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NUMERICAL INVESTIGATION APPROACH – COMPUTATIONAL FLUID DYNAMICS ON AERO-CONVERGED-SPIKE ROCKET NOZZLE

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ABSTRACT :

The Aero-Converged-Spike Nozzle presents a hybrid propulsion system that integrates the Convergent-Divergent (CD) Nozzle and the Aerospike Nozzle to optimize supersonic flow expansion, enhance thrust efficiency, and improve altitude adaptability. This design overcomes the limitations of conventional nozzles by combining the shock-free expansion of CD nozzles with the altitude-compensating characteristics of aerospike nozzles, resulting in superior aerodynamic performance.The nozzle geometry was formulated using the Method of Characteristics (MOC) in MATLAB, generating a precise supersonic contour. A 3D CAD model was developed in CATIA V5, and Computational Fluid Dynamics (CFD) simulations were performed in ANSYS Fluent to analyze pressure, velocity, and temperature distributions under operating conditions of 1 MPa chamber pressure and 1350 K combustion temperature.Simulation results confirm enhanced flow stability, with optimized supersonic expansion and reduced shock wave interactions. The pressure contours demonstrate effective altitude compensation, while velocity distributions indicate efficient kinetic energy conversion into thrust. Additionally, the temperature profiles reveal a significant reduction in aerothermal heating at the spike base, improving structural integrity and thermal resistance.Compared to conventional CD and aerospike nozzles, this hybrid design exhibits higher thrust efficiency, minimized energy losses, and extended operational lifespan, making it a promising advancement for next-generation aerospace propulsion systems.

KEY WORDS: Hybrid Nozzle, Aero-Converged-Spike Nozzle, Rocket Propulsion, Supersonic Flow,Thrust Optimization

INTRODUCTION:

Rocket propulsion systems depend on nozzle efficiency to maximize thrust by converting thermal energy into kinetic energy. While Convergent-Divergent (CD) Nozzles offer efficient supersonic expansion, they struggle with altitude-dependent performance, leading to thrust losses at varying atmospheric pressures. Aerospike Nozzles, on the other hand, provide altitude compensation by radially expanding exhaust gases, but they suffer from aerothermal heating and structural limitations.To address these challenges, the Aero-Converged-Spike Nozzle is introduced as a hybrid nozzle design that integrates the shock-free expansion of CD nozzles with the altitude adaptability of aerospikes. This study focuses on the design, optimization, and performance evaluation of this novel configuration. The nozzle contour is designed using MOC in MATLAB, followed by 3D modeling in CATIA V5 and CFD simulations in ANSYS Fluent. The research aims to demonstrate the improved aerodynamic performance, reduced thermal stress, and enhanced thrust efficiency of the Aero-Converged-Spike Nozzle, making it a potential candidate for next-generation aerospace propulsion systems.

CONVERGENT-DIVERGENT (CD) ROCKET NOZZLE BASICS AND WORKING:

It is a crucial component of propulsion systems, designed to accelerate exhaust gases to supersonic speeds. It consists of a convergent section where subsonic flow speeds up as it moves toward the throat, the narrowest point of the nozzle, where the gas reaches sonic velocity (Mach 1). Beyond this, the divergent section allows the gas to expand and accelerate further, reaching higher supersonic speeds, which maximizes thrust generation. This process efficiently converts thermal and pressure energy into kinetic energy, optimizing propulsion performance.

AEROSPIKE ROCKET NOZZLE BASIC AND WORKING:

An aerospike nozzle is a rocket nozzle designed for altitude compensation, where exhaust gases expand along a central spike rather than within a fixed chamber, allowing the surrounding atmospheric pressure to shape the flow efficiently at low altitudes while enabling free expansion at higher altitudes, ensuring optimal thrust, reducing flow separation, and improving propulsion efficiency across varying atmospheric conditions.

PURPOSE AND FUNCTIONALITY:

Aero-Converged-Spike Rocket Nozzle serves to ,

- **Acceleration to Supersonic Speeds:**The nozzle efficiently compresses and expands gases to achieve supersonic velocities, enhancing propulsion performance.
- **Efficient Energy Conservation:**By minimizing thermal losses and maximizing kinetic energy transfer, the nozzle ensures optimal energy utilization.
- **Optimal Flow Expansion:**The nozzle's geometry is designed to maintain uniform pressure distribution, reducing turbulence and improving exhaust streamlining.
- **Thrust Optimization**:The hybrid design maximizes propulsive force with minimal energy dissipation, ensuring higher efficiency.
- **Shock Wave Control:**The Aero-Converged-Spike Nozzle effectively manages shock waves, preventing unwanted disturbances and ensuring smooth supersonic expansion.

COMPONENTS AND MATERIALS:

The nozzle consists of the following key components:

Convergent Section: Compresses the subsonic flow toward Mach 1.

- **Divergent Section:** Expands the supersonic flow to generate thrust.
- **Aerospike Element:** Ensures altitude compensation by controlling external flow expansion.
- **Nozzle Wall Materials:** High-temperature-resistant alloys such as Inconel and titanium-based composites for durability.
- **Thermal Coatings:** Ceramic-based coatings to reduce thermal stresses on critical components.

PINTLE INJECTOR DESIGN :

The rocket engine industry uses the pintle injector as a simple innovative solution which performs effectively. Engineers use a central pintle element in this design to distribute propellant through the ring-shaped opening for fuel-oxidizer mixture effectiveness. Stable combustion and reduced combustion instability occur together with excellent thrust control through this design approach. The aerospace industry strongly favours pintle injectors because of their straightforward design which brings advantages of reliability and simplified production processes.

FLOW DYNAMICS:

The Aero-Converged-Spike Nozzle is a hybrid propulsion component that combines the features of traditional Convergent-Divergent (CD) nozzles and aerospike nozzles to enhance performance across various flight conditions. This design facilitates the acceleration of exhaust gases to supersonic speeds, optimizing thrust and energy efficiency. The nozzle's geometry ensures optimal flow expansion, effectively managing shock waves and minimizing energy losses. Additionally, the aerospike element provides altitude compensation, maintaining efficient thrust across different atmospheric pressures. This integration results in improved propulsion efficiency, reduced aerodynamic heating, and consistent performance from sea level to high altitudes.

PROPULSION SYSTEMS:

Propulsion systems are essential for generating thrust in aerospace applications, utilizing various propellant types, each with distinct characteristics.

