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A COMPARATIVE STUDY OF RUBAN'S COSMOLOGICAL MODELS IN SEAZ BALLESTER THEORY OF GRAVITATION

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Abstract: In this paper, we have investigated Ruban's cosmological model in Seaz-Ballester theory of gravitation under the influence of perfect fluid, cloud string and thick domain wall coupled with electromagnetic field. The general solution of Einstein's field equations for perfect fluid, cloud string of cosmological model in Seaz-Ballester theory of gravitation are obtained with the help of relation between metric coefficients and equation of state except Domain Wall, Also discussed some physical and kinematical properties of the models. The main aim of the paper is to compare the obtained results of perfect fluid, cloud string and thick domain wall within the framework of Saez-Ballester theory.

***keywords*- Ruban's Space Time; Saez-Ballester Theory; Perfect fluid; Cloud String; Domain Wall.**

1. INTRODUCTION

The study in astronomy, which deals with the origin, geometry, and evolution of the universe, is known as cosmology. Albert Einstein, in 1915, proposed a theory related to cosmology known as general relativity. It has been very successful in describing gravitational phenomena; it has also served as a basis for models of the universe. However, since Einstein first published his theory of gravitation, there have been many criticisms of general relativity because of the lack of certain 'desirable' features in the theory. For example Einstein himself pointed out that general relativity does not account satisfactory for inertial properties of matter; i.e. Mach's principle is not substantiated by general relativity. In the last few decades there has been much interest in alternative theories of gravitation, especially the scalar tensor theories proposed by Brans and Dicke [1], Nordvedt [2], Barber [3], Saez-Ballester [4], Lau and Prokhovnik [5] etc. out of which Saez-Ballester is one of the most important scalar tensor theory because this theories is capable of addressing the query of missing mass in the Friedmann-Roberson-Walker flat universe. In Seaz-Ballester theory [4], the metric is coupled with a dimensionless scalar field in a simple manner. This coupling gives a satisfactory description of the weak fields in which an anti-gravity regime appears despite the dimensionless behavior of the scalar field.

The field equations in the scalar tensor theory proposed by seaz and Ballester [4] are

$$G_{ij} - \omega\phi^n\left(\phi_{,i}\phi_{,j} - \frac{1}{2}g_{ij}\phi_{,k}\phi^{,k}\right) = -T_{ij} \tag{1}$$

and the Scalar field φ satisfies the equation

$$2\phi^n\phi^{;i}_{;i} + n\phi^{n-1}\phi_{,k}\phi^{,k} = 0 \tag{2}$$

Where $G_{ij} = R_{ij} - \frac{1}{2}Rg_{ij}$ is the Einstein tensor, R_{ij} is the Ricci tensor, R is the scalar curvature, n an arbitrary constant, ω a dimensionless coupling constant and T_{ij} is the matter energy-momentum tensor. Here comma and semicolon denote partial and covariant differentiation respectively.

Also, the equation of motion

$$T^{ij}_{,j} = 0 \tag{3}$$

is a consequence of the field equations (1) and (2).

In earlier literature, cosmological models within the framework of Saez-Ballester scalar-tensor theory of gravitation, have been studied by Wath et al.[6-7], Dabgar et al.[8], Quiros et al.[9], Sobhanbabu et al.[10], Karim [11], Daimary et al.[12], Shukla et al.[13], Vinutha et al.[14], Nimkar et al.[15], Santhi et al.[16], Mishra et al.[17], Sharma et al.[18], Shaikh et al.[19], Rao et al.[20],Mete et al.[21],Sahu et al.[22], Katore et al.[23], Adhav et al.[24-25], Rao et al.[26], Reddy et al.[27-28].

It is still a challenging problem to know the exact physical situation at very early stages of the formation of our universe. At very early stages of the formation of evolution of the universe, it is generally assumed that during the phase transition (as the universe passes through its critical temperature) the symmetry of the universe is broken spontaneously. It can give rise to topologically stable defects such as strings, domain wall and monopoles. Hatkar et al.[29] have proposed that domain wall Bianchi type VI_0 universe in $f(R,T)$ gravity. Ugale et al.[30] have studied Bianchi type IX cosmological models with perfect fluid in $f(R,T)$ theory of gravity. Raju et al. [31] discussed the Bianchi type-V string cosmological model with a massive scalar field. Shekh et al. [32] have investigated Accelerating Bianchi type dark energy cosmological model with cosmic string in $f(T)$ gravity. Mete et al. [33] have studied a five-dimensional cosmological model with one dimensional cosmic string coupled with zero mass scalar field in Lyra Manifold. Daimary et al.[34] have investigated Anisotropic L. R. S. Bianchi type-V cosmological models with Bulk Viscous String within the framework of Saez-Ballester theory in five-dimensional spacetime. Singh et al. [35] have obtained Bianchi Type-V domain walls and Quark Matter cosmological model with cosmological constant Λ in $f(R,T)$ gravity.

