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FIBONACCI SEQUENCE: ROLE OF INDIANS, MATHEMATICAL PROPERTIES , PRESENCE IN NATURE

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Abstract : The Fibonacci series is one of the most fascinated sequence of numbers among the Mathematicians, Artists and scholars. The aim of this research is to explain this wonderful sequence and correlate with nature, art and architecture with the help of golden ratio. This sequence also has an Indian aspect which impressed Fibonacci in discovering this sequence. Next this paper explores various mathematical properties and similar sequences and their sum with help of various papers

***Keywords*:** Fibonacci , golden ratio, square root of Fibonacci series, sum of reverse Fibonacci

1. Introduction

The great Italian mathematician "Leonardo Bonacci" popularly known as "Fibonacci" has given the famous Fibonacci sequence. In his book the Liber Abaci, he solved a problem involving the growth of a population of Rabbits with the help of a sequence of numbers later known as Fibonacci numbers.[1,2,7] The Fibonacci sequence is a series in which each element is the sum of the previous elements. Each number is denoted by F_n. The sequence starts with numbers 0 & 1.

$$F_n = F_{n-1} + F_{n-2} \quad ; \quad F_1=0,\ F_2=1 \tag{1}$$

Table 1Fibonacci Spiral

N	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
F_n	0	1	1	2	3	5	8	13	21	34	55	89	144	233	377	610	987	1597

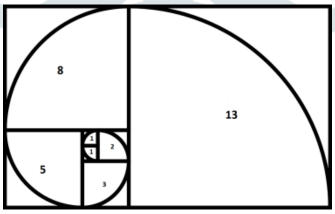

Figure 1

Golden ratio:- We can explain the golden ratio by referring two positive numbers a & b such that b>a. If the ratio between larger number and smaller number is same as the ration between their sum and larger number[5,8] i.e.

$$\frac{b}{a} = \frac{a+b}{b}$$
$$\frac{b}{a} = \frac{a}{b} + \frac{b}{b}$$

$$\frac{b}{a} = \frac{1}{b/a} + 1 \tag{2}$$

Here $\frac{b}{a} = \emptyset$\{ Greek letter Phi\} is known as golden ratio, now equation(2) becomes

$$\emptyset = \frac{1}{\emptyset} + 1$$
$$\emptyset^2 = 1 + \emptyset$$

$$\emptyset^2 - \emptyset - 1 = 0$$

By using quadratic formula

$$\emptyset = \frac{\sqrt{5}+1}{2} \approx 1.618$$

Relation between Fibonacci series & golden ratio:-

The relation between Fibonacci number is given by

$$F_n = F_{n-1} + F_{n-2}$$

Dividing by F_{n-1}

$$\frac{F_n}{F_{n-1}} = 1 + \frac{F_{n-2}}{F_{n-1}} \quad (3)$$

We assume that as number of Fibonacci terms approaches the limit $n \to \infty$ then

$$\lim_{n\to\infty} \frac{F_n}{F_{n-1}} = k \;\&\; \lim_{n\to\infty} \frac{F_{n-2}}{F_{n-1}} = \frac{1}{k}$$

Then equation(3) can be represented as

$$k = 1 + \frac{1}{k}$$

$$k^2 - k - 1 = 0 \quad (4)$$

Equation (4) has same solution as golden ratio

Here we can conclude that as number of Fibonacci series increases then Ratio of each term with its preceded term approaches to golden ratio.[9]

Indian aspects of Fibonacci sequences:- Fibonacci introduced "modus Indorum"(method of Indians), Today known as "Hindu Arabic numeral system" with ten digits including a zero and positional notation in his book Liber Abaci. He was very impressed with Indian mathematicians work.

Knowledge of Fibonacci sequence was expressed as early as Pingala(450 BC-200BC). Bharat Muni also expressed knowledge of sequence in NatyaShastra(100 BC-350 AD). However the clearest exposition of sequence arise in the work of Viranhanka(700 AD), whose own work is lost, but is available in a quotation by Gopala(1135AD).

Hemachandra is credited with knowledge of sequence as well, writing that " the sum of last and one before last is the number of next matra-vratta.[1]

2. MATHEMATICAL PROPERTIES OF FIBONACCI SEQUENCE

2.1 Sum of Fibonacci numbers squared [4]

Table 2 Squared Fibonacci

F_n	0	1	1	2	3	5	8	13	21	34	55	89	144	233
F_n^2	0	1	1	4	9	25	64	169	441	1156	3025	7921	20736	54289

$$1^2 + 1^2 = 1 \times 2$$

$$1^2 + 1^2 + 2^2 = 2 \times 3$$

$$1^2 + 1^2 + 2^2 + 3^2 = 3 \times 5$$

$$1^2 + 1^2 + 2^2 + 3^2 + \cdots \ldots \ldots + {F_n}^2 = F_n \times F_{n+1}$$

$$\sum_{i=1}^{n} {F_i}^2 = F_n \times F_{n+1}$$

Proof by geometry

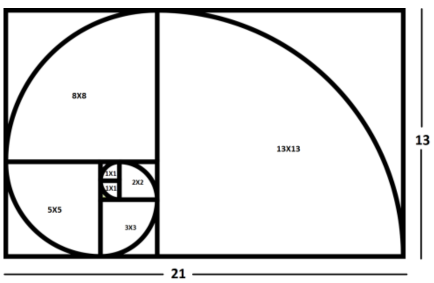

Figure 2

2.2 Even numbers of Fibonacci Series

0 , 1,1, **2** ,3,5, **8** ,13,21, **34** ,55,89, **144** ,233,377, **610** ,987,1597, **2584**