- **Solid Propellant Systems:**These systems use a solid mixture of fuel and oxidizer that burns rapidly upon ignition, producing high-pressure gases expelled through a nozzle to generate thrust. Advantages include simplicity and readiness for immediate use, making them ideal for military applications and launch vehicles. However, they lack the ability to be throttled or shut down once ignited, limiting flexibility.
- **Liquid Propellant Systems:**In these systems, fuel and oxidizer are stored separately as liquids and pumped into a combustion chamber where they mix and ignite. This configuration allows precise control over the engine's thrust, including the ability to start, stop, and throttle as needed. Common combinations include liquid oxygen with kerosene or liquid hydrogen. While offering high performance and control, liquid systems are complex due to the need for cryogenic storage and intricate plumbing.
- **Cold Gas Propellant Systems:**Cold gas propulsion utilizes compressed inert gases, such as nitrogen, expelled through a nozzle to produce thrust. These systems are simple, safe, and reliable, often used for attitude control

in spacecraft where minimal thrust is sufficient. However, they provide lower specific impulse compared to chemical propulsion methods, limiting their use to applications requiring small adjustments.

Hybrid Propellant Systems: Hybrid propulsion systems integrate aspects of both solid and liquid propellants, typically featuring a solid fuel and a liquid or gaseous oxidizer stored separately. During operation, the oxidizer flows over the solid fuel, initiating combustion. This design offers several advantages:

- ✓ **Safety:** Storing fuel and oxidizer separately reduces the risk of accidental ignition during handling and transportation.
- ✓ **Thrust Control:** Similar to liquid systems, hybrids can be throttled, stopped, and restarted by regulating the oxidizer flow.
- ✓ **Simplicity:** Compared to liquid systems, hybrids have a less complex design, as they eliminate the need for cryogenic storage of both propellants.

Each propulsion system type offers distinct benefits and limitations, influencing their suitability for specific missions. The choice depends on factors like mission requirements, desired performance, safety considerations, and cost constraints.

CONCEPTUAL DESIGN AND OPTIMIZATION:

Designing and optimizing hybrid rocket propulsion systems involve several key technical steps.

Propulsion Analysis: Evaluate the performance of potential propellant combinations by analyzing parameters such as specific impulse and thrust. This involves modeling the combustion process and exhaust characteristics to identify optimal fuel and oxidizer pairs.

Structural Design: Design the motor casing and internal components to withstand operational pressures and thermal loads. Material selection is crucial to ensure structural integrity while minimizing weight.

Aerodynamic Shaping: Optimize the external shape of the rocket to reduce drag and improve stability during flight. Computational fluid dynamics (CFD) simulations assist in refining the aerodynamic profile.

Nozzle Configuration: Design the nozzle to efficiently convert thermal energy into kinetic energy, maximizing thrust. This includes determining the optimal expansion ratio and contour to achieve desired performance.

Grain Geometry Optimization: Tailor the shape of the solid fuel grain to control the burn rate and ensure consistent thrust. Common geometries, such as wagon-wheel designs, are analyzed for their impact on regression rates and combustion efficiency.

Thermal Management: Implement cooling strategies and select materials capable of withstanding high temperatures to protect engine components from thermal degradation.

System Integration: Ensure all subsystems, including propulsion, structural, and control systems, work cohesively. This involves verifying compatibility and performance through system-level simulations.

Optimization Algorithms: Apply computational optimization techniques, such as genetic algorithms, to iteratively improve design parameters, balancing performance, cost, and safety considerations.

TYPES OF NOZZLES AND THEIR CONCEPTS:

Rocket nozzles are critical components in propulsion systems, designed to efficiently convert thermal energy from combustion into directed kinetic energy, thereby producing thrust. Various nozzle designs have been developed to optimize performance under different operating conditions:

1. **Conical Nozzle:**A conical nozzle features straight, tapering walls that converge to a throat and then diverge. While simple to design and manufacture, conical nozzles are less efficient compared to more advanced shapes due to higher divergence losses.
2. **Bell (Contour) Nozzle:**The bell or contour nozzle improves upon the conical design by incorporating a curved profile that more effectively directs exhaust gases, reducing divergence losses and improving overall efficiency. This design allows for a shorter nozzle length while achieving better performance.
3. **Aerospike Nozzle:**Aerospike nozzles utilize a central spike or plug around which exhaust gases flow, allowing for altitude compensation by adjusting the expansion of gases relative to ambient pressure. This design maintains optimal efficiency across a wide range of altitudes but is more complex and challenging to manufacture.
4. **Expansion-Deflection Nozzle:**This nozzle design employs a center pintle that deflects exhaust gases outward, allowing for altitude compensation by adjusting the expansion of gases relative to ambient pressure. Expansion-deflection nozzles aim to maintain optimal performance across varying altitudes.
5. **Plug Nozzle:**Plug nozzles feature a solid center-body around which exhaust gases expand. This design allows for altitude compensation by adjusting the expansion of gases relative to ambient pressure, aiming to maintain optimal performance across varying altitudes.
6. **Dual-Bell Nozzle:**The dual-bell nozzle incorporates two bell-shaped sections with different expansion ratios, allowing for efficient performance at both sea level and high altitudes. The nozzle transitions between the two sections based on ambient pressure, optimizing thrust throughout the rocket's ascent.

DESIGN THEORY:

CD Nozzle

The Convergent-Divergent (CD) Nozzle is a classic supersonic nozzle design used in rocket propulsion. Its design is based on isentropic flow relations, where the flow accelerates subsonically through the converging section, reaches sonic velocity at the throat, and transitions to supersonic velocity in the diverging section.

- Design Methodology: Governed by the area ratio (Ae/AtA_e/A_tAe/At) for desired Mach number and using isentropic flow relations.

Throat Conditions:

- Flow is choked (Mach 1) at the throat.
- Downstream pressure controls supersonic flow expansion.

Aerospike Nozzle

An Aerospike Nozzle provides altitude compensation by expanding exhaust gases radially along the spike's surface.
Advantages:

- Adapts to external pressure changes at varying altitudes.
- Reduces energy losses due to flow separation at low atmospheric pressures.

Limitation:

- Exposed spike structure suffers from aerothermal heating, limiting its application.

Aero-Converged-Spike Nozzle

The Aero-Converged-Spike Nozzle combines the best features of both designs:

1. Reduces thermal stress on the aerospike by partially shielding it within the CD nozzle.
2. Provides efficient supersonic flow expansion and minimizes shock wave formation.
3. Offers excellent thrust performance across altitudes.

WHY THIS NOZZLE?

