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Significant Contributions of Bhaskara II in Indian Knowledge System

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Abstract

Bhaskara II, also known as Bhaskaracharya, was a renowned Indian mathematician and astronomer who made significant contributions to the fields of mathematics and astronomy in the 12^{th} century. His works have played a critical role in the development of Indian Knowledge System (IKS), influencing both Indian and global scientific thought. His contributions are found primarily in his two major texts: "Lilavati" (a treatise on arithmetic and geometry) and "Bijaganita" (a treatise on algebra), along with "Siddhanta Shiromani" (a comprehensive work on astronomy). His work not only advanced scientific thought in his own time but also laid the foundation for future generations of scholars in India and beyond. We study the specific ways Bhaskara II influenced the Indian knowledge system in Algebra.

I. Introduction to Indian Knowledge System

Indian knowledge system is an ancient and rich collection of beliefs, practices, and philosophies that have been passed down from generation to generation in India. It encompasses various fields such as science, spirituality, art, literature, and social norms, and has played a significant role in shaping Indian society and culture. The foundation of the Indian knowledge system lies in the ancient texts of the Vedas, which are considered to be the oldest scriptures in the world. The Vedas contain a vast amount of knowledge on subjects ranging from medicine, astronomy, mathematics, and politics, to spirituality and philosophy. They provide insights into the Indian way of life, highlighting the importance of balance, harmony, and unity in society [1,2].

One of the key aspects of the Indian knowledge system is its holistic approach towards life. It acknowledges the interconnectedness of all aspects of existence from the individual to society, from human beings to nature, and from the physical to the spiritual. This holistic approach is reflected in various Indian practices, such as Ayurveda, Yoga, and Vastu Shastra, which focus on maintaining balance and harmony within and with the environment. Another significant aspect of the Indian knowledge system is its emphasis on seeking knowledge through observation and personal experience. This approach is reflected in the teachings of ancient Indian sages and philosophers, who encouraged critical thinking and self-reflection as a means to gain wisdom and understanding. It also highlights the value of oral tradition, with knowledge being passed down through storytelling, discussions, and debates [5,6].

II. Historical Development of IKS

Bhaskara II's contributions laid the foundation for further developments in the fields like algebra, trigonometry, and astronomy. His innovative methods and insights influenced later mathematicians and astronomers in India and beyond. While he is not as widely known in the Western world as figures like Euclid or Ptolemy. Bhaskara II's profound contributions remain a significant part of the rich intellectual heritage of India. His work, particularly in the realms of algebra and astronomy, had far-reaching consequences for the advancement of global scientific knowledge [4,5].

The Indian knowledge system, also known as the Indian school of thought or Hindu philosophy, refers to the vast body of knowledge, beliefs and practices that have been developed and passed down from ancient times in the Indian subcontinent. This knowledge system is deeply rooted in the ancient Vedic scriptures and has evolved over thousands of years, shaping the cultural, intellectual and spiritual landscape of India.

The origins of the Indian knowledge system can be traced back to the ancient Vedic period, which began around 1500 BCE. The Vedas, the oldest scriptures of Hinduism, were composed during this time and are considered the foundation of Indian knowledge and philosophy. The Vedas are a collection of hymns, rituals, and mantras that were transmitted orally from generation to generation for centuries before being written down. They contain a wide range of knowledge regarding rituals, sacrifices, cosmology, ethics, and spirituality.

The early Vedic period was followed by the emergence of the Upanishads, which are philosophical texts that expound upon the deeper meaning and significance of the Vedas. The Upanishads introduced the concept of self-realization, or the realization of one's true nature as being intertwined with the divine and the universe. They also laid the foundation for the concept of karma, the law of cause and effect that governs the cycle of birth, death, and rebirth.

Around 500 BCE, the period of the Upanishads gave way to the emergence of several schools of thought in India, each with its own interpretations and philosophical systems. These include Vedanta, Samkhya, Yoga, and Jainism, among others. These schools of thought brought new developments and interpretations to the original Vedic teachings, leading to a diversification of Indian knowledge and philosophies.

One of the most influential schools of thought was Vedanta, which emerged in the post-Vedic period and focused on the concepts of self-realization and the ultimate reality. It emphasized the non-dual nature of the universe and the belief in the existence of a universal consciousness. Another important school of thought, Buddhism, emerged in the 6th century BCE and spread throughout Asia, greatly influencing Indian knowledge and philosophy.

With the emergence of Buddhism and Jainism, the caste system, which had been prevalent in ancient India, began to weaken. These new religions challenged the traditional Brahmanical order and brought about significant social and religious reforms. As a result, the caste system gradually transformed into a class system, opening up opportunities for people from lower castes to access education and knowledge [2,3].

In the medieval period, with the rise of Islamic invasions and the arrival of European colonial powers, Indian knowledge faced challenges and underwent significant changes. The Islamic rulers brought their own traditions and practices, and the European colonizers introduced Western education and ideas, leading to a fusion of Indian and Western philosophies. Even with these changes, the Indian knowledge system continued to thrive and adapt. The Bhakti movement of the 15th and 16th centuries brought an emphasis on devotion and love for the divine, while the rise of Sikhism in the late 15th century synthesized elements of Hinduism and Islam.

