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Multiplicative Neutrosophic Expansion Map and Fixed point Result for Multiplicative Neutrosophic Expansion on MNMS

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Abstract: Kumar M. [33] has introduced the concept of neutrosophic expansion and proved a fixed point result for neutrosophic expansion. This result is a platform for new concept of neutrosophic expansion map on neutrosophic metric space (NMS) as Kirişci M., Simsek N., Akyigit M. [21] have established fixed point result platform for neutrosophic Banach contraction. Also Kumar M. [32] put the notion of multiplicative neutrosophic metric space (MNMS) and multiplicative neutrosophic contraction on MNMS. The aim of this paper is to establish the concept of multiplicative neutrosophic expansion on MNMS and proved a fixed point result for multiplicative neutrosophic expansion on MNMS.

Key Words: Fixed Point, Neutrosophic Contraction, Generalized Neutrosophic Contraction, Neutrosophic Metric Space, multiplicative neutrosophic metric space, multiplicative neutrosophic contraction, multiplicative neutrosophic expansion.

Introduction:

The concept of Fuzzy Sets introduced by Zadeh [1] has attracted all the scientific fields since its starting. It is seen that this concept remained failed for real-life situations, to provide enough solution to some problems in time.Atanassov [2] put the idea of Intuitionistic fuzzy sets for such cases. Neutrosophic set (NS) is a new version of the idea of the classical set which is defined by Smarandache [3]. Some of other generalizations are FS [1] interval-valued FS [4], IFS [2], interval-valued IFS [5], the sets paraconsistent, dialetheist and tautological [6], Pythagorean fuzzy sets [7].
Combining the concepts Probabilistic metric space and fuzziness, fuzzy metric space (FMS) is introduced in [8]. Kaleva and Seikkala [9] have defined the fuzz metric as the nearness between two points with respect to a real number to be a non-negative fuzzy number. In [10] some basic properties of FMS studied and the Baire Category Theorem for FMS proved. Further, some properties of metric structure like separability, countabilityetc are given and Uniform Limit Theorem is proved in [11]. Afterward, FMS has used in the applied sciences such as fixed point theory, image and signal processing, medical imaging, decision-making et al. After itroduction of the intuitionistic fuzzy set (IFS), it was used in all areas where FS theory was studied. Park [12] defined IF metric space (IFMS), which is a generalization of FMSs. Park used George and Veeramani's [10] idea of applying t-norm and t-conorm to the FMS meanwhile defining IFMS and studying its basic features. Fixed point theorem for fuzzy contraction mappings is initiated by Heilpern [13]. Bose and Sahani [14] extended the Heilpern's study. Alaca et al. [15] are given fixed point theorems related to intuitionistic fuzzy metric spaces(IFMSs). Fixed point results for fuzzy metric spaces and IFMSs are studied by many researchers [16], [17], [18], [19], [20].

Kirisci et al. [21, 23] defined neutrosophic contractive mapping and gave a fixed point results in complete neutrosophic metric spaces. In [22], Mohamad studied fixed point aprroach in intuitionistic fuzzy metric spaces. Kumar M [32] introduced the notion of Multiplicative Neutrosophic Metric Space (MNMS) and define topology on it and at last having defined multiplicative neutrosophic contraction and proved some fixed point result on MNMS. Kumar M[33] put the concept of neutrosophic expansion on NMS.
In this paper first we introduce the concept of multiplicative neutrosophic expansion (MNE) and then prove a fixed point result for MNE on MNMS.

Preliminaries: Triangular norms (t-norms) (TN) were initiated by Menger [27]. In the problem of computing the distance between two elements in space, Menger offered using probability distributions instead of using numbers of distance. TNs are used to generalize with the probability distribution of triangle inequality in metric space conditions. Triangular conforms (t-conorms) (TC) know as dual operations of TNs. TNs and TCs are very significant for fuzzy operations (intersections and unions).

Definition 2.1. Give an operation $\odot$:[0,1]×[0,1]→[0,1]. If the operation $\odot$ is satisfying the following conditions, then it is called that the operation $\odot$ is continuous TN (CTN): For s,t,u,∈[0,1],
i) $s\odot 1=s$,
ii) If $s\leq u$ and $t\leq v$, than $s\odot t\leq u\odot v$,
iii)$\odot$ is commutative and associate,
iv)$\odot$ is continuous.

Definition 2.2. Give an operation $\boxdot$:[0,1]×[0,1]→[0,1]. If the operation $\boxdot$ is satisfying the following conditions, then it is called that the operation $\boxdot$ is continuous TC (CTC):
i)$\boxdot 0=s$,
ii) If $s\leq u$ and $t\leq v$, than $s\boxdot t\leq u\boxdot v$,
iii)$\boxdot$ is commutative and associate,
iv) $\boxdot$ is continuous.

Remark 2.3.[23] Take $\odot$ and $\boxdot$ are CTN and CTC, respectively. For s,t,v,∈[0,1],
a. If $s>t$, then there are u, such that $s\odot u\geq t$ and $s\geq t\boxdot v$.
b. There are p, such that $t\odot t\geq s$ and $s\geq p\boxdot p$.