The Aero-Converged-Spike Nozzle is designed to enhance propulsion efficiency by integrating the advantages of traditional Convergent-Divergent (CD) nozzles and aerospike nozzles. This hybrid design addresses several key performance aspects:

1. Efficiency of Flow Expansion
By combining the efficient supersonic expansion characteristics of CD nozzles with the altitude-compensating properties of aerospike nozzles, the Aero-Converged-Spike Nozzle ensures optimal exhaust flow expansion across various altitudes. This results in improved thrust and overall propulsion efficiency.

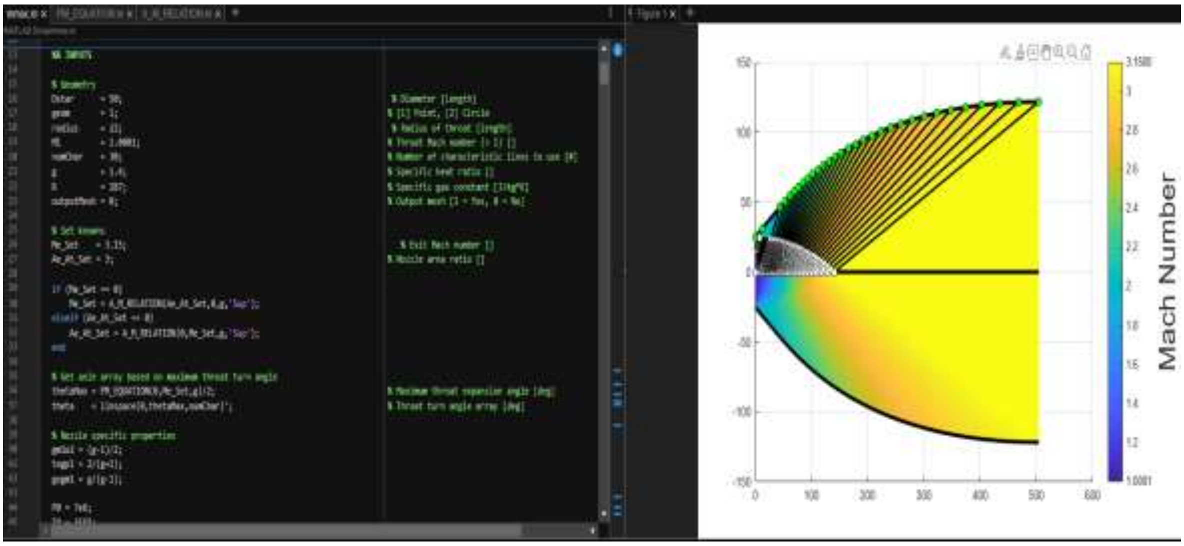
2. Adaptation to Ambient Pressure
The aerospike component of the nozzle allows for passive altitude compensation, enabling the nozzle to adjust to varying atmospheric pressures during ascent. This adaptability maintains consistent performance and efficiency throughout the flight trajectory.

3. Thermal and Structural Characteristics
The hybrid design aims to mitigate the thermal challenges associated with traditional aerospike nozzles by optimizing the nozzle contour and incorporating advanced materials. This enhances the thermal resistance and structural integrity of the nozzle under varying operational conditions.

4. Dependability
By integrating the proven designs of CD and aerospike nozzles, the Aero-Converged-Spike Nozzle offers a reliable propulsion solution. Its ability to maintain efficiency across different altitudes and conditions contributes to the overall dependability of the propulsion system.

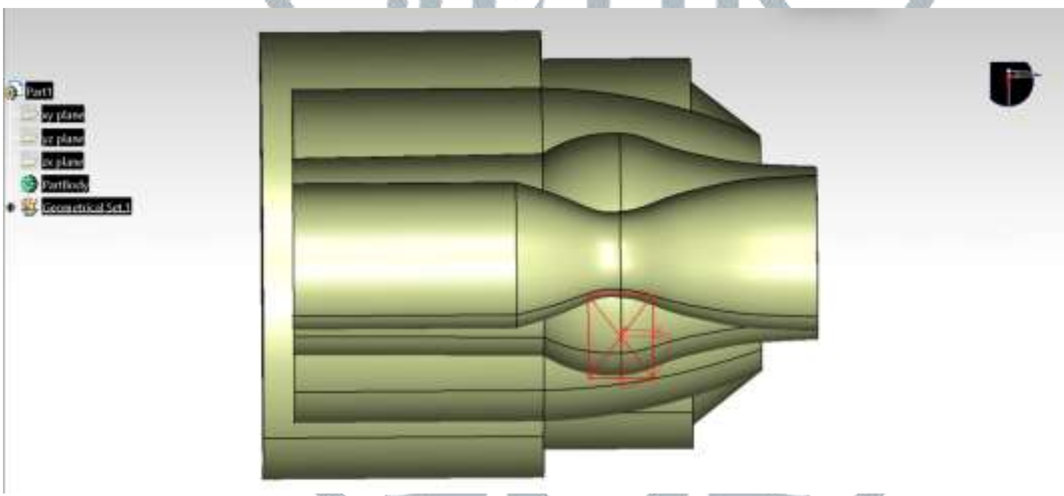
METHODOLOGY:

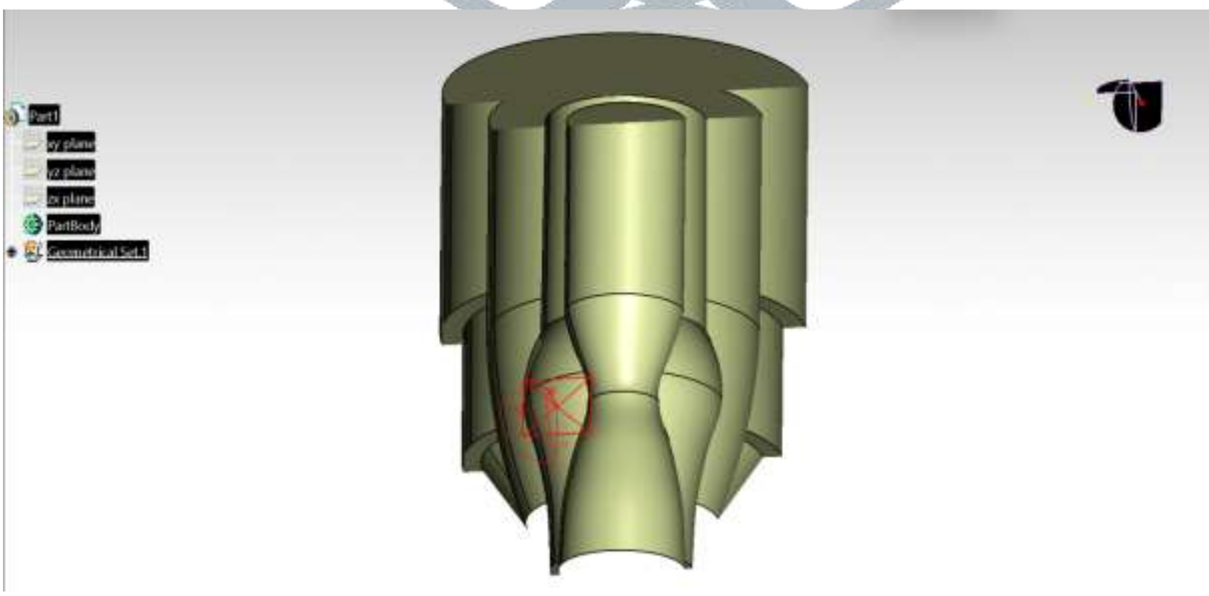
The nozzle contour is derived using the Method of Characteristics (MOC) in MATLAB.Design constraints include chamber pressure of 1 MPa and temperature of 1350 K.The nozzle geometry is optimized for uniform pressure distribution and minimal shock formation.