The electromagnetic field E_{ij} which is a part of the energy momentum tensor is considered as

$$E_{ij}=\frac{1}{4\pi}\left(-F_{i\alpha}F_{j\beta}g^{\alpha\beta}+\frac{1}{4}g_{ij}F_{\alpha\beta}F^{\alpha\beta}\right) \tag{4}$$

Here $F_{\alpha\beta}$ is the electromagnetic field tensor.

The cosmological model in the presence of electromagnetic fields plays an important role in the evolution of the universe and the formation of large scale structures like galaxies and other stellar bodies. The present phase of accelerated expansion of the universe is due to the presence of a cosmological electromagnetic field generated during inflation. Mete et al.[36], Hatkar et al.[37], Dewri et al.[38], Bhoyar et al.[39], Singh et al.[40], Patil et al.[41] are some of the authors who have investigated with electromagnetic field.

There are a few studies in the literature about the Ruban's universe model. Nimkar et al.[42-43], Aktas [44], Pund et al.[45-46], Mete et al.[47-48], Pund et al.[49], Lima et al.[50].In recent years, Caglar et al.[51] have investigated Domain wall with quark matters cosmological models in $f(R,T)$ theory. Vijayasanthi et al.[52] has studied Viscous ricci dark energy cosmological models in Brans-Dicke theory. Wath et al.[53] have obtained the Stability of macroscopic body cosmological model in Ruban's Background.

Motivated from the study of above literatures, in this paper we discuss a comparitive study of Ruban's cosmological models in Seaz Ballester theory of gravitation. The solutions of Einstein's field equations are obtained using various energy momentum tensors such as perfect fluid, cloud string and thick Domain wall interacting with electromagnetic fields and discussed in detail all aspects of physical and kinematic properties. The paper is organized as follows, in section 2, the metric and field equation is presented for perfect fluid, cosmic string and thick domain wall.In section 3, Physical and Kinematical properties for perfect fluid and cosmic string. Finally, Section 4, contains conclusion.

2. Metric and Field Equations

We consider the space-time of Ruban's [50] in the form

$$ds^2=dt^2-Q^2(x,t)dx^2-R^2(t)\left(dy^2+h^2dz^2\right) \tag{5}$$

Where

$$h(y)=\frac{\sin\sqrt{k}\,y}{\sqrt{k}}=\begin{cases}\sin y & \text{if } k=1\\ y & \text{if } k=0\\ \sinh y & \text{if } k=-1\end{cases}$$

and k is the curvature parameter of the homogeneous 2-spaces x and t constants. The function Q and R are free and will be determined.

The field equations in Seaz Ballester theory of gravitation are given by using equation (1),(2),(3), and equation (5) is given by

$$2\frac{\ddot{R}}{R}+\frac{\dot{R}^2}{R^2}+\frac{k}{R^2}-\frac{\omega}{2}\phi^n\phi_4^2=T_1^1 \tag{6}$$

$$\frac{\dot{R}\dot{Q}}{RQ}+\frac{\ddot{R}}{R}+\frac{\ddot{Q}}{Q}-\frac{\omega}{2}\phi^n\phi_4^2=T_2^2 \tag{7}$$

$$\frac{\dot{R}\dot{Q}}{RQ}+\frac{\ddot{R}}{R}+\frac{\ddot{Q}}{Q}-\frac{\omega}{2}\phi^n\phi_4^2=T_3^3 \tag{8}$$

$$2\frac{\dot{R}\dot{Q}}{RQ}+\frac{\dot{R}^2}{R^2}+\frac{k}{R^2}+\frac{\omega}{2}\phi^n\phi_4^2=T_4^4 \tag{9}$$

$$\phi_{44}+\left(\frac{\dot{Q}}{Q}+2\frac{\dot{R}}{R}\right)\phi_4+\frac{n}{2}\frac{\phi_4^2}{\phi}=0 \tag{10}$$

Where overhead $\cdot$ indicates differentiation with respect to time t only

2.1 Perfect Fluid

The energy momentum for perfect fluid distribution in the presence of electromagnetic field is given by

$$T_{ij}=(p+\rho)u_iu_j-pg_{ij}+E_{ij} \tag{11}$$

Here ρ is the energy density and p is the pressure of the perfect fluid respectively, together with co-moving coordinates $u^iu_i=1$

Where $u_i=(0,0,0,1)$

For solving equation (4), In co-moving coordinates, if we take the electromagnetic field along $z-axis$ then F_{12} it is only non-vanishing component of the electromagnetic field tensor F_{ij} .

