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"KAS Sequences: A Methodological Approach to Solving Equations and Analyzing Varg Prakriti Structures"

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Abstract

This research introduces a novel mathematical framework—KAS Sequences—as a robust methodology for solving equations, with particular emphasis on Varg Prakriti (quadratic indeterminate) equations. Rooted in historical mathematical principles and enriched with modern computational techniques, the approach aims to unravel the intricate patterns and structures inherent in these equations. The paper outlines the formulation of KAS Sequences, their theoretical underpinnings, and their application to diverse mathematical problems. By integrating historical insights with innovative algorithms, the proposed methodology provides a comprehensive toolkit for addressing both classical and contemporary challenges in equation solving. The findings are expected to contribute to the fields of number theory, algebra, and mathematical problem-solving, offering fresh perspectives for researchers and practitioners alike.
Key Words- KAS Sequences, Varg Prakriti

Introduction

A sequence, which comes from the Latin "sequens," which means "following," is a group of items, such as letters, numbers, and symbols, that can be arranged as you wish, regardless of repetitions or order. Every sequence has a length that is determined by how many components it contains. The length can be either infinite (meaning the sequence has no end) or finite (meaning the sequence has a fixed number of pieces). Notation-The location of each element is determined by its rank, which is sometimes referred to as its index. The first element in each sequence after this has a rank of either 0 or 1, depending on the sequence. This is how a sequence's notation would typically look:"a_n"={1,4,9,16} The sequence begins with the lowercase letter "a" enclosed in parenthesis, expressing the sequence's name or identity. One of the items in the sequence might also be used to indicate the sequence. The sequence will then be titled by the rank of that particular element. Next

is the identification of the subscripted letters "I" or "n," which, depending on the notation, might indicate two different things with and without braces.
The "index" or counter, represented by the notation "a_2," which is devoid of braces, merely indicates the particular phrase in the sequence—in this case, the second term. In contrast, the braced notation refers to the complete sequence. (When highlighting sequences, either curly brackets or standard brackets are utilized, and both have the same meaning.) The sequence's actual contents follow the equals sign, which makes it obvious what the sequence is.
The beginning and ending terms at that moment can also be defined by the notation. For instance, the sequence would be expressed as follows:

$$\{a_i\}^{n}_{i=1}$$

the first term is 1 and the last term is n.
As an additional illustration, a sequence that began with the index 2 and continued indefinitely would be expressed as follows:

$$\{a_i\}^{\infty}_{i=1}$$

This sequence is categorized as infinite since it lacks a definite termination value because it is infinity. Keep in mind that the lower index of the majority of infinite sequences is finite.
The lower index (i) represents the counter's starting value, whereas the upper index (outside of brackets and printed in superscript) represents the final value. This is logical given their position in the notation. The lower index could possibly be zero.

Example: - $\{a_n\} = \{1, 7, 49, 343, \dots\}$

Sequences frequently include a rule that determines the next term in a sequence. The same rule governs each successive term. A rule is a mathematical method that calculates the next term while repeating the previous computation. A rule is only present in a sequence if its words are not in random order, allowing the next term to be determined. A sequence's rule is frequently embedded in its notation. It is called indexing of sequence.
Thus, in mathematics, a sequence is an enumerated collection of items where recurrence is permitted and order is important.

History

The most famous post-Vedic Indian mathematician is Pingala, (fl. 300–200 BCE), a music theorist who wrote the Chhandas Shastra (chandaḥ-śāstra, or Chhandas Sutra chhandaḥ-sūtra), a Sanskrit treatise on prosody. The fundamental concepts of sequence, or maatraameru, are also included in Pingala's work.
Indian mathematics makes reference to the Fibonacci sequence in relation to Sanskrit prosody. There was interest in listing every pattern of long (L) syllables with a length of two units, in contrast to short (S) syllables with a duration of one unit in the Sanskrit literary tradition. Fibonacci numbers are obtained by counting the various patterns of successive L and S with a particular total time: F_{m+1} is the number of patterns of duration m units.
Fibonacci sequence knowledge was evident as early as Pingala (c. 450 BC–200 BC). Singh quotes Pingala's mysterious formula misrau cha ("the two are mingled"), which academics

understand to mean that adding one [S] to the Fm cases and one [L] to the Fm−1 instances yields the number of patterns for m beats (Fm+1). The sequence is also mentioned by Bharata Muni in the Natya Shastra (c. 100 BC–c. 350 AD). Virahanka's (c. 700 AD) treatise, which has been lost, but is quoted by Gopala (c. 1135), provides the most lucid explanation of the series.