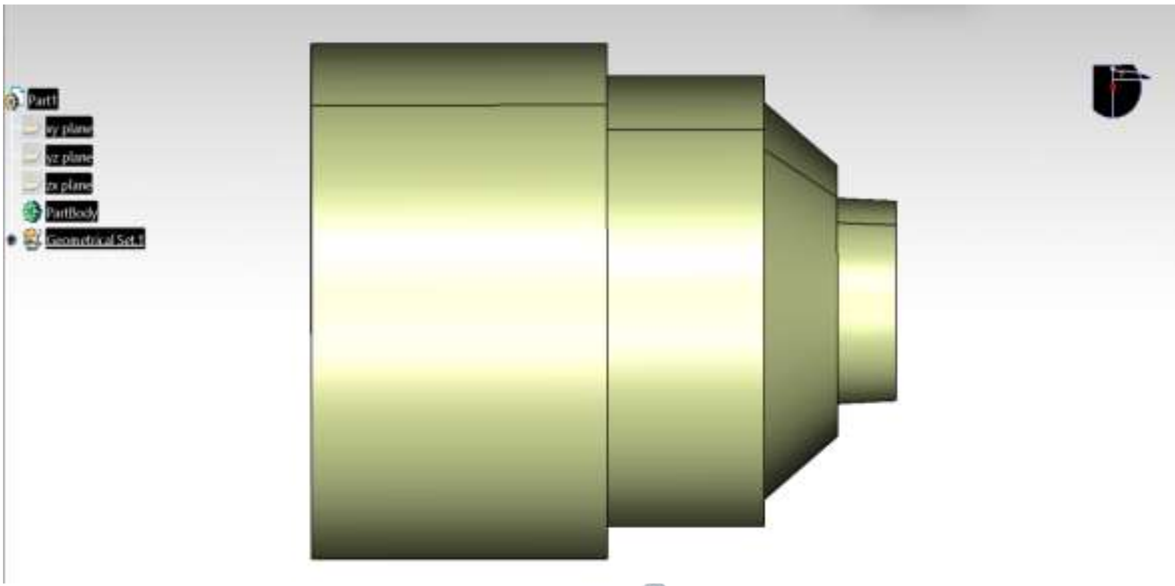


1.3D Modeling in CATIA V5

MATLAB-generated coordinates are imported into CATIA V5 for precise 3D modeling.
The model ensures smooth transitions between the CD nozzle and aerospike regions.
Virtual assembly and interference checks validate design integrity.







2.Material Selection:
Titanium: Selected for the nozzle and combustion chamber due to its outstanding thermal resistance and durability.
Graphite: Graphite can endure high temperatures up to approximately 2600 K, making it suitable for short-duration applications. (Emerson Vargas Niño)
Carbon-Carbon Composites: These materials offer high strength-to-weight ratios and thermal resistance, beneficial for aerospike nozzles.
The injector along with the spark plug housing uses stainless steel due to its strength properties which help withstand operational stresses.

3. Simulation Using ANSYS Fluent:
The analysis used enhanced mesh boundaries and narrow mesh structures to enhance simulation flow prediction accuracy.
Boundary Conditions: Inlet: Chamber pressure of 1MPa at a temperature of 1350 K.
Outlet: Atmospheric pressure (~101325Pa) at a temperature of 216.65 K 20 km above sea level.
The analysis of walls included an assumption of adiabatic conditions to evaluate thermal performance.

ANALYSIS:

