# A Cesaro Summable convergent paranormed sequence space with Orlicz Function

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**Abstract:** In this paper, we introduce and investigate the sequence space $C[\sigma_{n,l,M}]$, defined via Cesàro means and an Orlicz function $M$. Specifically, the space consists of all sequences whose Cesàro means converge to a finite limit . We establish that $c_{\sigma,\ell,M}$ forms a linear space and equip it with a paranorm, under which it is shown to be a complete paranormed linear space. The topological structure of this space is analyzed, and foundational properties such as closure under addition and scalar multiplication are proven. Our results contribute to the broader theory of sequence spaces defined by Orlicz-type functions and their modular convergence, with potential applications to summability theory and functional analysis.

**Keywords :** Sequence space , Cesaro summable, Orlics function, Paranormed, Dual.

## 1 Introduction

Sequence spaces have been a central object of study due to their role in summability theory, functional analysis, and applications involving convergence behavior [6, 2]. The idea of statistical convergence [6], and extensions involving filters and ideals [1, 8], give rise to refined concepts of convergence in topological and functional frameworks [7, 9].

The development of sequence spaces through Orlicz functions provides a flexible tool for generalizing classical results [4, 5]. Similarly, the use of multiplier sequences enables the definition of bounded, null, and convergent sequence spaces with additional algebraic structure [12, 10]. These constructions are particularly relevant in FK-spaces [2] and their Cesàro sections.

Moreover, recent works explore the matrix domain approach, where new sequence spaces are gener- ated via matrices acting on classical spaces [3]. Tripathi and sharma introduced, generalizing conver- gence vector valued paranormed sequence spaces[11] under broader notions of summability and normed structures.

This paper continues in this direction by focusing on the convergence properties of the sequence space $C[\sigma_{n,l,M}]$, leveraging the flexibility of Orlicz functions and the analytical tools of Cesàro summability.

# 2   Research Methodology

## 2.1   Definition of the Sequence Space

The paper introduces a new sequence space, denoted as $C[\sigma_n, \ell, M]$, constructed using Cesàro means and an Orlicz function $M$. The space consists of all sequences $(x_k)$ such that the modular limit

$$\lim_{n \to \infty} M\left(\frac{1}{\epsilon} \, \frac{1}{n} \sum_{k=1}^{n} x_k - \ell\right) = 0$$

for some $\ell \in \mathbb{R}$ and $\epsilon > 0$.

## 2.2   Construction of the Paranorm

A paranorm $p(x)$ is defined on this space as

$$p(x) = \sup_{n \in \mathbb{N}, \epsilon > 0} M\left(\frac{1}{\epsilon} \, \frac{1}{n} \sum_{k=1} x_k - \ell\right)$$

This paranorm is shown to satisfy the properties required for a paranormed space: non-negativity, symmetry, subadditivity, and continuity under scalar multiplication.

## 2.3   Verification of Linear Space Structure

The authors verify that $C[\sigma_n, \ell, M]$ is a linear space by demonstrating closure under addition and scalar multiplication. Specifically, if $x, y \in C[\sigma_n, \ell, M]$ and $\alpha, \beta \in \mathbb{R}$, then

$$\alpha x + \beta y \in C[\sigma_n, \ell, M]$$

by using the properties of the Orlicz function and Cesàro means.

## 2.4   Proof of Completeness

The completeness of the space is established by considering a Cauchy sequence $(x_m)$ in $C[\sigma_n, \ell, M]$ with respect to the paranorm $p$. The authors show that for each index $k$, the sequence $(x_{k_m})$ converges in $\mathbb{R}$, and the limit sequence $x = (x_k)$ belongs to $C[\sigma_n, \ell, M]$. Furthermore, $x_m \to x$ in the paranorm, ensuring completeness.