"Two prior meters' variations [is the variation]... For instance, when [a meter of length] four is blended with variations of meters of two [and] three, five occurs. [solves instances 8, 13, 21]... In all mātrā-vṛttas [prosodic combinations], the procedure should be followed in this manner."

Hemachandra, who lived around 1150, is also credited with knowing the sequence, noting that "the number... of the next mātrā-vṛtta is the total of the last and the one before the last."

They bear the name of Leonardo of Pisa, also called Fibonacci, an Italian mathematician who originally presented the series to Western European mathematics in his book Liber Abaci in 1202.

The Fibonacci sequence was initially employed to determine the increase of rabbit populations in Fibonacci's book Liber Abaci (The Book of Calculation, 1202). Assuming that: a newborn breeding pair of rabbits is placed in a field; each breeding pair mates at the age of one month, and at the end of their second month, they always produce another pair of rabbits; rabbits never die, but breed indefinitely, Fibonacci examines the growth of an idealized (biologically unrealistic) rabbit population. The rabbit math problem was introduced by Fibonacci: how many pairs would there be in a year?

One straightforward classical example is the Fibonacci sequence, which is determined by the recurrence relation.

$$a_n=a_{n-1}+a_{n-2}$$

with initial terms $a_0=0$ and $a_1=1$. From this, a simple computation shows that the first ten terms of this sequence are 0, 1, 1, 2, 3, 5, 8, 13, 21, and 34.

Development of KAS Sequence

We have also developed a sequence whose name is KAS sequence.

"The KAS sequence is defined as a sequence with constant order that is the difference between the sequence's third and first terms divided by the second term."

KAS sequence is a collection of different terms or elements or objects that have been arranged in a specific order.

(K_1, K_2, K_3, K_4, K_5, K_6 K_7....... K_n.......) are the member of sequency where the subscript n refers to the nth element of the sequence.

Let us find out specific order i.e. KAS Order

$$KAS_0= \left(\frac{Current\ term+one\ step\ previous\ term}{middle\ term}\right)$$

$$= \left(\frac{K3+K1}{K2}\right)= \left(\frac{K4+K2}{K3}\right) = \left(\frac{K5+K3}{K4}\right) \ldots. = S$$

In general KAS Order-

$$KAS_0=[(K_{n+2}+K_n)/K_{n+1}]$$

KAS recurrence relation is-

$$K_{n+2}=S\ K_{n+1}-K_n$$

Where-

K_{n+2} =(n+2) th term of sequence
K_{n+1} =(n+1) th term of sequence
K_n = (n)th term of sequence
S = KAS Order

Example: 1

$\{0, \alpha, \alpha\beta, \alpha(\beta^2-1), \alpha\beta(\beta^2-2), \alpha(\beta^4-3\beta^2+1) \ldots\}$
$KAS_0=[(K_{n+2}+K_n)/K_{n+1}]$
$=[(\alpha\beta+0)/\alpha]=[\alpha(\beta^2-1)+\alpha\beta)/\alpha]=\beta$

Example: 2

$\{0,\alpha\beta, \alpha\beta\gamma, \alpha\beta(\gamma^2-1), \alpha\beta\gamma(\gamma^2-2)\ldots\ldots\}$
$KAS_0=[(\alpha\beta\gamma+0)/\alpha\beta]=[(\alpha\beta(\gamma^2-1)+\alpha\beta)/\alpha\beta\gamma]=\gamma$

Example: 3

(1,2,5,13,34,89,233,610,1597 …………………)
As KAS_0 = (5+1)/2=(2+13)/5=(60+5)/13=(3027+441)/1156=3

Example: 4

0,7,35,168,805,3857,18480,88543,424235…….
KAS_0= (35+0)/7=5

KAS sequency is very useful for finding out infinite integer solutions of any type of equation if first three solution is known.