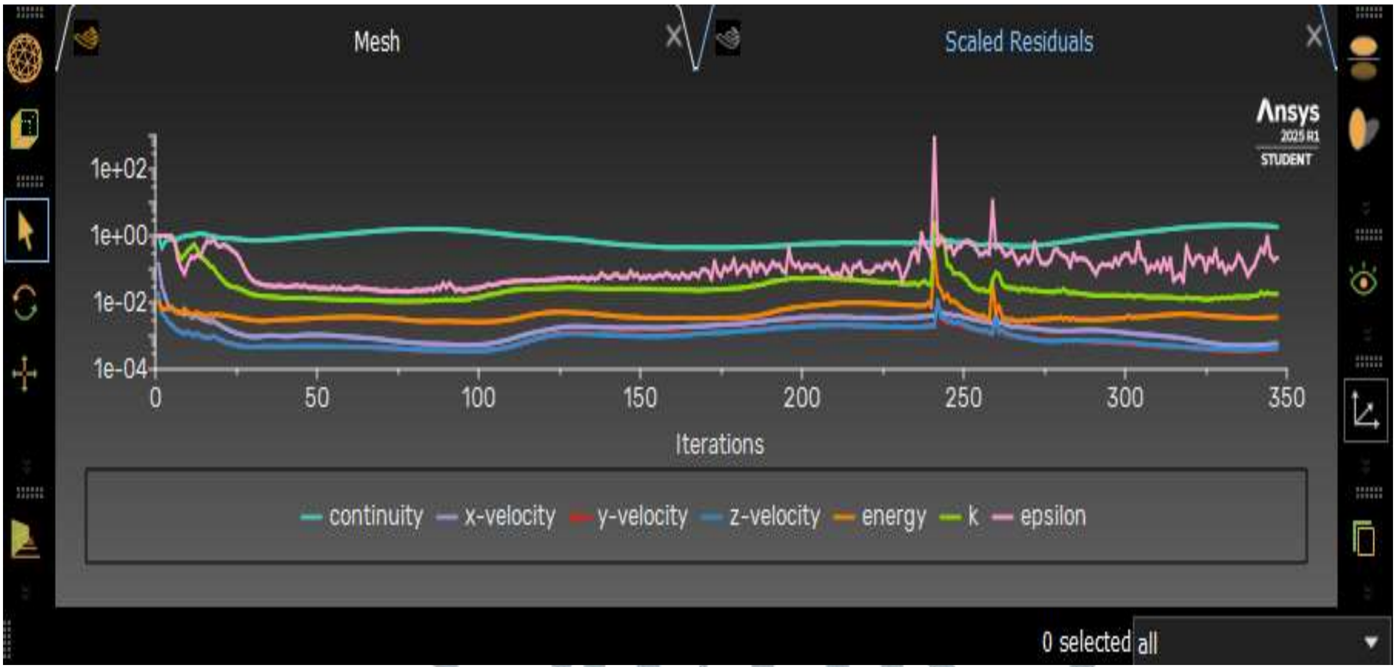


Fig 1: The graph of the Aero converged spike nozzle

Velocity:

