



# An Inventory Model of Deteriorating Items with Exponentially Declining Demand, variable Holding Cost and Shortages

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**Abstract :** This paper deals with an inventory model under exponentially declining demand and constant deterioration. The variable holding cost is a linear function of time. There are fully backlogged shortages assumed in the present model. The items (like food grains, fashion apparels and electronic equipments) have fixed shelf-life which decreases with time during the end of the season. The inventory model is minimized to the total cost by finding optimal values. The developed model is illustrated by a numerical example and lastly the sensitivity analysis of the optimal solutions towards the changes in the values of key parameters has been presented.

**IndexTerms** - Deteriorating items, exponentially declining demand, variable holding cost and shortages.

**Subject classification:** AMS Classification No. 90B05

## 1. INTRODUCTION:

In tradition, the inventory model is assumed in which depletion of inventory is assumed as demand rate in constant nature. Whitin is the founder researcher of this development. Our focus of the proposed model is to develop an inventory model for a deteriorating item having a time-dependent exponential declining demand rate and variable time dependent holding cost. The literature review of the present paper is discussed as follows:

Normally, deteriorating items refer to the items that become damaged, decayed, spoiled, lost of utility or lost of its marginal value, evaporative, devaluation, invalid, degradation and so on through time. There are two process of deterioration happening in two categories of items. The first category refers to the items that become damaged, spoiled, decayed, loss of utility, evaporative or expired through time like food grain, food stuffs, fruits, flowers, vegetables, films, medicines and so on, while the other category refers to the items that loss their parts or their total values through time because of the in grounding a new technology or the alternatives like fashion and seasonal goods, electronic equipment, computer chips and mobile phones and so on. In the first category, a short life cycle of items is seen whereas in the second category, a short market life of items is seen. The literature surveys by Raafat [1], Wee [2], Shah and Shah [3], Goyal and Giri [4] and Li *et al.* [5] discussed on such category of deteriorating inventory model. Harris [6] was the first researcher who studied the inventory model. Wilson [7] explained Harris' model and gave a formula to obtain (EOQ) economic order quantity. Whitin [8] developed the inventory model on fashion items deteriorating at the end of the storage period. Then Ghare and Schrader [9] assumed in their study that the consumption of the deteriorating items which was considered a negative exponential function of time. They established the inventory model with deterioration rate with constant nature and without shortages. Dave and Patel [10] discussed firstly the inventory model on deteriorating items with linear increasing demand when shortages were not allowed. Some research works related to this field has been established by Chung and Ting [11], Wee [12] and Mishra *et al.* [13]. Chang and Dye [14] established an inventory model with time-varying demand and partial backlogging. On the basis of demand variations, Ouyang and Cheng [15] developed an inventory model for deteriorating items with exponential declining demand. Abad [16,17], Shah and Raykundaliya [18] studied the case of partial backlogging in their inventory model. Singh and Pattnayak [19] also studied an inventory model for deteriorating items with demand in leaner trended, variable deterioration and partial backlogging. Aliyu[20], Dr Biswaranjan Mandal [21] also studied an inventory model on deteriorating items.

Usually, in inventory models, inventory holding cost and the demand rate are considered to be constant in nature. But sometimes in reality, we may observe that inventory holding cost and demand rate for physical goods are time-independent. The paper examines an inventory model with linearly time varying holding cost. The objective function is to minimize the total cost of an inventory system based on inventory holding cost, ordering cost and deterioration cost. In reality, the demand rate and the inventory holding cost for physical goods may be time-dependent. As time is an important factor in the inventory system, we consider that demand rate and inventory holding cost are time-dependent. Researchers have developed several EOQ inventory model for deteriorating items with exponentially declining demand when the deterioration rate follows a constant rate with time-dependent linear holding cost and shortages are not allowed.

In an inventory model, a "shortage" refers to a situation where the demand for a product exceeds the available inventory, meaning there are not enough items on hand to meet customer orders, resulting in a "stockout" where customers cannot purchase the product immediately; typically, these shortages are either backlogged (customers wait for the item to be restocked) or lost sales if the customer chooses to buy elsewhere. So shortages can lead to lost sales, unhappy customers, and damage to a company's reputation. There are many researchers like [M. Maragatham](#) [22], Rabeya Sarker [23] and many more who have developed several inventory models assuming shortages with fully backlogged.