As a result, the electromagnetic field E_{ij} are obtained as follows

$$E_1^1=E_2^2=-\frac{D^2}{8\pi R^2Q^2},E_3^3=E_4^4=\frac{D^2}{8\pi R^2Q^2} \tag{12}$$

From equation (11) and equation (12) gives

$$T_1^1=T_2^2=-p-\frac{D^2}{8\pi R^2Q^2},T_3^3=-p+\frac{D^2}{8\pi R^2Q^2},T_4^4=\rho+\frac{D^2}{8\pi R^2Q^2} \tag{13}$$

The field equations of Seaz -Ballester theory of gravitation with perfect fluid is obtained by using equation (13) in RHS of equation (6) to equation (9) we get

$$2\frac{\ddot{R}}{R}+\frac{\dot{R}^2}{R^2}+\frac{k}{R^2}-\frac{\omega}{2}\phi^n\phi_4^2=-p-\frac{D^2}{8\pi R^2Q^2} \tag{14}$$

$$\frac{\dot{R}\dot{Q}}{RQ}+\frac{\ddot{R}}{R}+\frac{\ddot{Q}}{Q}-\frac{\omega}{2}\phi^n\phi_4^2=-p-\frac{D^2}{8\pi R^2Q^2} \tag{15}$$

$$\frac{\dot{R}\dot{Q}}{RQ}+\frac{\ddot{R}}{R}+\frac{\ddot{Q}}{Q}-\frac{\omega}{2}\phi^n\phi_4^2=-p+\frac{D^2}{8\pi R^2Q^2} \tag{16}$$

$$2\frac{\dot{R}\dot{Q}}{RQ}+\frac{\dot{R}^2}{R^2}+\frac{k}{R^2}+\frac{\omega}{2}\phi^n\phi_4^2=\rho+\frac{D^2}{8\pi R^2Q^2} \tag{17}$$

$$\phi_{44}+\left(\frac{\dot{Q}}{Q}+2\frac{\dot{R}}{R}\right)\phi_4+\frac{n}{2}\frac{\phi_4^2}{\phi}=0 \tag{18}$$

$$\dot{\rho}+(\rho+p)\frac{\dot{Q}}{Q}+2(\rho+p)\frac{\dot{R}}{R}=0 \tag{19}$$

Where the overhead $\bullet$ denote partial differentiation with respect to time t

Now we discuss some physical and kinematical parameters which are important in cosmological observations. The spatial volume V and average scale factor $a(t)$ are defined as

$$V = [a(t)]^3 = QR^2 \tag{20}$$

The mean generalized Hubble parameters H for this model is given by

$$H = \frac{1}{3}\left(\frac{2\dot{R}}{R} + \frac{\dot{Q}}{Q}\right) \tag{21}$$

Where H_1, H_2, H_3 are the directional Hubble parameters defined by

$$H_1 = H_2 = \frac{\dot{R}}{R}, \;\; H_3 = \frac{\dot{Q}}{Q} \tag{22}$$

The average anisotropic parameter is

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2 \tag{23}$$

Where $\Delta H_i = H_i - H.$

The expansion scalar θ and shear scalar σ are given by

$$\theta = 3H = \frac{2\dot{R}}{R} + \frac{\dot{Q}}{Q} \tag{24}$$

$$\sigma^2 = \frac{1}{2}\sigma^{ij}\sigma_{ij} \tag{25}$$

2.2 Solution of the Field Equations for Perfect Fluid

The above system of the equations (14), (15) and (17) with five unknown R, Q, ϕ, p and ρ. In order to solve this undetermined system one additional constraint are required. Hence to find a determinate solution we assume the relations as the equation of state as

$$\rho = p \tag{26}$$

From equations (14)-(18) we obtain,

$$4\frac{\dot{R}\dot{Q}}{RQ} + \frac{\ddot{Q}}{Q} = 0 \tag{27}$$

To solve equation (27), assuming the relation between metric coefficients

$$Q = x^n R^n.$$

This gives an exact solution as

$$R = M(c_2 t + c_3)^{\frac{1}{n+2}} \tag{28}$$

$$Q = N(c_2 t + c_3)^{\frac{n}{n+2}} \tag{29}$$

Where, $M = (n+2)^{\frac{1}{n+2}}$ and $N = x^n M^n$

Using equation (28) and equation (29), the Ruban's cosmological model in equation (5) takes the form

$$ds^2 = dt^2 - N^2 (c_2 t + c_3)^{\frac{2n}{n+2}} dx^2 - M^2 (c_2 t + c_3)^{\frac{2}{n+2}} (dy^2 + h^2 dz^2)$$

Through a proper choice of coordinates and constants the model can be written as

$$ds^2 = dt^2 - N^2 T^{\frac{2n}{n+2}} dx^2 - M^2 T^{\frac{2}{n+2}} (dy^2 + h^2 dz^2) \tag{30}$$

Where $T = c_2 t + c_3$.