Definition 2.4.[32] Take F be an arbitrary set, $\Omega=\{\langle a,HU(a), MU(a),SU(a)\rangle:a\in F\}$ be a NS such that Ω:$F\times F\times\mathbb{R}+\to[0,1]$. Let $\odot$ and $\boxdot$ show the CTN and CTC, respectively. The four tuple $V=(F,\Omega,\odot,\boxdot)$ is called Multiplicative Neutrosophic Metric Space(MNMS) when the following conditions are satisfied. $\forall a,b,c\in F$,
1. $0\leq H(a,b,\lambda)\leq 1$, $0\leq M(a,b,\lambda)\leq 1$, $0\leq S(a,b,\lambda)\leq 1\,\forall\lambda\in\mathbb{R}^+$,
2. $H(a,b,\lambda)+M(a,b,\lambda)+S(a,b,\lambda)\leq 3$, $(for\lambda\in\mathbb{R}+)$,
3. H$(a$,b,$\lambda)$ =1 $(for\lambda>0)$ $ifandonlyifa=b$,
iv) H$(a$,b,$\lambda)$=$H(b,a,\lambda)$ $(for\lambda>0)$,
4. $H(a,b,\lambda)\odot H(b,c,\mu)\leq H(a,c,\lambda\mu)$ $(for\lambda,\mu>1)$,
vi) H$(a$,b,.):[0,∞)→[0,1] is continuous,
vii) lim$\lambda\to\infty$ H(a,b,λ) =1 $(\forall\lambda>0)$,
viii) M$(a$,b,$\lambda)$=0 $(for\lambda>0)$ $ifandonlyifa=b$,
ix) M$(a$,b,$\lambda)$ =$M(b,a,\lambda)$ $(for\lambda>0)$,
x) $M(a,b,\lambda)\boxdot M(b,c,\mu)\geq M(a,c,\lambda\mu)$ $(for\lambda,\mu>1)$
xi) M$(a$,b,.):[0,∞)→[0,1] is continuous,
xii) lim$\lambda\to\infty$M(a,b,λ)=0 $(\forall\lambda>0)$,
xiii) S$(a$,b,$\lambda)$=0 $(for\lambda>0)$ $ifandonlyifa=b$
xiv) S$(a$,b,$\lambda)$ =$S(b,a,\lambda)$ $(for\lambda>0)$,
xv) $S(a,b,\lambda)\boxdot S(b,c,\mu)\geq S(a,c,\lambda\mu)$ $(for\lambda,\mu>1)$,
xvi) S$(a$,b,.) :[0,∞)→[0,1] is continuous,

xvii) $\lim_{\lambda\to\infty}$ S(a,b,λ) =0 ($\forall\lambda$>0)

xviii) If $\lambda\leq 0$, then $H(a,b,\lambda)=0, M(a,b,\lambda)=1, S(a,b,\lambda)=1$.

Then Ω =(H,M,S) is called MultiplicativeNeutrosophic Metric (MNM) on F.

The functions $H(a,b,\lambda)$,$M(a,b,\lambda)$, $S(a,b,\lambda)$ denote the degree of nearness, the degree of neutralness and the degree of non-nearness between a and b with respect to λ, respectively.

MULTIPLICATIVE NEUTROSOPHIC CONTRACTIVE MAPPING :

Definition 2.5[32]. Let V be a MNMS. The mapping f:$F\to F$ is called multiplicative neutrosophic contraction (MNC) if there exists $k\in(0,1)$ such that

$$\frac{1}{H(f(a),f(b),\gamma)}-1 \leq \left(\frac{1}{H(a,b,\gamma)}-1\right)^k$$

$$M(f(a),f(b),\lambda)\leq (M(a,b,\lambda))^k,$$

$$S(f(a),f(b),\lambda)\leq (S(a,b,\lambda))^k$$

for each a, $b\in F$ and λ>0.

Definition 2.6[32]. Let V be a MNMS and let f:$F\to F$be a NC mapping. If there exists $c\in F$ such that f(c)= c . Then$c$ is called multiplicative neutrosophic fixed point (MNFP) of f.

Theorem [32]: Let V be a complete NMS with (2) in which a NC sequence is a Cauchy sequence. Let f:$F\to F$ is a generalized neutrosophic contraction satisfying conditions of Definition 3.9. Then f has a unique fixe point in V.

Main Result:

MULTIPLICATIVE NEUTROSOPHIC EXPANSION MAPPING :

Definition 3.1. Let V be a MNMS. The mapping f: $F\to F$ is called multiplicative neutrosophic expansion (MNE) if there exists $k\in(1,\infty)$ such that

$$\frac{1}{H(f(a),f(b),\gamma)}-1 \geq \left(\frac{1}{H(a,b,\gamma)}-1\right)^k$$

$$M(f(a),f(b),\lambda) \geq (M(a,b,\lambda))^k,$$

$$S(f(a),f(b),\lambda) \geq (S(a,b,\lambda))^k$$

for each a, $b\in F$ and λ>0.