On the above facts and features, efforts have been made to develop with an inventory model under exponential declining demand and constant deterioration. The variable holding cost is assumed as a linear function of time. Shortages are allowed with fully backlogged. The model is minimized to the total cost by finding optimal values. The developed model is illustrated by a numerical example and lastly the sensitivity analysis of the optimal solutions towards the changes in the values of key parameters has been presented.

## 2. ASSUMPTIONS AND NOTATIONS:

### 2.1 Assumptions:

The present inventory model is developed on the basis of the following assumptions

- i. Lead time is zero.
- ii. Replenishment rate is infinite but size is finite.
- iii. The time horizon is finite.
- iv. There is no repair of deteriorated items occurring during the cycle.
- v. The deterioration rate is considered as constant.
- vi. The demand rate is deterministic and is an exponential declining function of time.
- vii. Holding cost follows time-varying linear trended.
- viii. Shortages are allowed and completely backlogged.

### 2.2 Notations:

The following notations are used in the proposed model:

- i.  $Q$  : The size of initial inventory.
- ii. The constant rate of deterioration of the on-hand inventory is  
 $\theta(t) = \theta, \theta > 0$ .
- iii.  $t_1$  : The time length in which the stock is completely diminished.
- iv.  $T$  : The fixed length of each production cycle.
- v.  $A_0$  : Ordering cost of inventory.
- vi.  $c_d$  : The deteriorating cost per unit item.
- iv.  $h(t)$  : Holding cost  $h(t) = \alpha + \beta t, \alpha, \beta > 0$ .
- vii.  $c_s$  : The shortage cost per unit item.
- viii.  $D(t)$  : Demand rate  $D(t) = ke^{-\nu t}, k, \nu > 0$ .
- ix.  $q(t)$  : The level of inventory  $q(t) = \begin{cases} q_1(t), 0 \leq t \leq t_1 \\ q_2(t), t_1 \leq t \leq T \end{cases}$
- x.  $TC$  : total cost per unit time.

## 3. FORMULATION AND SOLUTION:

In this model, we consider the variation of the inventory level during the period  $[0, T]$ . The inventory level is depleted only due to demand and deterioration and ultimately falls to zero at  $t = t_1$ . The shortages occur during time period  $[t_1, T]$  which are completely backlogged. The differential equations pertaining to the above situations are given by

$$\frac{dq_1(t)}{dt} + \theta(t)q_1(t) = -D(t), 0 \leq t \leq t_1 \quad (3.1)$$

$$\text{And } \frac{dq_2(t)}{dt} = -D(t), t_1 \leq t \leq T \quad (3.2)$$

$$\text{The initial conditions are } q_1(0) = Q, q_1(t_1) = 0 \text{ and } q_2(t_1) = 0 \quad (3.3)$$

Putting the values of  $\theta(t) = \theta$  and  $D(t) = ke^{-\gamma t}$ , and then solving the equations (3.1) and (3.2), using the initial condition (3.3), we get the following

$$q_1(t) = Qe^{-\theta t} + \frac{k}{\theta - \gamma}(e^{-\theta t} - e^{-\gamma t}), \quad 0 \leq t \leq t_1 \quad (3.4)$$

$$\text{And } q_2(t) = \frac{k}{\gamma}(e^{-\gamma t} - e^{-\gamma t_1}), \quad t_1 \leq t \leq T \quad (3.5)$$

Since  $q_1(t_1) = 0$ , we get the following expression of initial inventory from the equation (3.4)

$$Q = \frac{k}{\theta - \gamma} \{e^{(\theta - \gamma)t_1} - 1\} \quad (3.6)$$

#### 4. COST COMPONENTS:

The total cost over the period  $[0, T]$  consists of the following cost components:

- The ordering cost for raw materials is fixed at  $A_0$ . (4.1)
- The cost of deterioration of items ( $C_D$ ) over entire cycle  $(0, T)$  is given by

$$\begin{aligned} C_D &= [c_d [Q - \int_0^{t_1} D(t) dt]] = c_d [Q - k \int_0^{t_1} e^{-\gamma t} dt] \\ &= c_d [Q + \frac{k}{\gamma}(e^{-\gamma t_1} - 1)] \\ &= c_d k [\frac{1}{\theta - \gamma} \{e^{(\theta - \gamma)t_1} - 1\} + \frac{1}{\gamma} \{e^{-\gamma t_1} - 1\}] \end{aligned} \quad (4.2)$$

