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APPLICATIONS OF DIFFERENTIAL EQUATIONS IN REAL WORLD

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ABSTRACT

Differential equations is important part of mathematics for understanding the physical sciences. Most differential equations arise from problems in physics, engineering and other sciences and these equations serve as mathematical models for solving numerous problems in science and engineering and also technology. This study introduced real life application of first order differential equation. In this paper We basically discussed about different types of differential equation, solution of first order differential equation and application of first order differential equation in different field of science and technology. Newton's law of cooling and orthogonal trajectory has been studied.

Keywords: Differential Equations, Orthogonal Trajectory, Newtons law of cooling

Introduction:

The equation which contain derivative of one or more dependent variable with respect to one or more independent variables. They are used in different fields such as,Physics, engineering,biology, economics,chemistry. The differential equation which contains only first order derivative is known as first order differential equation. Basically, we divide differential equation into two categories that are ordinary differential equation and partial differential equation. The ordinary differential equation contains unknown functions of one independent variable whereas the partial differential equation contains unknown function of more than one independent variable.T his paper is going to discussed about a applications of first order differential equations.Hassan and Zakari Karthikeyan and Jayaraja (2016) found the use of newton's law of cooling in solving some practical problem of first order ordinary differential equations. Rehan Hsu (2018) studied the first order differential equations and newtons law of cooling. Keryzig Rehan (2020), Caronongan Hsu (n.d.), Michael Keryszig (2006). carried the solution of first order and applications of differential equations has been considerd.

Types of differential equation:

Ordinary differential equations: The equations where the derivatives are taken with respect to only one variable is known as ordinary differential equations.

For e. g. dy/dx = cosx

Partial differential equations: The equation in which one variable depends on more than one variable is known as partial differential equations.

$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial x}$

Linear differential equations: The linear differential equation is of the form dy/dx+p(x)y=Q(x)

Non-linear differential equations:A non-linear differential equation that is not a linear equation in the unknown function and its derivatives.

It is of the form . dy/dx+P y=Qy^n —

Homogeneous and non-homogeneous differential equations: A homogeneous equation does have zero on the right side of the equality while a non-homogeneous equations have a function on right side of the equations.

Applications of differential equations:

- Newtons law of Cooling:-

Newton's law of cooling, the formula of the rate of loss of heat is **– dQ/dt** of the body is directly corresponding to the difference in temperature is, **ΔT = (T2 – T1)** of the body and its surrounding environment.

Hence, this statement can be written as:

– dQ/dt = k(T2 – T1) The above **k term** is a positive constant; it depends upon the environment and surroundings of the surface of the body. Newton's law of cooling is only for determining the small temperature differences.

Method To Apply The Newton Law Of Cooling

For the application of the Newton law of Cooling, i.e., the loss of heat is considered. Mathematically, it is expressed as the term, dQ/dt

furthermore, the rate in the change of the temperature is considered between the body and the surrounding environment, which is expressed as,

[q-qs] (where q represents the temperature of the body and qs represent the temperature of the surrounding environment)

According to Newton's Law of Cooling, both equations are straight corresponding to each other,

$dQ/dt \propto [q-qs]$ $dQ/dt=k [q-qs]$

Here k is termed is donated as the constant.

However, the temperature difference between the body can be defined as the terms of the initial and final temperature of the body.

$q=(qi-qf)/2$

Here, qi term is the initial temperature of the body, whereas qf term is the final temperature of the body.

For example:A body of of temp of 40^{0c} is kept in room temp of 20^{0c} .It is observed that the temp of the body falls to 30^{0c} im 10 minutes Find how much more time will be required the body to attain a temperature 25^{0C}.