Theorem: Let V be a complete MNMS with (2) in which a MNC sequence is a Cauchy sequence. Let f: $F\to F$ is a surjective neutrosophic expansion satisfying conditions of Definition 3.1. Then f has a unique fixed point in V.

Proof: Let $a_0 \in V. Since\ f\ is\ surjective, we\ can\ define\ a\ sequence\ \{a_n\}\ by\ a_n = f(a_{n+1})\ for\ all\ n\in \mathbb{N}$. For each$\gamma$ >0,

$$\frac{1}{H(a_{n-1},a_n,\gamma)}-1=\frac{1}{H(f(a_n),f(a_{n+1}),\gamma)}-1$$

$$\geq\left(\frac{1}{H(a_n,a_{n+1},\gamma)}-1\right)^k$$

which implies that

$$\left(\frac{1}{H(a_n,a_{n+1},\gamma)}-1\right) \leq \left(\frac{1}{H(a_{n-1},a_n,\gamma)}-1\right)^\alpha \quad (3.1)$$

Where $\alpha=\frac{1}{k}\ so\ that\ \alpha\in(0,1)$.

Repeating (3.1) we get

$$\left(\frac{1}{H(a_n,a_{n+1},\gamma)}-1\right) \le \left(\frac{1}{H(a_0,a_1,\gamma)}-1\right)^{\alpha^n}$$

which implies that

In the same way

$$M(a_n,a_{n+1},\gamma) \le (M(a_0,a_1,\gamma))^{\alpha^n},$$
$$S(a_n,a_{n+1},\gamma) \le (S(a_0,a_1,\gamma))^{\alpha^n}.$$

$$\frac{1}{H(a_n,a_{n+p},\gamma)}-1 \le \frac{1}{*_{i=n}^{p} H\left(a_i,a_{i+1},\gamma^{\frac{1}{2^{i+1-n}}}\right)}-1$$

$$\le *_{i=n}^{p}\left(\frac{1}{H\left(a_i,a_{i+1},\gamma^{\frac{1}{2^{i+1-n}}}\right)}\right)-1$$
$$\le *_{i=n}^{p}\left(\frac{1}{H\left(a_0,a_1,\gamma^{\frac{1}{2^{i+1-n}}}\right)}\right)^{\alpha^i}-1$$

Which tends to 0 as $n \to \infty$. So that $\lim_{n\to\infty} H(a_n,a_{n+p},\gamma)=1$. In the same way $\lim_{n\to\infty} M(a_n,a_{n+p},\gamma)=0$. And $\lim_{n\to\infty} S(a_n,a_{n+p},\gamma)=0$. Therefore it is a Cauchy sequence in complete MNMS V.Hence $\{a_n\}$ is convergent and converges to some $c \in V$. Now we show that this point c is a neutrosophic fixed point of f. For

.

$$\frac{1}{H(a_{n+1},f(c),\gamma)}-1=\frac{1}{H(f(a_n),f(c),\gamma)}-1$$
$$\le\left(\frac{1}{H(a_n,c,\gamma)}-1\right)^{\alpha} \to 0 \ \ as \ \ n\to\infty.$$

So that $\frac{1}{H(c,f(c),\gamma)}-1=0$ *and thus* $H(c,f(c),\gamma)=1$.

In the same way, we can have

$M(c,f(c),\gamma)=0$ *and* $S(c,f(c),\gamma)=0$.

Therefore $f(c)=c$.
To show the uniqueness, let $f(b)=b$ for some $b\in V$. Then for all $\gamma>0$, we have

$$\frac{1}{H(c,b,\gamma)}-1=\frac{1}{H(f(c),f(b),\gamma)} \le\left(\frac{1}{H(c,b,\gamma)}-1\right)^{\alpha}.$$

Which on repeating yields

$$\frac{1}{H(c,b,\gamma)}-1\le\left(\frac{1}{H(c,b,\gamma)}-1\right)^{\alpha^n}\to 0 \quad as \ \ n\to\infty.$$

Also

$M(c,b,\gamma)=\mathrm{M}(f(c),f(b),\gamma)\le(M(c,b,\gamma))^{\alpha}\le(M(c,b,\gamma))^{\alpha^n}\to 0$ *as* $n\to\infty$ *so that* $M(c,b,\gamma)=0$.

$S(c,b,\gamma) = S(f(c), f(b), \gamma) \leq (S(c,b,\gamma))^{\alpha} \leq (S(c,b,\gamma))^{\alpha^n} \to 0 \quad as \quad n \to \infty \; so \; that \; S(c,b,\gamma) = 0.$
Thus $H(c,b,\gamma) = 1$ and $M(c,b,\gamma) = S(c,b,\gamma) = 0$ and hence $c = b$.

Conclusion: The above theorem is an extension to the one Kirisci et al [21] on MNMS and Kumar [32] for expansion mapping . Also it opens an era to establish a fixed point theory for expansion mapping on MNMS.

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