III. Contributions of Bhaskara II in Algebra

Bhaskara II's works are also known for their emphasis on practical problem-solving. He often framed mathematical problems with real-world contexts, such as trade, astronomy, and construction, making his ideas accessible to a wide range of audiences, not just scholars. He used poetic forms (verses) to express complex mathematical concepts, which made learning more engaging and memorable. Bhaskara II is known for his significant advancements in algebra. In *Bijaganita*, he systematically described methods to solve quadratic, cubic, and even higher degree equations. He provided solutions for indeterminate equations (now called Diophantine equations), where the solution could be in integers, which was an important precursor to later developments in number theory

a. On Linear Equation in more than two unknowns

To solve a linear equation involving more than two unknowns the usual Hindu method is to assume arbitrary values for all the unknowns except two and then to apply the method of the pulveriser. Thus ,Brahmagupta remarks, "The method of the pulveriser (should be employed), if there be present many unknowns (in an equation)." Similar directions have been given by Bhaskara II and others [1,6].

One of the astronomical problems proposed by Brahmagupta leads to the equation

$197x - 1644y - z = 6302.$

Hence

$x = (1644y + z + 6502)/197$

The commentator assumes $z = 131$ then

$x = (1644y + 6453)/197$

Hence the rule of Bhaskara II

"Subtract the resulting *varna* from the higher *varna* and diminish it by the lower *varna*, the remainders

multiplied by an optional number will be the weights of gold of the lower and higher *varna* respectively."

b. Simultaneous Indeterminate Equations of the First Degree

Bhaskara II's Rule[3,6]. He writes:

"Remove the first unknown from the second side of an equation and the others as well as the absolute number from the first side. Then on dividing the second side by the coefficient of the first unknown, its value will be obtained. If there be found in this way several values of the same unknown, from them, after reduction to a common denominator and then dropping it, values of another unknown should be determined. In the final stage of this process, the multiplier and quotient obtained by the method of the pulveriser will be the values of the unknowns associated with the dividend and the divisor (respectively). If there be several unknowns in the dividend, their values should be determined after assuming values of all but one arbitrarily. Substituting these values and proceeding reversely, the values of the other unknowns can be obtained. If on so doing there results a fractional value (at any stage), the method of the pulveriser should be employed again.

Then determining the (integral) values of the latter unknowns accordingly and substituting them, the values of the former unknowns should be found proceeding reversely again.

Example (from Bhaskara II): (*Four merchants), who have horses 5, 3, 6 and 8 respectively; camels 2, 7, 4 and 3 ; whose mules are 8,2,1 and 3 ; and oxen 7, 1, 2 and 1 in number ; are all owners of equal wealth. Tell me instantly the price of a horse, etc.*

If x, y, z, w denote respectively the prices of a horse ,a camel, a mule and an ox, and W be the total wealth of each merchant, we have

$5x + 2y + 8z + 7w = W$ (1)

$3x + 7y+2z+w= W$ (2)

$6x+ 4y + z + 2w = W$ (3)

$8x+y+ 3z + w=W$ (4)

Then

$x = (5y-6z-6w)/2$ from (1) and (2)

$= (3y+z-w)/3$ from (2) and (3)

$= (3y-2z+w)/2$ from (3) and (4)

From the first and second values of x we get

$y=(20z+16w)/9$

and from the second and third values, we have

$y=(8z-5w)/3$

Equating these two values of y and simplifying,

$20z+16w=24z-15w$

Therefore

$z=31w/4$, Take $w=4t$;then $z=31t$, $y=76t$, $x= 85 t$.

Special Rules: Bhaskara II observes that the physical conditions of problems may sometimes be such that the ordinary method of solving simultaneous indeterminate equations of the first degree, which has been just explained, will fail to give the desired result.

One such problem has been described by him as follows

"*Tell quickly, O algebraist, what number is that which multiplied by 23 and severally divided by 60*

and 80 leaves remainders whose sum is 100."

Let the number be denoted by x ; the quotients by u, v ; and the remainders by s, t. Then we have

$(23x-s)/60 = u$, $(23x-t)/80= v$; $s + t=100$.Therefore,

$x=(60u+s)/23 = (80v+t)/23$

Hence, $x=(60u +80 v + s + t)/46$

or $x=(30u+40v+50)/23$

For the solution of the above he observes:

"Here, (although) there is more than one quotient (u, v) in the dividend, the value of any should not be arbitrarily assumed ; for on so doing the process will fail." "In a case like this," continues he, "the (given) sum of the remainders should be so broken up that each remainder will be less than the divisor corresponding to it and further that impossibility will not arise ; then must be applied the usual method."

In the present example we thus suppose s = 40, t = 60. Hence we have

$(60 u + 40= 80 v + 60)$

$u= (80v+20)/60 = (4v +1)/3$;

Whence by the method of the pulveriser, we get

$v=3w+2$, $u=4w+3$. Therefore $x=(240w+220)/23$.

Again, applying the method of the pulveriser in order to obtain an integral value of x, we have

$w=23m+1$, $x=240m+20$,

If we take $s=30$, $t=70$, we shall find, by proceeding in the same way, another value of x as $240m+90$

IV. Conclusion

Bhaskara II was a brilliant polymath whose groundbreaking contributions have left a lasting imprint on both Indian and global knowledge systems. His innovative work in mathematics and astronomy is still celebrated and continues to influence modern scientific thought. His contributions to the Indian knowledge system are profound and varied, especially in the realms of mathematics

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