Algorithm Of KAS Sequence for Integral Solution of Equations

Let us understand algorithm for integer solution of equation-

1) Find out three solution of given equation i.e. in order pairs (α_1, β_1), (α_2, β_2) and (α_3, β_3).
2) In these order pairs, first elements in order pairs show one sequence $(\alpha_1, \alpha_2\ \alpha_3)$ and second elements in order pairs show second sequence $(\beta_1, \beta_2, \beta_3)$.

$$X=(\alpha_1, \alpha_2, \alpha_3)$$
$$Y=(\beta_1, \beta_2, \beta_3)$$

3) Find out KAS order of sequences i.e. $KAS_0=[(K_{n+2}+K_n)/K_{n+1}]$.

$$(KAS_0)_x=[(x_{n+2}+x_n)/x_{n+1}] = [(\alpha_3+ \alpha_1)/ \alpha_2)$$
and
$$(KAS_0)_y=[(y_{n+2}+y_n)/y_{n+1}] = [(\beta_3+ \beta_1)/ \beta_2)$$

4) By using KAS recurrence relation $K_{n+2}= (S\, K_{n+1}- K_n)$,

$$(x_{n+2})= (S\, x_{n+1}- x_n)$$
and
$$(y_{n+2})= (S\, y_{n+1}- y_n),$$

we will find out value of $(\alpha_4, \alpha_5, \alpha_6 \ldots\ldots\ldots\ldots\infty)$ and $(\beta_4, \beta_5, \beta_6 \ldots\ldots\ldots\ldots\infty)$

5) Order pairs (α_4, β_4),((α_5, β_5), (α_6, β_6),……… will be required solution of equation.

Example: 2x+3y=1

1) First three solution of above equation in order pair (2,-1),(5,-3) and (8,-5)
2) Sequence of both elements-

$$X=(2,5,8) \text{ and } Y= (-1,-3,-5)$$

3) Now let us find KAS order-

$$(KAS_0)_x=[(K_{n+2}+K_n)/K_{n+1}]$$
$$(KAS_0)_x=[(8+2)/5]=2 \text{ and } (KAS_0)_y=[\{-5+(-1)\}/(-3)]=2$$

4) Now By using KAS recurrence relation

$$K_{n+2}= (S\, K_{n+1}- K_n),$$
$$(x_{n+2})= (S\, x_{n+1}- x_n) \text{ and } (y_{n+2})= (S\, y_{n+1}- y_n),$$

$X_4= (2*8-5)=11$ $y_4= \{2*(-5)-(-3)\}= -7$
$X_5= (2*11-8) =14$ $y_5= \{2*(-7) -(-5)\} = -9$
$X_6= (2*14-11) =17$ $y_6= \{2*(-9) -(-7)\} = -11$

So,
X= (2,5,8,11,14,17,……..) Y=(-1,-3,-5,-7,-9,-11…….)

5) Order pairs (2,-1),(5,-3),(8,-5)(11,-7),(14,-9)(17,-11)…..are required solution.

Analyzing of Brahmagupta's varg prikriti equation $Dx^2 + 1 = y^2$ utilizing my own recently discovered KAS sequence approach

Brahmagupta provided a partial answer to the issue of discovering integer solutions to Dx2+1=y2. Later, Indian algebraists used a cyclic approach known as cakravāla in ancient India to uncover the entire integer solution. Jayadeva, a mathematician, is credited with the first description of the cakravāla method, which is quoted in the work Sundarī by

Udayadivākara, written in 1073 CE. Jayadeva, a mathematician, lived from the seventh to eleventh century, following Brahmagupta and preceding Udayadivākara.

Famous astronomer and mathematician Bhaskra II (1150 CE), often called Bhaskaracarya, defines the cakravāla and provides challenging numerical examples like D=61 and D=67. Using the cases D=97 and D=103, the mathematician Narayana (c.1350 CE) explains the cakravāla and demonstrates the procedure. He demonstrates how to use the solutions to produce logical estimates of √D.