- Holding cost for carrying inventory ( $C_H$ ) over the period  $[0, T]$

$$C_H = \int_0^{t_1} (\alpha + \beta t) q_1(t) dt$$

Putting the values of  $q_1(t)$  from (3.4), and integrating and then substituting the value of  $Q$  from (3.6), we get the following expression,

$$C_H = \frac{k}{\theta(\theta - \gamma)} (\alpha + \frac{\beta}{\theta}) \{e^{(\theta - \gamma)t_1} - e^{-\gamma t_1}\} + \frac{\beta k}{\theta \gamma} t_1 e^{-\gamma t_1} + \frac{k}{\gamma(\theta - \gamma)} (\alpha + \frac{\beta}{\gamma}) \{1 - e^{-\gamma t_1}\} \quad (4.3)$$

- Cost due to shortage ( $C_S$ ) over the period  $[0, T]$

$$C_S = -c_s \int_{t_1}^T q_2(t) dt$$

Putting the values of  $q_2(t)$  from (3.5), and integrating, we get the following expression

$$\text{Or, } C_S = \frac{c_s k}{\gamma} [\frac{e^{-\gamma T} - e^{-\gamma t_1}}{\gamma} + (T - t_1)e^{-\gamma t_1}] \quad (4.4)$$

Thus the total cost per unit time of the system during the cycle  $[0, T]$  will be

$$\begin{aligned} \text{TC}(t_1) &= \frac{1}{T} [A_0 + C_D + C_H + C_S] \\ &= \frac{1}{T} [A_0 + c_d k [\frac{1}{\theta - \gamma} \{e^{(\theta - \gamma)t_1} - 1\} + \frac{1}{\gamma} \{e^{-\gamma t_1} - 1\}] + \\ &\quad [\frac{k}{\theta(\theta - \gamma)} (\alpha + \frac{\beta}{\theta}) \{e^{(\theta - \gamma)t_1} - e^{-\gamma t_1}\} + \frac{\beta k}{\theta \gamma} t_1 e^{-\gamma t_1} + \frac{k}{\gamma(\theta - \gamma)} (\alpha + \frac{\beta}{\gamma}) \{1 - e^{-\gamma t_1}\}] \\ &\quad + \frac{c_s k}{\gamma} [\frac{e^{-\gamma T} - e^{-\gamma t_1}}{\gamma} + (T - t_1)e^{-\gamma t_1}]] \end{aligned} \quad (4.5)$$

To minimize the total cost, the necessary condition is  $\frac{dTC(t_1)}{dt_1} = 0$

This gives

$$(c_d + \frac{\alpha}{\theta} + \frac{\beta}{\theta^2})(e^{\theta t_1} - 1) - (\frac{\beta}{\theta} - c_s)t_1 - c_s T = 0 \tag{4.6}$$

For minimum, the sufficient condition  $\frac{d^2TC(t_1)}{dt_1^2} > 0$  would be satisfied.

Let  $t_1 = t_1^*$  be the optimum value of  $t_1$ .

The optimal values  $Q^*$  of Q and  $TC^*$  of TC are obtained from the expressions (3.6) and (3.7) by putting the value  $t_1 = t_1^*$ .

**5. NUMERICAL ANALYSIS:**

To exemplify the above model numerically, let the values of parameters be as follows:

$A_0 = \$500/\text{order}$ ;  $k = 250$ ;  $\gamma = 0.02$ ;  $\theta = 0.8$ ;  $\alpha = 0.5$ ;  $\beta = 0.2$ ;  $c_d = \$5/\text{unit/year}$ ;  $c_s = \$10/\text{unit/year}$  and  $T = 1$  year.

Solving the equation (4.6) with the help of computer using the above values of parameters, we find the following optimum outputs

$$t_1^* = 0.631 \text{ year}; Q^* = 203.79 \text{ units and } TC^* = \text{Rs. } 802.17$$

It is checked that this solution satisfies the sufficient condition for optimality.