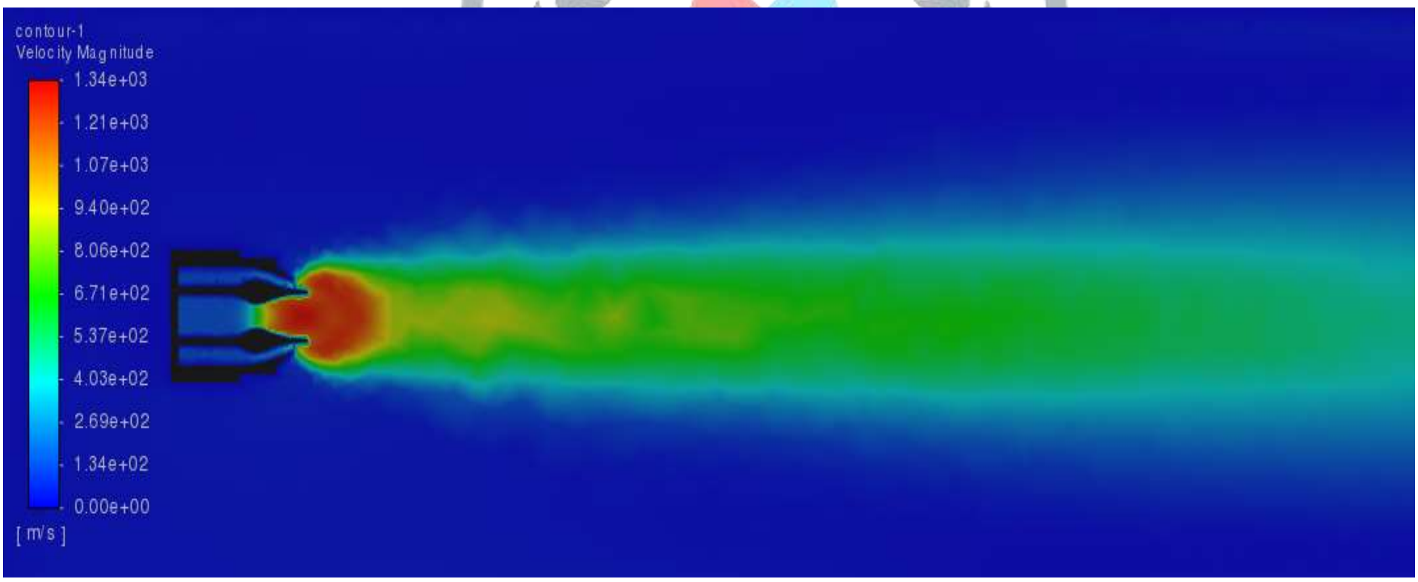


Fig:2- Velocity

Maximum Velocity : 1441.476 m/s

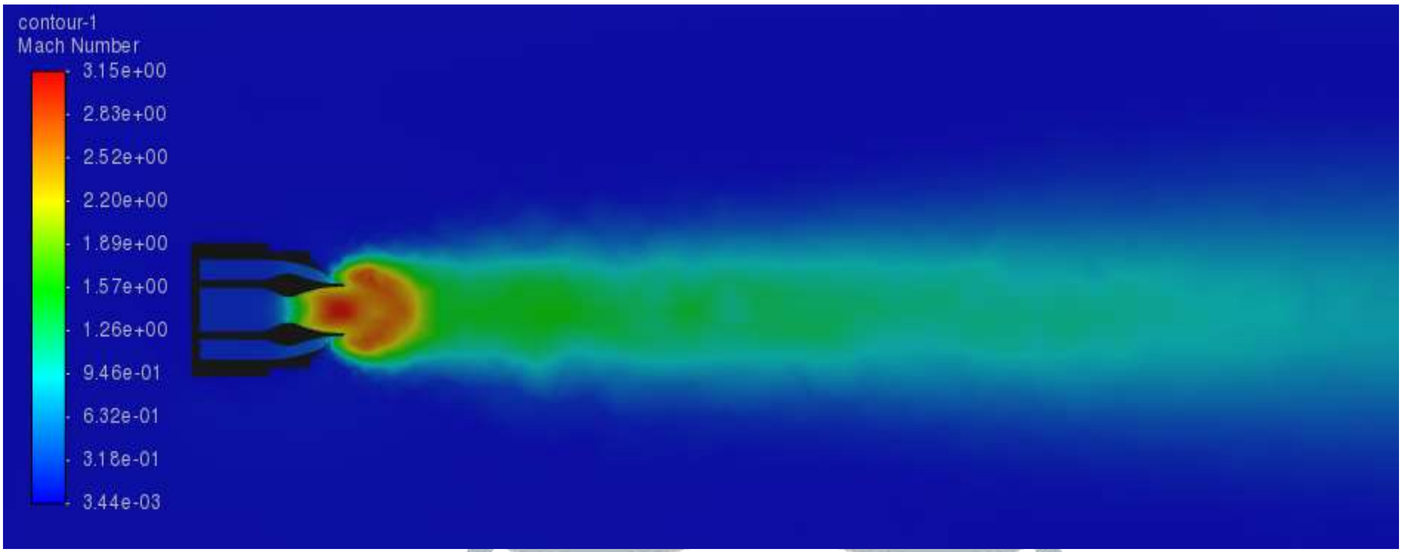


Fig:3- Mach Number

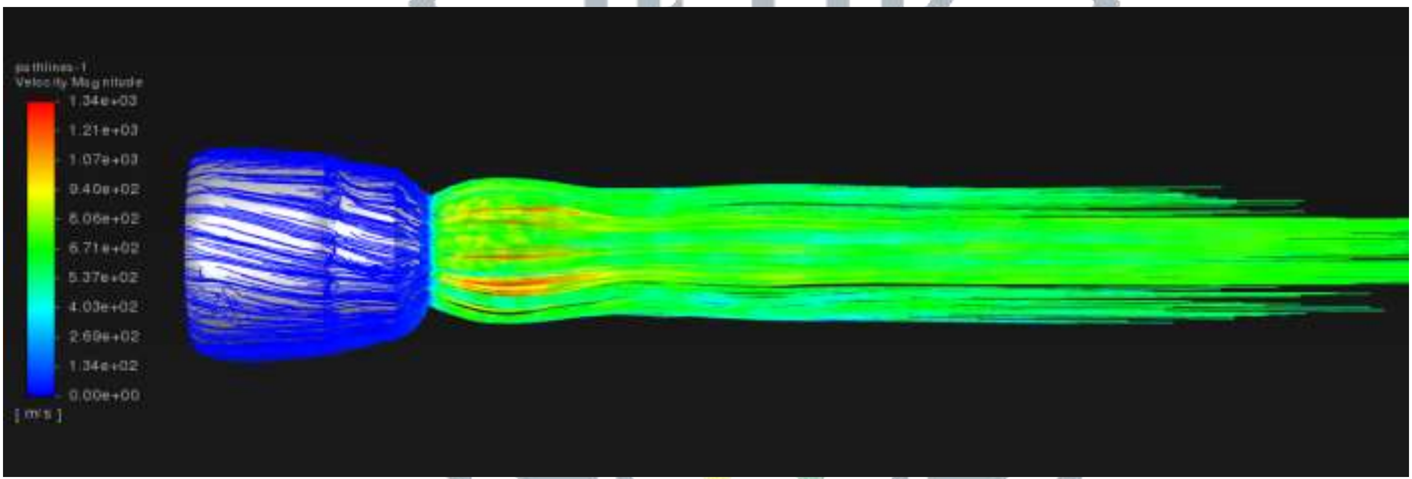


Fig:4 – Velocity Pathline

The velocity contour reveals how the exhaust gases accelerate through the nozzle and beyond:

1. **Subsonic to Supersonic Transition:**
   - In the convergent section, the flow remains subsonic and reaches Mach 1 at the throat.
   - As the flow enters the divergent section, it accelerates due to the pressure drop, achieving supersonic velocities.
2. **Aerospike Influence:**
   - The aerospike generates a controlled expansion of exhaust gases, reducing turbulence and maintaining high velocity.
   - Near the aerospike's surface, flow vectors indicate a well-formed supersonic jet, minimizing kinetic energy losses.
3. **Exit Velocity:**
   - The exit velocity reaches peak values, indicative of efficient energy conversion from thermal energy to kinetic energy.
   - These high velocities directly translate to increased thrust and propulsion efficiency.

Pressure :