Sol. According to Newton's Law of Cooling,

$T2-T1 = 4e^{-kt}$Where T1=20,T2=40 when t=0

$\therefore$ 40-20=Ae^0 = A $\therefore$A=20

$T2-T1=20e^{-kt}$

Now when t=10,T2 =30 $\therefore$3 0 - 20 = $20e^{-10k}$,

$\therefore e^{-10k}$,=1/2 now we have to find t whenT2=25

$25-20=20e^{-kt}$ $\therefore$ $5=20e^{-kt}$

$\therefore e^{-kt}$ =1/4 $\therefore(e^{-10k})^{t/10}$ = $1/4=(1/2)^2$

$\therefore(1/2)^{t/10}$ =2 $\therefore$t=2

After 20 minutes the temp will be 25^{oc} . 10 minutes more will be required for a day to attain temperature25^{0c}.

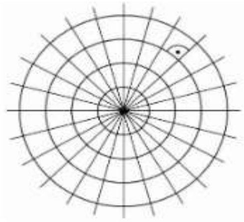
Applications Of Newton's Law Of Cooling

- The Newton law of cooling is used to predict how much time will be taken for a warm object to cool down at a constant temperature of its surrounding environment.
- Through the Newton law of cooling any person can determine the temperature of the body or of any object.

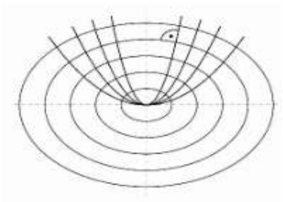
Limitations Of Newton's Law Of Cooling

- The difference between the temperatures of the surrounding environment and the object should be small.
- The loss of heat from the object should only be via radiation only.
- The temperature of the surrounding environment should remain constant during the cooling down of the object.

Orthogonal Trajectory : Any curve which cuts every member of given family or curves at right angles ,is called orthogonal trajectory of the family



Concentric circles with trajectories (1. Example)



Parabolas with orthogonal trajectories (2. Example)

For finding orthogonal trajectory of cartesian family of curves f(x,y,c)=0

1. First differentiate given equation f(x,y,c)=0 w.r.t. x and find dy/dx .
2. Elliminate c from two relations .
3. Replace dy/dx by - dx/dy
4. Solve the new differential equation
5. This solution of the new differential equation give the orthogonal trajectory of the given family of curves .

For e.g : Find orthogonal trajectory of the curve $x^2 - y^2 = c^2$

Sol. Diff w.r.t x

2x - 2y dy/dx=0 ∴ dy/dx=x/y

Replacing dy/dx by - dx/dy

-dx/dy=x/y ydx+xdy=0

∴d(xy)=0

Integrating,we get the ortogonal trajectory as xy=a

∴ Xy=a is orthogonal trajectory

Orthogonal trajectory of polar curves f(r,θ,c) = 0

Find orthogonal trajectory of f(r,θ,c) = 0

1. First diff w.r.t.θ and find dr/dθ
2. Eliminate the parameter c between the equations
3. Replace dr/dθ by $- r^2 d\theta/dr$ and form the differential equation of orthogonal trajectory.
4. The solution of new differential equation gives the orthogonal trajectory of given family o curves

For e.g.: Find orthogonal trajectory of the curve r = a (cosθ +sinθ)

dr/dθ = - asinθ

Eliminating a between two equations,

dr/dθ = - sinθ / (r/(1+cosθ)

5. Replacing dr/dθ by $- r^2 d\theta/dr$,the differential equation of the required orthogonal trajectory is

$-r^2$ dθ/dr = - sinθ / (r/(1+cosθ)

dr/r = (1+cosθ/sinθ) dθ

dr/r = sinθ (1+cosθ) /{ ((1+cosθ) ((1-cosθ) } dθ

∴dr/r = sinθ/((1-cosθ) dθ Integrating we get Logr=log(1-cosθ)+ logb

i.e. r=b(1-cosθ) is required orthogonal trajectory

Conclusion: Newton's law of cooling describes us that the rate of heat loss from a body or from an object is directly corresponding to the difference between the temperatures of the body and the environment surrounding it. Newton has developed a formula through which we can calculate the temperature of a material as it loses heat and Orthogonal trajectories are used in mathematics and physics to describe curves that intersect another family of curves at right angles. They find applications in areas like curved coordinate systems (e.g., elliptical coordinates) and in describing electric fields and their equipotential curvestransfers the heat from an object to the surrounding environment.

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