**6. SENSITIVITY ANALYSIS AND DISCUSSION:**

We now study the effects of changes in the system parameters  $k = 250$ ;  $\gamma = 0.02$ ;  $\theta = 0.8$ ;  $\alpha = 0.5$ ;  $\beta = 0.2$ ;  $c_d = 5$  and  $c_s = 10$  on the optimum amount of initial inventory.  $Q^*$  and optimum total cost per unit time ( $TC(t_1)$ ) in the present inventory model. The sensitivity analysis is performed by changing each of the parameters by  $-50\%$ ,  $-20\%$ ,  $+20\%$  and  $+50\%$ , taking one parameter at a time and keeping remaining parameters unchanged. The results are furnished in table A.

**Table A: Effect of changes in the parameters on the model**

Changing parameter	% change in the system parameter	The optimum time length ( $t^*$ )	% change in	
			$Q^*$	$TC^*$
k	-50	0.631	- 49.99	- 18.83
	-20	0.631	- 24.99	- 9.42
	+20	0.631	25.00	9.42
	+50	0.631	50.00	18.83
$\gamma$	-50	0.631	0.34	- 0.65
	-20	0.631	0.17	- 0.33
	+20	0.631	- 0.17	0.33
	+50	0.631	- 0.34	0.67
$\theta$	-50	0.773	10.17	5.39
	-20	0.698	5.47	2.40
	+20	0.573	- 5.68	- 2.36
	+50	0.523	- 11.15	- 5.67
$\alpha$	-50	0.643	2.48	- 0.32
	-20	0.637	1.22	- 0.12
	+20	0.625	- 1.19	0.12
	+50	0.619	- 2.35	0.31
$\beta$	-50	0.632	0.27	1.29
	-20	0.631	0.13	0.65
	+20	0.630	- 0.13	- 0.65
	+50	0.629	- 0.27	- 1.31
$c_d$	-50	0.747	24.58	- 20.11
	-20	0.683	10.82	- 10.97
	+20	0.587	- 8.14	12.23
	+50	0.549	- 15.89	25.37
	-50	0.475	- 29.45	2.98
	-20	0.566	- 12.59	1.66

$c_s$	+20	0.678	9.76	- 1.60
	+50	0.715	17.53	- 3.05

Analyzing the results of table A, the following observations may be made:

(i) The optimum initial inventory  $Q^*$  increase or decrease with the increase or decrease in the values of the system parameters  $k$  and  $c_s$ . On the other hand  $Q^*$  increase or decrease with the decrease or increase in the values of the system parameters  $\gamma$ ,  $\theta$ ,  $\alpha$ ,  $\beta$  and  $c_d$ . The results obtained show that  $Q^*$  is very highly sensitive to changes in the value of parameters  $k$ ,  $c_d$  and  $c_s$ ; moderate sensitive to the changes of parameters  $\theta$  and  $\alpha$ ; and less sensitive to the changes of parameters  $\gamma$  and  $\beta$ .

(ii) The optimum expected average total cost  $TC^*$  increase or decrease with the increase or decrease in the values of the system parameters  $k$ ,  $\gamma$ ,  $\alpha$  and  $c_d$ . On the other hand  $TC^*$  increase or decrease with the decrease or increase in the values of the system parameters  $\theta$ ,  $\beta$  and  $c_s$ . The results obtained show that  $TC^*$  is very highly sensitive to changes in the value of parameters  $k$  and  $c_d$ ; moderate sensitive for  $\theta$  and  $c_s$ ; and less sensitive to the changes of parameters  $\gamma$ ,  $\alpha$  and  $\beta$ .

From the above analysis, it is seen that  $k$  and  $c_d$  are highly sensitive parameters in the sense that any small changes in the estimation of these parameters result with significant effect in the optimal cost solution. Hence estimation of such parameters needs adequate attention.

#### SCOPE OF FUTURE WORK:

The present paper deals with an inventory model for deteriorating in which shortages are allowed with fully backlogged. The deteriorating items follows constant in nature. The demand pattern is assumed to be exponentially declining function of time. The model is minimized to the total cost by finding optimal values. Eventually, a researcher can extend this model considering ramp type demand under stochastic behaviour along with salvage value and permissible delay in payments.

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