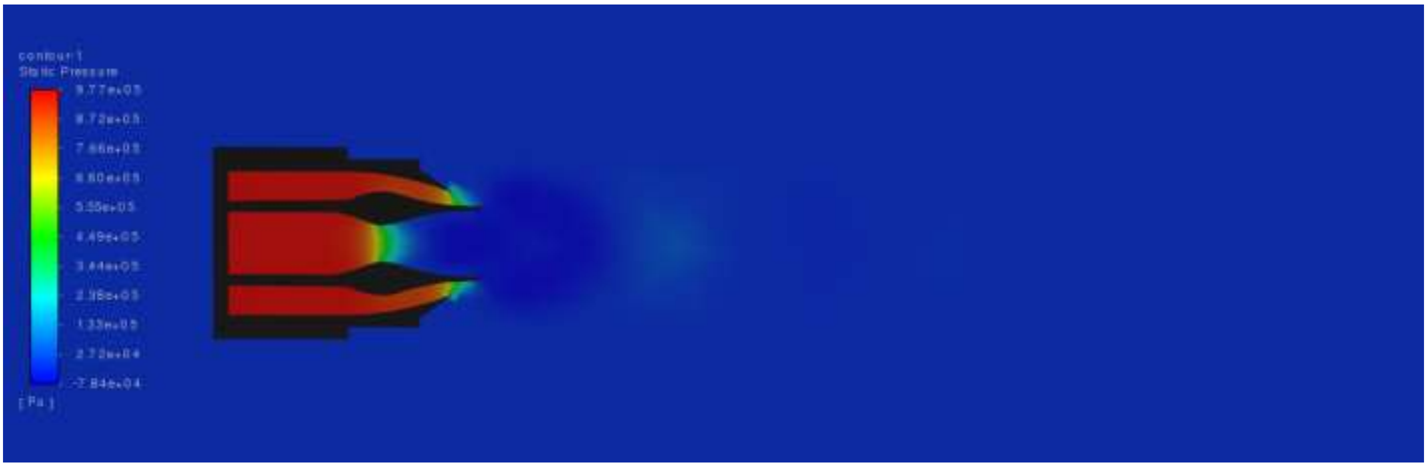


Fig:5 – Static Pressure

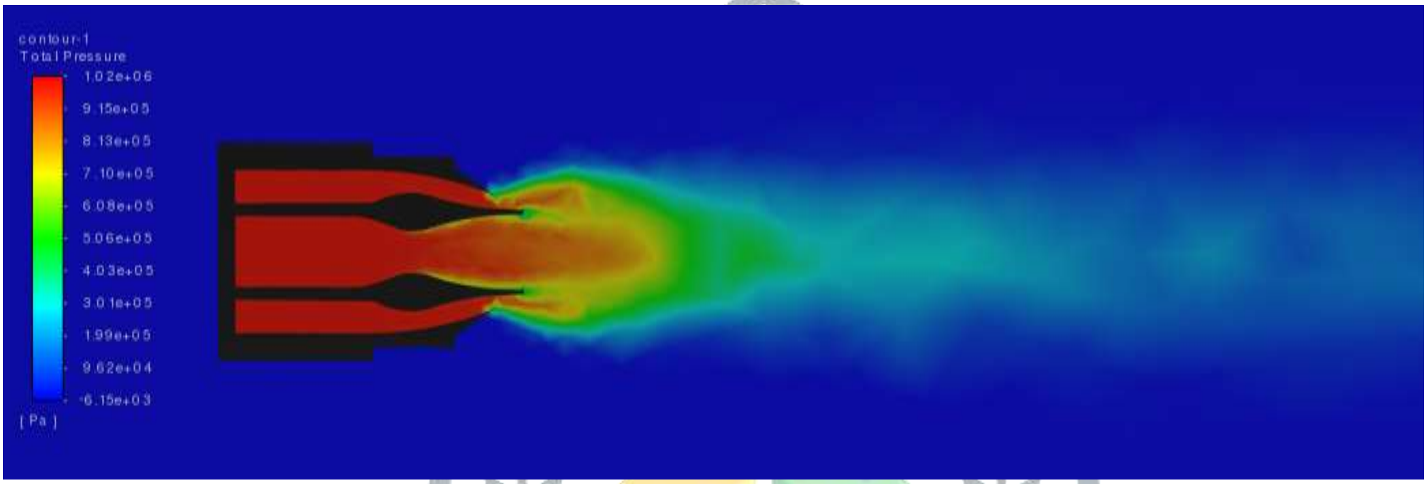


Fig:6 – Total Pressure

Maximum Pressure : 4893440 Pa
Minimum Pressure : -89751.73 Pa

The pressure contour illustrates how the exhaust gases expand and interact with the nozzle walls and the external atmosphere.

1. **Chamber Pressure:**
   - At the inlet (combustion chamber), the pressure is maintained at 1 MPa.
   - This high pressure ensures that the exhaust gases are forced through the convergent section into the throat, where the flow reaches Mach 1 (choked flow).
2. **Throat Region:**
   - The pressure drops significantly as the flow accelerates from subsonic to supersonic speeds.
   - At this point, the nozzle's geometry is critical, as any irregularities could create unwanted shock waves.
3. **Divergent Section:**
   - Smooth expansion is observed, with the pressure decreasing as the gases accelerate to supersonic velocities.
   - The aerospike plays a key role in maintaining uniform pressure distribution across the exhaust plume.
4. **External Plume:**
   - The aerospike nozzle's altitude-compensating design ensures that the exhaust flow adapts to atmospheric pressure variations.
   - This results in a near-optimal pressure ratio at all altitudes, reducing flow separation and improving thrust efficiency.

Temperature :

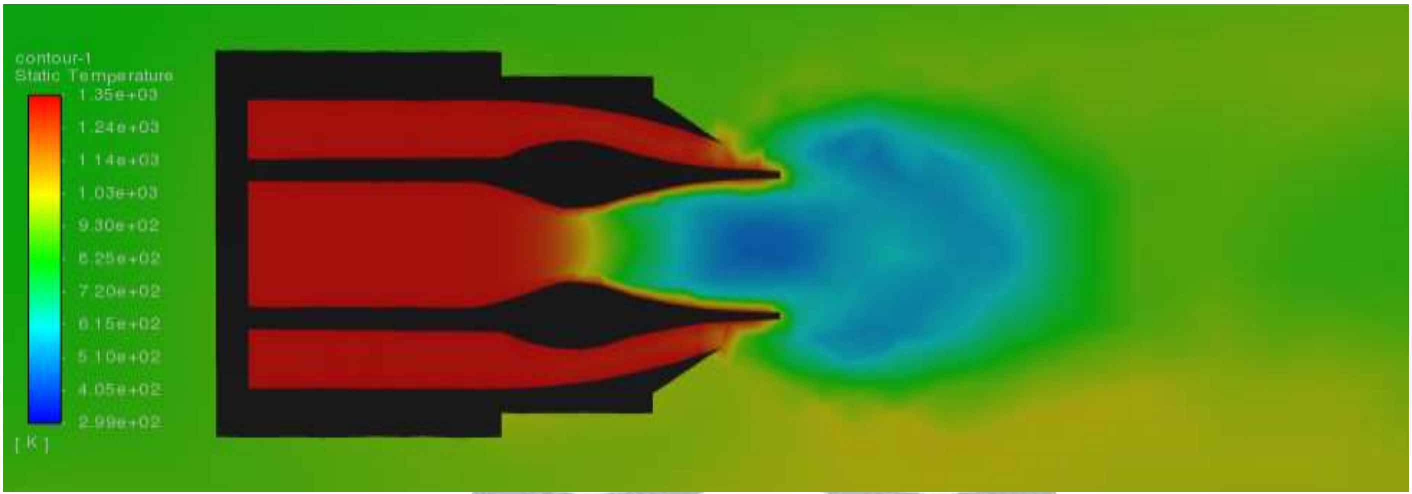


Fig:7- Static Temperature

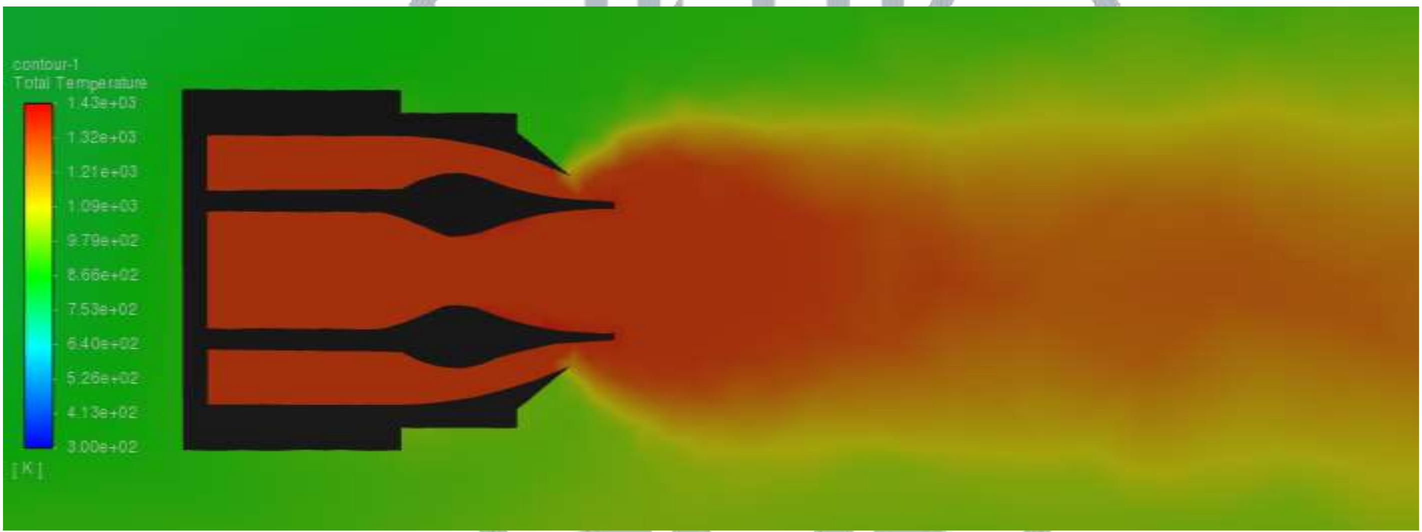


Fig:8 - Total Temperature

Maximum Temperature : 1256.675 K
Minimum Temperature : 215.8036 K

Temperature contours highlight the thermal characteristics of the nozzle, essential for understanding material performance and durability.