Certain mathematicians, like Bhāskara II, use simultaneous and indeterminate equations in their writings. While Jayadeva's results on varga-prakṛti are presented by K.S. Shukla in [15], Datta-Singh and other indeterminate equations examine the Indian results in [8].

I am currently utilizing a novel idea (The KAS sequence approach) to determine the integral solution of Bharamgupt's indeterminate equations (Varg prikriti equation).It takes very little time and is quite simple. Additionally, it is highly helpful to determine the integral solution of a linear equation. This newly discovered approach is entirely conceptual.

As the creator of modern number theory, Pierre de Fermat (1601–65) uses the issue of finding integer solutions to a Dx2+1=y2 to illustrate the beauty and complexity of number theory. The two instances D=61 and D=109 are two particular instances that Fermat presents to Frenicle in 1657 CE. The smallest positive integer solution to 109x2+1=y2 is provided by y=158070671986249 and X=15140424455100.

Brouncker discovers a broad solution to Fermat's dilemma in 1657. Brouncker uses his approach to determine the (smallest) answer of $313x^2+1=y^2$ in response to a challenge from Frenicle. The solution is X=1819380158564160 y=32188120829134849

The two most influential mathematicians of the eighteenth century, J.L. Lagrange (1736–1813) and L. Euler (1707–83), tackled Fermat's issue once more in the following century. Around 1730, Euler starts to show interest in the issue, and around 1768, Lagrange takes it up. In order to prove that if D is not a perfect square, √D has an infinite but periodic continued fraction expansion, and that all solutions (p,q) to the equation $Dx^2+1=y^2$ are provided by specific convergents qn/pn of the expansion, they develop the theory of continued fractions and examine the problem within its framework. Furthermore, Dx2+1=y2 has a positive integer solution (x1,y1) such that all positive integer solutions are given by (x_n,y_n, where x_n,y_n are specified by the connection $y_n+\sqrt{D}x_n=(y_1+\sqrt{D}x_1)^n$.

The English mathematician John Pell (c. 1611–85) is credited by Euler with creating the equation $Dx^2+1=y^2$, however there is no proof that Pell really looked into it. A superficial study of Wallis' Algebra, which primarily focuses on the work of five English mathematicians—Oughtred, Harriot, Pell, Newton, and Wallis himself—could have caused Euler's confusion. Regardless, the moniker "Pell's equation" remained. According to Weil ([18], p. 174):

Pell's name appears often in Wallis's Algebra, but never in relation to the equation x2-Ny2=1, to which his name has remained attached due to Euler's incorrect attribution; we will continue to use it because its traditional designation as "Pell's equation" is clear and practical, despite the fact that it historically incorrect. These days, the equation $Dx^2+1=y^2$ is rapidly being dubbed the Brahmagupta–Pell equation because Brahmagupta was the first mathematician to study this significant equation in a comprehensive context.

Now, I am currently utilizing new discovered novel idea (The KAS sequence approach) to determine the integral solution of Bharamgupt's indeterminate equations (Varg prikriti equation). It takes very little time and is quite simple. Additionally, it is highly helpful to determine the integral solution of a linear equation. This newly discovered approach is entirely conceptual.

As Brahmagupta's pioneering work on the varga-prakṛti developed the composition law bhāvanā on the solution space of the equation $Dx^2+1=y^2$. He found

$(0,1),(2ab,b^2+Da^2),[4ab(b^2+Da^2),(3ab^2+Da^3,D3a^2b+b^3), (4ab^3+D4a^3b),D6a^2b^2+D^2a^4+b^4)$…………

This process can be repeated to generate infinitely many distinct solutions.
Let us analysis Bharamgupt indeterminate equation with help of KAS sequence-
Let first three solution of this equation $Dx^2 + 1 = y^2$ are (0,1), (α, β) and $(2\alpha\beta, \beta^2+D\alpha^2)$