## 2.5   Analysis of Topological Properties

The paper investigates several topological and structural properties of the new space:

**Monotonicity:** If $x \in C[\sigma_n, \ell, M]$ and $y$ satisfies $|y_k| \leq |x_k|$ for all $k$, then $y \in C[\sigma_n, \ell, M]$.

**Solidity:** If $x \in C[\sigma_n, \ell, M]$ and $(\alpha_k)$ is a sequence with $|\alpha_k| \leq 1$, then $(\alpha_k x_k) \in C[\sigma_n, \ell, M]$.

**Non-symmetry:** The space is not symmetric; permuting the sequence elements can change the Cesàro means and thus the membership in the space.

## 2.6    Computation of Dual Spaces

The methodology includes the identification of the $\alpha$-, $\beta$-, and $\gamma$-duals of $C[\sigma_n, \ell, M]$ by analyzing the conditions under which sequences $(a_k)$ interact with sequences in $C[\sigma_n, \ell, M]$ via summability and boundedness of partial sums:

    $\alpha$-**dual:** $\ell_1$ (absolutely summable sequences)

    $\beta$-**dual and** $\gamma$-**dual:** $\ell_\infty$ (sequences with bounded partial sums)

# 3    Results and Discussion

## 3.1    Property : The space $C[\sigma_{n,l,M}]$ is a linear space

**Proof :**    Let us consider the sequence space

$$c_{\sigma,\ell,M} = \left\{ (x_k) \in \omega(X) : \lim_{n\to\infty} M\left(\frac{\frac{1}{n}\sum_{k=1}^{n} x_k - \ell}{\epsilon}\right) = 0 \text{ for some } \ell \in \mathsf{R}, \epsilon > 0 \right\}$$

We show that this space is a linear space by verifying closure under addition and scalar multiplication.

Let $x, y \in C[\sigma_{n,l,M}]$. For $x \in C[\sigma_{n,l,M}]$, we know that:

$$\lim_{n\to\infty} \frac{M\left(\frac{1}{n}\sum_{k=1}^{n} x_k - l\right)}{e} = 0$$

Now,

$$\lim_{n\to\infty} \frac{M\left(\frac{1}{n}\sum_{k=1}^{n}(\alpha x_k) - E\right)}{e},$$

where $E = \alpha l$,

$$= \lim_{n\to\infty} \frac{M\left(\alpha\frac{1}{n}\sum_{k=1}^{n} x_k - \alpha l\right)}{e}$$

$$= \lim_{n\to\infty} \alpha \frac{M\left(\frac{1}{n}\sum_{k=1}^{n} x_k - l\right)}{e}$$

$$= \alpha \lim_{n\to\infty} \frac{M\left(\frac{1}{n}\sum_{k=1}^{n}(x_k) - l\right)}{e} \tag{1}$$

Similarly, for $y \in C[\sigma_{n,l,M}]$,

$$\lim_{n\to\infty} \frac{M\left(\frac{1}{n}\sum_{k=1}^{n}(\beta y_k) - P\right)}{e},$$

where $P = \beta l$,

$$= \lim_{n\to\infty} \frac{M\left(\beta\frac{1}{n}\sum_{k=1}^{n} y_k - \beta l\right)}{e}$$

$$= \lim_{n\to\infty} \beta \frac{M\left(\frac{1}{n}\sum_{k=1}^{n} y_k - l\right)}{e}$$

$$= \beta \lim_{n\to\infty} \frac{M\left(\frac{1}{n}\sum_{k=1}^{n}(y_k) - l\right)}{e} \tag{2}$$

From equations (1) and (2), we get

$$\lim_{n \to \infty} \frac{M \, |\alpha x_k + \beta y_k|}{e} \leq \alpha \lim_{n \to \infty} \frac{M \, \frac{1}{n} \sum_{k=1}^{n} (x_k) - l}{e} + \beta \lim_{n \to \infty} \frac{M \, \frac{1}{n} \sum_{k=1}^{n} (y_k) - l}{e}$$

$$= \alpha x + \beta y$$

This implies $\alpha x + \beta y \in C[\sigma_{n,l,M}]$, or $x + y \in C[\sigma_{n, l, M}]$ .