1. **Combustion Chamber:**
   - The temperature is highest at the chamber inlet, around 1350 K, due to the combustion of propellant.
   - This high temperature drives the expansion of gases and contributes to thrust.
2. **Throat Region:**
   - A drop in temperature is observed as the gases accelerate and expand through the throat. This follows the principles of isentropic flow, where velocity increases at the expense of temperature.
3. **Aerospike Thermal Impact**:
   - Conventional aerospike nozzles suffer from aerothermal heating, especially at the spike tip. However, the Aero-Converged-Spike design mitigates this by partially shielding the spike within the CD nozzle.
   - The contours reveal a reduction in thermal gradients near the spike base, extending the material's lifespan.
4. **External Plume:**
   - The temperature continues to drop as the gases mix with the atmosphere, showcasing effective expansion and cooling.

PERFORMANCE IMPROVEMENT OVER CONVENTIONAL NOZZLES

1. **Compared to CD Nozzles:**
   - While CD nozzles are efficient in ideal conditions, they struggle at varying altitudes due to the mismatch between nozzle exit pressure and ambient pressure.
   - The Aero-Converged-Spike Nozzle adapts to altitude changes, minimizing thrust loss.
2. **Compared to Aerospike Nozzles:**
   - Aerospike nozzles excel at altitude compensation but are prone to aerothermal heating.
   - The hybrid design mitigates this issue by partially integrating the aerospike with the CD nozzle, reducing thermal stresses.
3. **Efficiency Gains:**
   - The hybrid design achieves higher overall thrust efficiency due to better flow stability and thermal management.

GRAPHICAL REPRESENTATION:

1. **Pressure Distribution:**
   - Graphs show a smooth pressure drop from the chamber to the nozzle exit.
   - External pressure adapts seamlessly with the ambient pressure, confirming the altitude-compensating nature.
2. **Velocity Profiles:**
   - Velocity rises sharply in the divergent section, with peak values at the nozzle exit.
   - Flow vectors near the aerospike confirm minimal turbulence and effective energy utilization.
3. **Temperature Profiles:**
   - Gradual cooling of exhaust gases from chamber to exit demonstrates effective thermal expansion

RESULTS:

The Aero-Converged-Spike Nozzle was analyzed using Computational Fluid Dynamics (CFD) to evaluate its aerodynamic performance, with key parameters such as pressure distribution, velocity profile, and temperature contours examined under operating conditions of 1 MPa chamber pressure and 1350 K temperature.

1. Pressure Distribution: The chamber pressure remains at 1 MPa, ensuring effective supersonic expansion.At the throat, pressure drops significantly as the flow transitions to Mach 1.The divergent section ensures smooth expansion with minimal shock formation.The aerospike structure provides uniform pressure adaptation, reducing thrust losses at varying altitudes.

2. Velocity Profile: The flow remains subsonic until it reaches the throat, where Mach 1 is achieved.The velocity increases sharply in the divergent section, reaching peak supersonic values at the exit.The aerospike design further improves flow uniformity and reduces turbulence, leading to higher kinetic energy conversion into thrust.

3. Temperature Contours: The combustion chamber temperature remains at 1350 K.As the flow expands through the throat, a temperature drop is observed, consistent with isentropic expansion.The aerospike shielding effect significantly reduces thermal stress, particularly at the spike base, enhancing material durability.

DISCUSSION:

The results validate the superior aerodynamic and thermodynamic efficiency of the Aero-Converged-Spike Nozzle, showcasing its advantages over conventional nozzles.

1.Improved Flow Stability and Shock Reduction: The smooth pressure transition prevents shock wave formation, a common issue in traditional CD nozzles.The aerospike integration ensures continuous supersonic expansion, minimizing flow separation losses.

2. Enhanced Thrust Efficiency: Higher exit velocities indicate improved kinetic energy conversion, leading to greater thrust production.The altitude-compensating nature of the aerospike allows optimal performance across varying atmospheric conditions.

3. Reduced Thermal Stress and Material Durability: Traditional aerospike nozzles suffer from excessive aerothermal heating at the spike tip, leading to material degradation.The hybrid design mitigates this issue by partially shielding the aerospike, significantly reducing thermal stress and extending component lifespan.

4. Comparative Performance Analysis:

Parameter	CD Nozzle	Aerospike Nozzle	Aero-Converged Spike Nozzzle
Altitude Adaptation	Poor	Excellent	Excellent
Thrust Efficiency	Moderate	High	Higher than both
Thermal Stress	Low	High(at Spike tip)	Reduced(due to shielding)
Flow Stability	Moderate	High	Superior
Shock Wave Losses	Present	Minimal	Negligible

ACKNOWLEDGMENT:

I extend my heartfelt gratitude to Prime Toolings for their invaluable support and technical guidance throughout the course of this project. The design of the Aero-Converged Spike Nozzle was provided by Prime Toolings, and their contributions played a crucial role in enhancing the technical quality and practical relevance of this work. The hands-on experience and real-time exposure I gained at Prime Toolings significantly deepened my understanding of nozzle design, computational simulations, and propulsion system analysis.I am also thankful to my institution for providing access to essential software tools like MATLAB, CATIA V5, and ANSYS Fluent, which were critical in carrying out the design validation and CFD analysis.I sincerely appreciate the constant encouragement and insightful discussions shared with my friends and teammates, which helped me stay motivated and focused. Most importantly, I am deeply grateful to my family for their unwavering support, patience, and motivation throughout the project journey.This project would not have been possible without the valuable involvement and encouragement from everyone mentioned above, especially the significant contribution of Prime Toolings, which added practical depth and industry insight to my academic research.

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