And Scalar field is given by

$$\phi = \left[\left(\frac{n+2}{2} \right) \int \frac{c_1}{NM^2 (c_2 t + c_3)} dt \right]^{\frac{2}{n+2}} \tag{31}$$

Also, Pressure and energy density of obtained model is

$$p = \rho = \frac{c_4}{(n+2)^2 (c_2 t + c_3)^2} \tag{32}$$

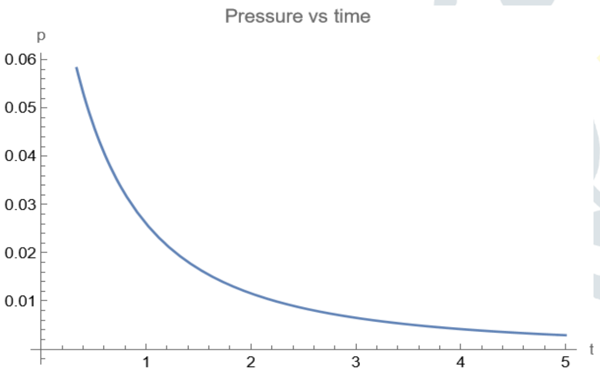

fig.1: pressure vs. time for $n = 1.1, c_2 = c_3 = c_4 = 1$

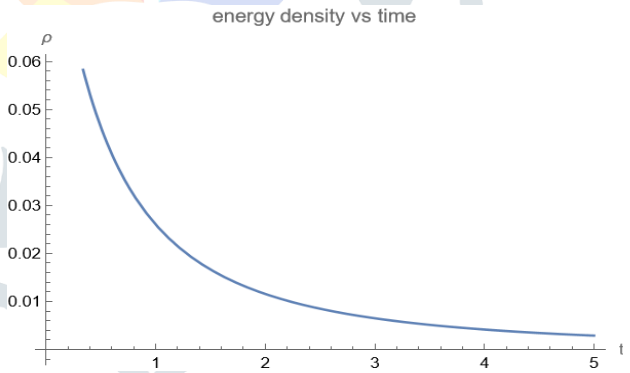

fig.2: energy density vs. time for $n = 1.1, c_2 = c_3 = c_4 = 1$

From fig.1 and fig.2 it is observed that as time increases the pressure density and Energy density decreases.

2.3 *Cloud String*

The energy momentum tensor for a cosmic string in presence of magnetic field may be defined as

$$T_{ij} = \rho u_i u_j - \lambda x_i x_j + E_{ij} \tag{33}$$

With $u^i u_i = -x^i x_i = -1,$ (34)

and $u^i x_i = 0$ (35)

Here ρ rest is the energy density of the cloud of strings with particles attached to them and λ is the tension density of the strings. As pointed out Letelier [54] λ may be positive or negative, u^i describes the system four velocity and x^i represents a direction of anisotropy, i.e. direction of the strings.

We consider,

$$\rho = \rho_p + \lambda \tag{36}$$

Where ρ_p is the rest energy density of the particles attached to the string. Here ρ and λ are the functions of t only.

In this paper, we have obtained a cosmological model corresponding to cosmic string with magnetic field along $z-axis$, so that F_{12} is only non-vanishing component F_{ij} because a cosmological model which contains a global magnetic field is necessary anisotropic since the magnetic field vector is prepared specifies a preferred special direction.