Since the space $c_{\sigma,\ell,M}$ is closed under both addition and scalar multiplication, it is a linear space.

## 3.2   Theorem 1 : show that the space $c_{\sigma,\ell,M}$ is a paranormed Linear Space with the paranormed

$$p(x) = \sup_{n \in \mathbb{N}, \, \epsilon > 0} M \, \frac{\frac{1}{n} \sum_{k=1}^{n} x_k - \ell}{\epsilon}$$

**Proof :**

Let $C[\sigma_{n,l,M}$    $] =$    $(x_k) \in \omega : \lim_{n \to \infty} M \, \frac{\frac{1}{n} \sum_{k=1}^{n} x_k - \ell}{\epsilon} = 0,$  for some $\ell \in \mathbb{R}, \epsilon > 0$ , where $\omega$ is the space

of all sequences, and $M$ is an Orlicz function.

   Define the paranorm:

$$p(x) = \sup_{n \in \mathbb{N}, \, \epsilon > 0} M \, \frac{\frac{1}{n} \sum_{k=1}^{n} x_k - \ell}{\epsilon}$$

We show that $C[\sigma_{n,l,M}]$ is a paranormed linear space.

## 1. $p(x) = 0 \iff x \in c_{\sigma,\ell,M}$ and converges to $\ell$

Assume $p(x) = 0$. Then for all $\epsilon > 0$,

$$\lim_{n \to \infty} M \, \frac{\frac{1}{n} \sum_{k=1}^{n} x_k - \ell}{\epsilon} = 0.$$

Since $M$ is continuous and increasing with $M(0) = 0$, this implies

$$\lim_{n \to \infty} \frac{1}{n} \sum_{k=1}^{n} x_k = \ell,$$

i.e., the Cesàro means of $x$ converge to $\ell$, hence $x \in c_{\sigma,\ell,M}$.

## 2. Subadditivity: $p(x + y) \leq p(x) + p(y)$

Let $x = (x_k), \, y = (y_k)$. Then

$$\frac{1}{n} \sum_{k=1}^{n} (x_k + y_k) = \frac{1}{n} \sum_{k=1}^{n} x_k + \frac{1}{n} \sum_{k=1}^{n} y_k.$$

Using the convexity of $M$, we get:

$$M\left(\frac{\frac{1}{n}\sum_{k=1}^{n}(x_k+y_k)-\ell}{\epsilon}\right) \leq M\left(\frac{\frac{1}{n}\sum_{k=1}^{n}x_k-\ell}{\epsilon}\right) + M\left(\frac{\frac{1}{n}\sum_{k=1}^{n}y_k}{\epsilon}\right).$$

Taking supremum over $n \in \mathrm{N}$, $\epsilon > 0$, we get:

$$p(x+y) \leq p(x)+p(y).$$

## 3. Continuity under scalar multiplication: $p(\alpha x) \to 0$ as $\alpha \to 0$

We have:

$$p(\alpha x) = \sup_{n\in\mathrm{N},\,\epsilon>0} M\left(\frac{\frac{1}{n}\sum_{k=1}^{n}\alpha x_k-\ell}{\epsilon}\right) = \sup_{n,\epsilon>0} M\left(\alpha\frac{\frac{1}{n}\sum_{k=1}^{n}x_k-\frac{\ell}{\alpha}}{\epsilon}\right).$$

As $\alpha \to 0$, the argument of $M$ tends to zero. Since $M$ is continuous and $M(0)=0$, we get:

$$\lim_{\alpha\to 0} p(\alpha x) = 0.$$

The function $p(x)$ is a paranorm. Since $C[\sigma_{n,l,M}]$ is closed under addition and scalar multiplication, the space

$$(C[\sigma_{n,l,M}])$$

is a **paranormed linear space**.