2 X 4+ 0 = 8

8 X 4 + 2 =34

34 X4 + 8 =144

144 X 4 + 144 = 610

610 X 4 + 610 = 2584

$[\ F_{3n+1} \text{ X } 4 + F_{3n-2} = F_{3n+4}\ ]\ \ n\epsilon N$

2.3 Sum of reverse Fibonacci with golden ratio

Let a series which follows golden ratio $1, \emptyset, \emptyset^2, \emptyset^3, \dots \dots \dots ..$

If we reciprocate this series, we get previous terms of '1'

$$1, \frac{1}{\emptyset}, \frac{1}{\emptyset^2}, \frac{1}{\emptyset^3}, \dots \dots \dots.$$

$$\text{Sum } S_\infty = \frac{1}{1 - {}^1/_\emptyset}$$

$$S_\infty = \frac{\emptyset}{\emptyset - 1}$$

$\{\emptyset^2 = \emptyset + 1$

$\emptyset^2 - 1 = \emptyset$

$\emptyset - 1 = \frac{\emptyset}{\emptyset+1}\}$

$$S_\infty = \frac{\emptyset}{{}^{\emptyset}/_{(\emptyset+1)}}$$

$$S_\infty = \emptyset + 1 \quad \approx 1.618 + 1$$

$$[\ \boldsymbol{S_\infty} \approx 2.618\ ]$$

2.4 Fibonacci with Square root

Fibonacci series $\mathbf{F_n = F_{n-1} + F_{n-2}}$

Table 3 Square root Fibonacci

F_n	0	1	1	2	3	5	8	13	21	34	55	89	144	233
$f_n = \sqrt{Fn}$	0	1	1	$\sqrt{2}$	$\sqrt{3}$	$\sqrt{5}$	$\sqrt{8}$	$\sqrt{13}$	$\sqrt{21}$	$\sqrt{34}$	$\sqrt{55}$	$\sqrt{89}$	$\sqrt{144}$	$\sqrt{233}$

$$\boldsymbol{f^2{}_n = f^2{}_{n-1} + f^2{}_{n-2}}$$

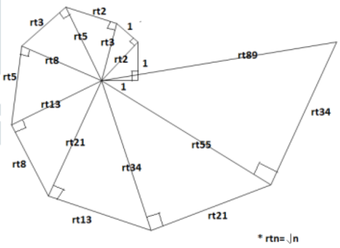

Figure 3

3. PRESENCE OF FIBONACCI SERIES AROUND US

3.1 Nature

3.1.1 Plants and Flowers: We commonly identify Fibonacci numbers in leaf arrangement on plants such as Pear tree, cherry tree[4]. Many flowers follow Fibonacci sequence in Petal arrangement. Example **1 petal:** White lily, **3 petals:** Lily, iris, **5 petals:** buttercup, wild rose,**8 petals:** delphiniums,**13 petals:** corn marigold,**21 petals:** aster, **34 petals:** plantain [6]

Spiral pattern of flowers also can be seen on the surface of some flowers. Where alternative spirals goes clockwise and counterclockwise such as sunflower. Pinecones are the most popular example of Fibonacci series. Where three and five or eight and thirteen steep spirals.

3.1.2 Animals and other living organism: Spiral can be seen in the nature at various places. One of the most popular golden spiral found in the shell of the chambered nautilus, snail shell, Elephant tusk, Bighorn sheep.[4]Human body also shows Fibonacci pattern. The bones of finger spiral pattern of fingerprint, human ear form a golden spiral. The human face which follows golden ratio is more beautiful & attractive. Fibonacci discovered the series by studying the exponents & rabbit breeding.[10]

3.1.3 Architecture: In Europe, many historic monuments and building follows Fibonacci sequence and golden ratio. The great Pyramid at Giza is one of earliest example. Let base of a triangle y which goes from mid-point of a side of Pyramid to the center of the square base. Let x be the diagonal up the side of the pyramid from the same mid-point of the side to very top of pyramid. Approximate lengths of x & y are 612 feet and 377.9 feet respectively.

x/y = 612/377.9 ≈ **1.62** which is very close to golden ratio.[4]

Other popular examples are Parthenon of ancient Greece, Florence Cathedral, Tajmahal in India.

3.1.4 Art: Golden ratio is popular in the field of art. It is commonly used by various artist in their art. Famous painting of Monalisa, middle ages paintings of Modonna, Indian statues of Buddha, the famous scholar Leonardo da vinci, utilized the golden ratio in majorityof his work. In his well known sketch of vitruvian man, the ratio of the side of the square which corresponds to man's arm span and height to the radius of circle which contains his outstretched arms and legs in the golden ratio.[3,4]

4. CONCLUSION

The greatest mathematician Fibonacci has given us one of the very fascinating and interesting sequence. He has also acknowledged Indian mathematicians work by showing their work in his book. In god's world, there are many things which follows these series and golden ratio. Some mathematician worked on this series and tried to make it more interesting but series has amazing results and properties which are still unlocked.

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