Therefore, for solving equation (4), the nontrivial components of the electromagnetic field E_{ij} are obtained as follows

$$E_1^1 = E_2^2 = -\frac{H^2}{8\pi R^2 Q^2}, E_3^3 = E_4^4 = \frac{H^2}{8\pi R^2 Q^2} \tag{37}$$

Substituting eq. (37) and eq. (33) in eq. (6) to eq. (9), the field equation of seaz Ballester theory of gravitation as follows:

$$2\frac{\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} + \frac{k}{R^2} - \frac{\omega}{2}\phi^n \phi_4^2 = -\lambda + \frac{H^2}{8\pi R^2 Q^2} \tag{38}$$

$$\frac{\dot{R}\dot{Q}}{RQ} + \frac{\ddot{R}}{R} + \frac{\ddot{Q}}{Q} - \frac{\omega}{2}\phi^n \phi_4^2 = \frac{H^2}{8\pi R^2 Q^2} \tag{39}$$

$$\frac{\dot{R}\dot{Q}}{RQ} + \frac{\ddot{R}}{R} + \frac{\ddot{Q}}{Q} - \frac{\omega}{2}\phi^n \phi_4^2 = -\frac{H^2}{8\pi R^2 Q^2} \tag{40}$$

$$2\frac{\dot{R}\dot{Q}}{RQ} + \frac{\dot{R}^2}{R^2} + \frac{k}{R^2} + \frac{\omega}{2}\phi^n \phi_4^2 = -\rho - \frac{H^2}{8\pi R^2 Q^2} \tag{41}$$

$$\phi_{44} + \left(\frac{\dot{Q}}{Q} + 2\frac{\dot{R}}{R}\right)\phi_4 + \frac{n}{2}\frac{\phi_4^2}{\phi} = 0 \tag{42}$$

The conservation law for energy momentum tensor gives

$$-\dot{\rho} + (\lambda - \rho)\frac{\dot{Q}}{Q} - 2\rho\frac{\dot{R}}{R} = 0 \tag{43}$$

Here an overhead dot denotes a derivative with respect to cosmic time t.

2.4 Solution of Field Equations for Cloud String

The set of above equations (38), (39) and (41) contains five unknown R, Q, ϕ, λ and ρ to get a determinate solution we assume a physical or mathematical condition. In the literature Letelier [54], we have equation of state for the string model $\rho = \lambda$ (Geometric string or Nambu String)

We get the equation,

$$2\frac{\dot{R}\dot{Q}}{RQ}+\frac{\ddot{Q}}{Q}=0 \quad (44)$$

Solving above equation, consider the relation between metric potential

$$Q=x^{n}R^{n} \quad (45)$$

Using this relation, the field equations (45) to equation (44) admit the exact solution.

$$R=M\left(c_1t+c_2\right)^{\frac{1}{(n+2)}} \quad (46)$$

$$Q=N\left(c_1t+c_2\right)^{\frac{n}{(n+2)}} \quad (47)$$

Where $M=\left(n+2\right)^{\frac{1}{(n+2)}}$ and $N=x^{n}M^{n}$

Using equation (46) and equation (47) in the metric (5) can be reduce to

$$ds^2=dt^2-N^2\left(c_1t+c_2\right)^{\frac{2n}{(n+2)}}dx^2-M^2\left(c_1t+c_2\right)^{\frac{2}{(n+2)}}\left(dy^2+h^2dz^2\right)$$

This metric can be transformed through a proper choice of coordinates to the form

$$ds^2=dt^2-N^2T^{\frac{2n}{(n+2)}}dx^2-M^2T^{\frac{2}{(n+2)}}\left(dy^2+h^2dz^2\right) \quad (48)$$

Where $T=\left(c_1t+c_2\right)$

Also, the Scalar field is as follows

$$\phi=\left[\left(\frac{n+2}{2}\right)\int\frac{c_4}{NM^2\left(c_1t+c_2\right)}dt\right]^{\frac{2}{n+2}} \quad (49)$$