Let us see sequence in x-coordinate:

$$(0, \alpha, 2\alpha\beta)$$

$$\text{KAS Order- } KAS_0=[(K_{n+2}+K_n)/K_{n+1}]$$

$$KAS_0= \left(\frac{2\alpha\beta+0}{\alpha}\right) = 2\beta$$

KAS recurrence relation is-

$$K_{n+2}=S\, K_{n+1}- K_n \text{ where } S= KAS_0$$

As given equation $Dx^2 + 1 = y^2$, if satisfy (α, β) then

$$D\alpha^2 + 1 = \beta^2$$
$$D\alpha^2 = \beta^2-1$$

So, sequence of x-coordinate is-

I^{st} element =0
II^{nd} element= α α³β
III^{rd} element=(2β)* α – 0 =2αβ
IV^{th} element= $(2\beta)(2\alpha\beta)- \alpha =4\alpha\beta^2- \alpha =3\alpha\beta^2+ \alpha\beta^2- \alpha$
$=3\alpha\beta^2+ \alpha(D\alpha^2 + 1)- \alpha$
$=3\alpha\beta^2+ D\alpha^3$
V^{th} element = $(2\beta)(3\alpha\beta^2+ Da^3)-2\alpha\beta$
$=(6\alpha\beta^2+ 2D\alpha^3\beta)-2\alpha\beta$
$=(4\alpha\beta^2+ 2D\alpha^3\beta) +2\alpha\beta^2-2\alpha\beta$
$=(4\alpha\beta^2+ 2D\alpha^3\beta) +2\alpha\beta(b^2-1)$
$=(4\alpha\beta^2+ 2D\alpha^3\beta) +2\alpha\beta D\alpha^2$
$=(4\alpha\beta^2+ 4D\alpha^3\beta)$

Continue, we can find infinite elements of this sequency.
We can also continue-

$$(0,a, 2\alpha\beta, 3\alpha\beta^2+ D\alpha^3, 4\alpha\beta^2+ 4D\alpha^3\beta \ldots\ldots)$$

Or

$$(0, \alpha, 2\alpha\beta, 4\alpha\beta^2- \alpha, 8\alpha\beta^3-4\alpha\beta, 16\alpha\beta^4-12\alpha\beta^2- \alpha, \ldots\ldots)$$

Let us see in y coordinate-

$(1, \beta, \beta^2+D\,\alpha^2, D3\,\alpha^2\,\beta+\beta^3, D6\,\alpha^2b^2+D^2\,\alpha^4+\beta^4)$,.........

K A Saini Order-

$KAS_0=[(K_{n+2}+K_n)/K_{n+1}]$ $2\alpha^2\beta$ 2 α β

$[(K_{n+2}+K_n)/K_{n+1}]$

$= [(\beta^2+D\,\alpha^2+1)/\,\beta]$

$= [(\beta^2+\beta^2-1+1)/\,\beta]$

$= 2\,\beta$

Similiarly,

$KAS_0=[(\,D3\,\alpha^2\,\beta+\beta^3+\beta)/\,\beta^2+D\,\alpha^2]$

$=[(\,3D\alpha^2\beta+\beta^3+\beta)/\,\beta^2+\beta^2-1]$

$=[\{\,3(\beta^2-1)\,\beta+\beta^3+\beta\,\}/\,\beta^2+\beta^2-1]$

$=[(\,4\beta^3-2\,\beta)/\,2\,\beta^2-1]$

$= 2\beta$

$KAS_0=[(\,D6\,\alpha^2b^2+D^2\,\alpha^4+\beta^4+\beta^2+D\,\alpha^2)/\,D3\,\alpha^2\,\beta+\beta^3]$

$=2\beta$

So, KAS_0 of y coordinate= 2β

Let first two y coordinate is known then other coordinate with help of this sequency-

I^{st} element =1

II^{nd} element= β

III^{rd} element= $2\,\beta\,(\beta)-1 = 2\,\beta^2-1= \beta^2+D\,\alpha^{2`}$

IV^{th} element= $(2\beta)\,(2\beta^2-1)-\,\beta = 4\beta^3-3\beta =2\,\beta\,(\beta^2+D\alpha^2)-\,\beta$

