit system and with the help of quantum protocol, we can make it use the like the classical communication. Although this is highly speculative and far-future, the principles of quantum teleportation could inspire trans- forming matter, although the energy and technical challenges are currently insurmountable.

5.1 Future Scope:

Its future scope spans across several transformative applications:

- **Space Exploration:** Quantum teleportation could enable instantaneous communication across astronomical distances, which would be a game-changer for deep-space missions and communication with interstellar probes.
- **Quantum Communication Network:** Quantum teleportation is foundational to the concept of a quantum internet, where qubits are transformed over large distances without physical transfer. This could lead to secure, instantaneous communication networks that are immune to eavesdropping due to the principles of quantum mechanics.
- **Distributed Quantum Computing:** Quantum teleportation allows the sharing of quantum states between distant quantum computers, enabling distributed quantum computing. This can potentially combine the power of multiple quantum processors to solve problems more efficiently.
- **Quantum Cryptography:** Beyond QKD, quantum teleportation may lead to the development of new cryptographic protocols that rely on quantum entanglement and superposition, offering even more robust security mechanisms.
- **Fundamental Science:** Continued experiments with quantum teleportation provide a way to test the boundaries of quantum mechanics and explore phenomena such as quantum entanglement and superposition at a larger scale. It also contributes to a deeper understanding of the foundation of quantum theory, potentially leading to new physics beyond the standard model.

5.2 Conclusion

In this paper, we proposed and analyzed a modified quantum teleportation protocol aimed at addressing reducing the need for classical communication. This modification is introduced in the protocol, such as density matrix and PPT criteria, demonstrating how to achieve quantum teleportation without the need for classical communication.

These findings contribute to the ongoing efforts to make quantum communication systems more reliable and efficient. However, challenges such as Fidelity and Noise, Bell state measurement, Quantum Decoherence, Synchronization, Scalability, etc remain and warrant further investigation. In future work, the proposed protocol can be further refined by incorporating a multi-qubit system to synchronize it with classical communication.

Additionally, exploring its performance in

multi-qubit could pave the way for its application in more complex quantum systems.

In conclusion, the modified quantum teleportation protocol represents a significant step toward realizing robust and scalable quantum communication, with potential implications for advancing quantum technologies and practical quantum internet development.

References

- [1] P. Grangier A. Aspect and G. Roger. Experimental tests of realistic local theories via bell's theorem. *Physical review letters* vol. 47, n. 7, p. 460, 1981.
- [2] B. Podolsky A. Einstein and N. Rosen. Can quantum-mechanical description of physical reality be considered complete. *Physical review*, vol. 47, n. 10, p. 777, 1935.
- [3] F. Arute, K. Arya, R. Babbush, D. Bacon, J.C. Bardin, R. Barends, R. Biswas, S. Boixo, F.G. Brandao, D.A. Buell, et al. Quantum supremacy using a programmable superconducting processor. *Nature*, 575(7779):505–510, 2019.
- [4] E. Bleuler and H. Bradt. Correlation between the states of polarization of the two quanta of annihilation radiation. *Physical review*, vol. 73, n. 11, p. 1398, 1948.
- [5] E. R. Jefferey J. B. Altepeter and P. G. Kwiat. Photonic state tomography. *Dept. of Physics, University of Illinois at Urbana-Champaign, Urbana IL 61801*.
- [6] E.P. Wigner J. Naumann and R. Hofstadter. Mathematical foundations of quantum mechanics. *Princeton University Press*, 1955.
- [7] M. Nielsen and I. Chuang. *Quantum Computation and Quantum Information*. Cambridge Series on Information and Natural Sciences (Cambridge University Press, Cambridge, 2000).
- [8] S. Prabhakar, T. Shield, A.C. Dada, M. Ebrahim, G.G. Taylor, D. Morozov, K. Erotikriou, S. Miki, Y. Yabuno, H. Terai, et al. Two photon quantum interference and entanglement at 2.1 um. *Science Advances*, 6(13), 2020.
- [9] R. Courtland. Google plans to demonstrate the supremacy of quantum computing. *IEEE Spectrum*, 2017.
- [10] Eleanor Rieffel and Wolfgang Polak. *Quantum Computing: a Gentle Introduction*. (The MIT Press Cambridge England, Massachusetts, 2011).
- [11] John Watrous. Basics of quantum information. *IBM Quantum Learning*, 2024.
- [12] C.S. Wu and I. Shaknov. The angular correlation of scattered annihilation radiation. *Physical review* vol. 77, n. 1, p. 136, 1950.