Also the Energy density and Tension density of obtained cosmological model is

$$\rho=\lambda=\frac{c_3}{M^2\left(c_1t+c_2\right)^{\frac{2}{n+2}}} \quad (50)$$

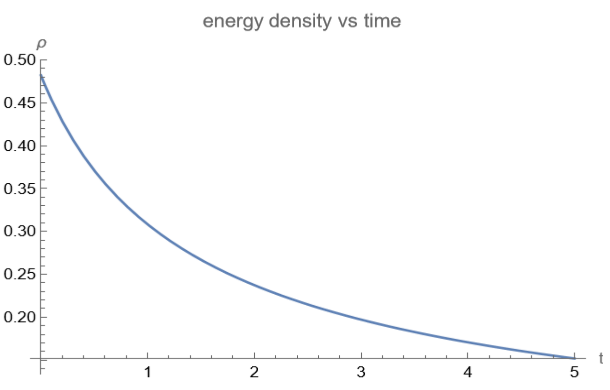

Fig.3 Energy density Vs. time for
$n=1.1, c_1=c_2=c_3=1$

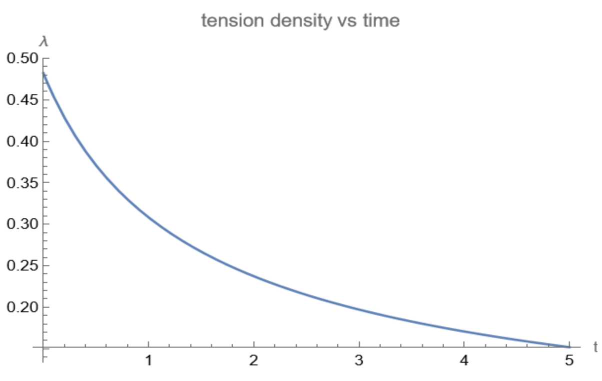

Fig.4 tension density Vs. time for
$n=1.1, c_1=c_2=c_3=1$

From fig.3 and fig.4 it is observed that as cosmic time increases energy density and tension density decreases.

2.5 *Domain Wall*

A thick domain wall can be viewed as a solution-like solution of the scalar field equations coupled with gravity. There are two ways of studying thick domain walls. One way is to solve gravitational field equations with an energy momentum tensor describing a scalar field ϕ with self-interactions contained in a potential $V(\psi)$ given by

$$\psi_{,i}\psi_{,j} - g_{ij}\left[\frac{1}{2}\psi_{,k}\psi^{,k} - V(\psi)\right] \tag{51}$$

Second approach is to assume the energy momentum tensor in the form

$$T_{ij} = \rho\left(g_{ij}+\omega_i\omega_j\right) + p\omega_i\omega_j + E_{ij} \tag{52}$$

With $\omega^i\omega_j = -1$

Where ρ is the energy density of the wall, p is the pressure in the direction normal to the plane of the wall and ω_i is a unit space-like vector in the same direction.

Here, using the second approach to study the thick domain walls in saez-ballester theory of gravitation

In this case we have obtained a thick domain wall cosmological model with magnetic field for solving equation (4), along $y-axis$ then F_{13} is only non-vanishing component of the electromagnetic field tensor F_{ij} . Therefore, from equation (4) the nontrivial components of the electromagnetic field E_{ij} are obtained as follows

$$E_1^1 = E_3^3 = -\frac{D^2}{8\pi R^2 Q^2 h^2}, E_2^2 = E_4^4 = \frac{D^2}{8\pi R^2 Q^2 h^2} \tag{53}$$

Using equation (53) and equation (52) in RHS of equation (6) to (9) gets the field equation of Saez-Ballester theory of gravitation with thick domain wall are

$$2\frac{\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} + \frac{k}{R^2} - \frac{\omega}{2}\phi^n\phi_4^2 = -p - \frac{D^2}{8\pi R^2 Q^2 h^2} \tag{54}$$

$$\frac{\dot{R}\dot{Q}}{RQ} + \frac{\ddot{R}}{R} + \frac{\ddot{Q}}{Q} - \frac{\omega}{2}\phi^n\phi_4^2 = \rho + \frac{D^2}{8\pi R^2 Q^2 h^2} \tag{55}$$

$$\frac{\dot{R}\dot{Q}}{RQ} + \frac{\ddot{R}}{R} + \frac{\ddot{Q}}{Q} - \frac{\omega}{2}\phi^n\phi_4^2 = \rho - \frac{D^2}{8\pi R^2 Q^2 h^2} \tag{56}$$

$$2\frac{\dot{R}\dot{Q}}{RQ} + \frac{\dot{R}^2}{R^2} + \frac{k}{R^2} + \frac{\omega}{2}\phi^n\phi_4^2 = \rho + \frac{D^2}{8\pi R^2 Q^2 h^2} \tag{57}$$

$$\phi_{44} + \left(\frac{\dot{Q}}{Q} + 2\frac{\dot{R}}{R}\right)\phi_4 + \frac{n}{2}\frac{\phi_4^2}{\phi} = 0 \tag{58}$$

$$\dot{\rho} + (p+\rho)\frac{\dot{Q}}{Q} = 0 \tag{59}$$

Where the overhead $\bullet$ denote partial differentiation with respect to time t

3. Physical and Kinematical Properties

In this section we have discussed some comparative study of physical and kinematical properties of Ruban's space time cosmological model for perfect fluid, Cosmic String in Seaz-Ballester theory of gravitation. The physical quantities that are important in cosmology are the average scale factor $(a(t))$, special volume (V), the Hubble parameter (H), the expansion scalar (θ), shear scalar (σ^2), and anisotropic parameter (A_m), are obtain as

The average scale factor $$(a(t)) = \left[hNM^2(c_1t+c_2)\right]^{\frac{1}{3}} \tag{60}$$