$=2\beta^3+2D\alpha^2\beta-\beta$

$=2D\alpha^2\beta+\beta^3+\beta^3-\beta$

$=2D\alpha^2\beta+\beta^3+\beta\,(\beta^2-1)$

$=2D\alpha^2\beta+\beta^3+D\,\alpha^2\beta$

$=3D\alpha^2\beta+\beta^3$

V^{th} element = $(2\beta)\,(4\beta^3-3\beta)-\,(2\beta^2-1) = 8\beta^4-6\beta^2-2\beta^2-1 = 8\beta^4-8\beta^2-1$

$=(2\beta)\,(3D\,\alpha^2\beta+\beta^3)-\,(\beta^2+D\,\alpha^2)$

$=D6\alpha^2\beta^{2`}+2\,\beta^4-(1+2D\alpha^2)$

$= D6\alpha^2\beta^{2`}+\beta^4+\beta^4-(1+2D\alpha^2)$

$= D6\alpha^2\beta^{2`}+\beta^4+(D\,\alpha^2+1)^2-(1+2D\alpha^2)$

$= D6\alpha^2\beta^{2`}+\beta^4+D^2\alpha^4+1+2D\alpha^2-(1+2D\alpha^2)$

$= 6D\alpha^2\beta^{2`}+\beta^4+D^2\alpha^4$

Continue we can find infinite elements of this sequency.

So, sequence will be

$(1, \beta, \beta^2+D\,\alpha^{2`}, 3D\,\alpha^2\,\beta+\beta^3, 6D\alpha^2\beta^{2`}+\beta^4+D^2\alpha^4$, ...)

Or

$(1, \beta, 2\beta^2-1, 4\beta^3-3\beta, 8\beta^4-6\beta^2-2\beta^2-1$, ...)

Thus, we conclude-
Solution of Bharamgupt indeterminate equation $Dx^2+1=y^2$

Sr. No.	X Coordinate		Y coordinate	
1	0	0	1	1
2	a	a	β	β
3	2ab	2ab	**$2β^2$-1**	$β^2+Dα^2$
4	$4ab^2-a$	$3ab^2+Da^3$	**$4β^3$-3β**	$3Da^2b+b^3$
5	$8ab^3-4ab$	$4ab^2+4Da^3b$	$8β^4-6β^2-2β^2-1$	$6Dα^2β^2$` + $β^4+D^2α^4$
6	….	…	…	…

As above we have observed on good thing that Order of KAS sequence in x-coordinate and x-coordinate is same.

Example

Example 1:[Brahmagupta] Solve, in integers, the equation $83x^2+1=y^2$.
First two solution of this equation- (0,1) and (9,82)
Other solution, with help of KAS sequency
As KAS_0=2b=2*82=164

Sr. No.	X	Y
1	0	1
2	9	82
3	(164*9-0)=1476	(164*82-1)=13447
4	(164*1476-9)=242055	(164*13447-82)=2205226
5	39695544	361643617
6	6509827161	29307347962
7	1067571958860	9726043422151
8	……	……

So, solution are -
(0,1), (9,82), (1476,13447),(242055, 2205226),(39695544, 361643617), (6509827161, 29307347962), (1067571958860, 9726043422151) ……

Conclusion

The study of KAS sequences presents a pioneering framework that intertwines modern mathematical methodologies with historical insights from Varg Prakriti structures. By systematically analyzing these sequences, this paper demonstrates their utility in solving intricate equations and uncovering patterns intrinsic to Vedic mathematical principles. The proposed methodological approach not only advances the computational efficiency of solving quadratic Diophantine equations but also bridges ancient wisdom with contemporary problem-solving techniques.

This research establishes the foundational groundwork for further explorations into sequence behaviors and their applications in algebraic and number-theoretic domains. The integration of KAS sequences within the context of Varg Prakriti underscores the enduring relevance of historical mathematical contributions while paving the way for innovative applications in modern mathematical discourse. Future research may focus on expanding the applicability of these sequences across broader mathematical paradigms and exploring their interdisciplinary potential.

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