The spatial volume is given by $$(V) = \sqrt{-g} = hNM^2(c_1t+c_2) \tag{61}$$

The Hubble parameter $$(H) = \frac{c_1}{3T} \quad \text{where } T = (c_1t+c_2) \tag{62}$$

Expansion Scalar $$(\theta) = \frac{c_1}{T} \quad \text{where } T = (c_1t+c_2) \tag{63}$$

Shear Scalar $$(\sigma^2) = \frac{c_1^2}{6T^2} \quad \text{where } T = (c_1t+c_2) \tag{64}$$

The anisotropic parameter $$(A_m) = \frac{2(n-1)^2}{(n+2)^2} \tag{65}$$

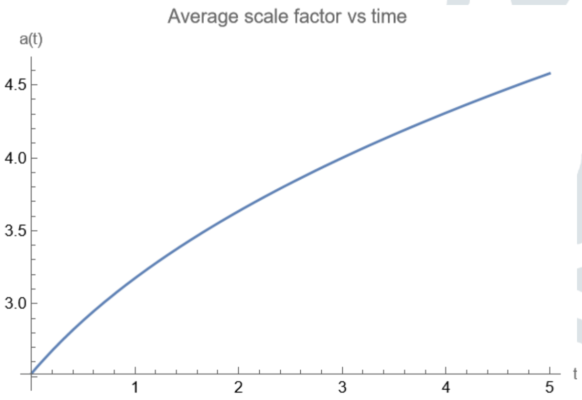

Fig.5: Average Scale factor Vs. time for $h = N = M = 2, c_1 = c_2 = 1,$

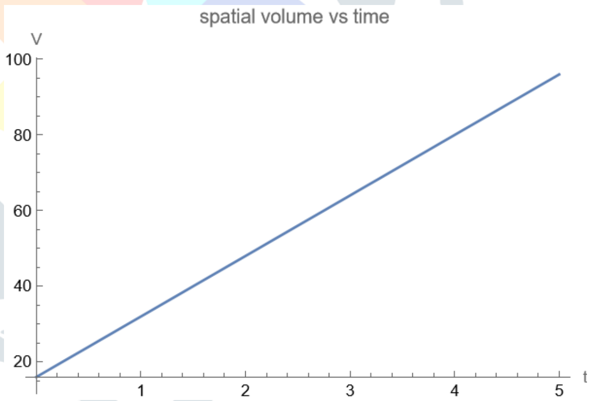

Fig.6: spatial volume Vs. time for $h = N = M = 2, c_1 = c_2 = 1,$

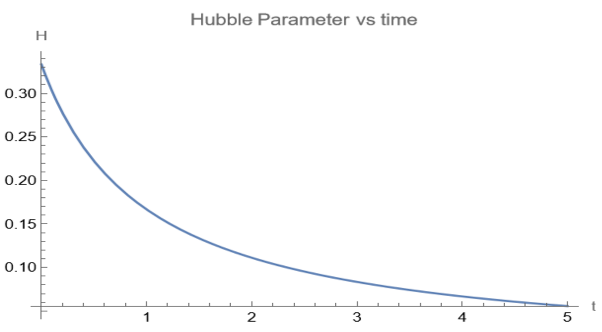

Fig.7 Hubble parameter Vs. time for $c_1 = c_2 = 1,$

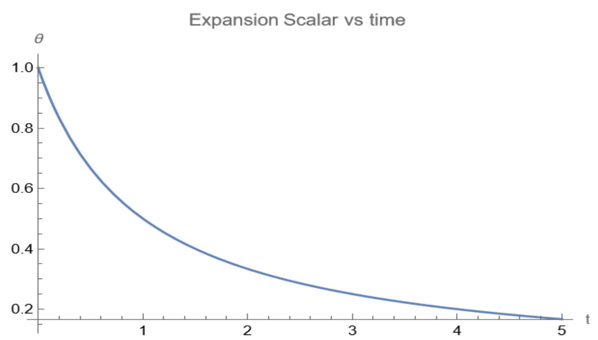

Fig.8 Expansion Scalar Vs. time for $c_1 = c_2 = 1,$

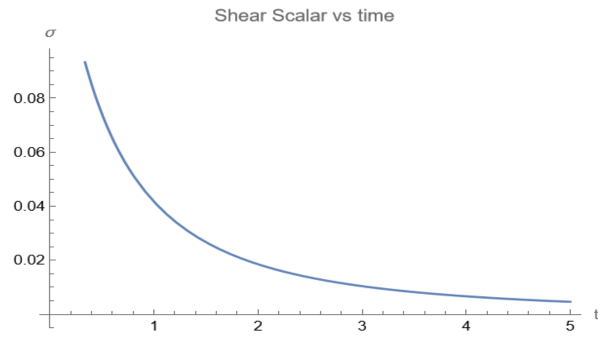

Fig.9 Shear Scalar Vs. time for $c_1 = c_2 = 1,$

From fig.5 it is observed that the average scale factor increases when time increases and approaches infinity, it shows that continuous expansion of the universe. And from fig.6 it is observed that initially spatial volume is constant and as time increases volume increases and approaches infinity. From fig.7, fig.8 and fig.9 it is observed that the Hubble parameter, expansion scalar and shear scalar decrease gradually as time increases.

3.1 Deceleration Parameter (q)

The deceleration parameter is define as

$$q = \frac{-\ddot{a}a}{\dot{a}^2} \tag{66}$$

That depends upon the scale factor and derivatives, the sign of deceleration parameter (q) shows whether the model either accelerates or decelerates. If $q > 0,$ the model exhibits decelerating expansion, if $q = 0$ a constant rate of expansion, and an accelerating expansion if $-1 < q < 0.$ The universe exhibits exponential expansion or de sitter expansion for $q = -1$ and super exponential expansion for $q < -1$. For perfect fluid and cosmic string we observed that the deceleration parameter is positive hence the model is decelerated.

3.2 Jerk Parameter $(j(t))$

A dimensionless cosmic jerk parameter is obtained by the third derivative of the average scale factor w.r.t. cosmic time‘t’ which is given by

$$j(t) = \frac{\dddot{a}}{aH^3} \tag{67}$$

Where a is the cosmic scale factor, H is the Hubble parameter and the dot denotes differentiation with respect to the cosmic time. It is believed that the transition of the universe from decelerating to accelerating phase of the universe is due to a cosmic jerk. For various models of the cosmos, there is a variation in the transition of the cosmos, whenever the jerk parameter lies in the positive region and the negative values of deceleration parameter (Visser[55]). The investigations of Rapetti et.al.[56] have shown that for a flat ΛCDM model, the value of jerk becomes unity. The jerk parameter for perfect fluid and cosmic string is given by the equation (72) can be written as

$$j(t) = q + 2q^2 - \frac{\dot{q}}{H} = 10 \text{ (Constant)} \tag{68}$$

This is positive throughout the evolution of the universe. The parameter $j(t)$ attains a constant value at late time. Hence the expansion in the model from decelerating phase to attain accelerating phase is smooth.

3.3 Stability Analysis

To examine the stability of the cosmological model by observing the ratio of sound speed is given by $\frac{dp}{d\rho} = c_s^2$, when the ratio $\frac{dp}{d\rho}$ is positive, i.e. $c_s^2 > 0$, shows a stable model whereas when the ratio $\frac{dp}{d\rho}$ is negative, i.e. $c_s^2 < 0$, shows an unstable model.

We determine the stability of the perfect fluid to solve equation (32) we get,

$$c_s^2 = \frac{dp}{d\rho} = 1 = \text{Constant.} \tag{69}$$

Based on the results of equation (69), it can be seen that the ratio of sound speed is positive. So, the model of the universe is stable.

Conclusion

The origin of the structure in the universe is one of the greatest cosmological mysteries even today the exact physical situation at very early stages of the formation of our universe is still known. Ruben's Perfect fluid, cosmic string and Domain Walls play a vital role in understanding the formation of large scale structure in the universe.

In this paper, we have studied Ruban's Perfect fluid (Stiff fluid) and Cosmic String (Geometric String) Coupled with Electromagnetic field cosmological models in Scalar tensor theory formulated by Seaz Ballester. Also observed that, in this particular theory, Domain Walls (Stiff Fluid) do not exist.

It is observed that, initially the average scale factors, spatial volume, pressure and energy density are finite and as time increases average scale factors and spatial volume increase infinitely while pressure and energy density reduces. Also the Hubble parameter (H) Scalar Expansion (θ) and Shear scalar (σ^2) are infinite at $t=0$ and approaches zero as $t \to \infty$. Here, the anisotropic parameter is zero for $n=1$ and the model is isotropic for $n=1$ and anisotropic for $n \neq 1$, it is observed that the ratio of sound speed is positive. i.e. $c_s^2 \geq 0$ so the given cosmological model is Stable for perfect fluid. In this model $\lim_{t\to\infty}\left(\frac{\sigma}{\theta}\right)^2 = \frac{1}{6} \neq 0$ and hence the model does not approach isotropy.

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