## 3.3   Theorem 2 : The space $C[\sigma_{n,l,M}]$ is a complete paranormed space

**Proof :**

Let $(x^m)$ be a Cauchy sequence in $C[\sigma_{n,l,M}]$, where the paranorm is defined by:

$$p(x) = \sup_{n\in\mathrm{N},\,\epsilon>0} M\left(\frac{\frac{1}{n}\sum_{k=1}^{n}x_k-\ell}{\epsilon}\right).$$

By the definition of a Cauchy sequence, for every $\delta > 0$, there exists $N \in \mathrm{N}$ such that for all $m, n \geq N$,

$$p(x^m - x^n) < \delta.$$

This implies:

$$\sup_{n\in\mathrm{N},\,\epsilon>0} M\left(\frac{\frac{1}{n}\sum_{k=1}^{n}(x_k^m-x_k^n)}{\epsilon}\right) < \delta.$$

Fix $k \in \mathrm{N}$. Then $(x_k^m)$ is a Cauchy sequence in R, and hence there exists $x_k \in \mathrm{R}$ such that

$$x_k^m \to x_k \quad \text{as } m \to \infty.$$

Define $x = (x_k) \in \omega$.

Now, we need to show that $x \in c_{\sigma,\ell,M}$, i.e., the Cesàro means of $x$ converge to some $\ell \in \mathrm{R}$ in the sense defined by $M$, and that $x^m \to x$ in the paranorm $p$.

Since $x_k^m \to x_k$ for each $k$, we have

$$\frac{1}{n}\sum_{k=1}^{n} x_k^m \to \frac{1}{n}\sum_{k=1}^{n} x_k \quad \text{as } m \to \infty.$$

Moreover, since $(x^m) \subset c_{\sigma,\ell,M}$, for each $m$, there exists $\ell_m \in R$ such that

$$\lim_{n\to\infty} M\left( \frac{\frac{1}{n}\sum_{k=1}^{n} x_k^m - \ell_m}{\epsilon} \right) = 0 \quad \text{for all } \epsilon > 0.$$

By the properties of Cesàro means and the continuity of $M$, along with the convergence $x_k^m \to x_k$, it follows that:

$$\lim_{n\to\infty} M\left( \frac{\frac{1}{n}\sum_{k=1}^{n} x_k - \ell}{\epsilon} \right) = 0 \quad \text{for some } \ell \in R.$$

Hence, $x \in C[\sigma_{n,l,M}]$.

Now, since $x_k^m \to x_k$ for all $k$, and the Cesàro means converge, we get:

$$\lim_{m\to\infty} p(x^m - x) = 0.$$

Every Cauchy sequence $(x^m)$ in $c_{\sigma,\ell,M}$ converges to a point $x \in c_{\sigma,\ell,M}$ with respect to the paranorm $p$. Therefore, the space $C[\sigma_{n,l,M}]$ is **complete**.

## 3.4   Theorem 3 : The Space $C[\sigma_{n,l,M}]$ is monotone

**Proof :**

Let the space $C[\sigma_{n,l,M}]$ be defined as:

$$C[\sigma_{n,l,M}] = \left( x = (x_k) \in \omega : \sup_{n\in N,\, \epsilon>0} M\left( \frac{1}{\epsilon}\left| \frac{1}{n}\sum_{k=1}^{n} x_k - \ell \right| \right) < \infty \right),$$

where $M$ is an Orlicz function and $\ell \in R$.

We will prove that the space $C[\sigma_{n,l,M}]$ is **monotone**. That is, if $x = (x_k) \in C[\sigma_{n,l,M}]$ and $y = (y_k) \in \omega$ such that $|y_k| \le |x_k|$ for all $k \in N$, then $y \in C[\sigma_{n,l,M}]$.

Define the averages:

$$A_n(x) = \frac{1}{n}\sum_{k=1}^{n} x_k, \quad A_n(y) = \frac{1}{n}\sum_{k=1}^{n} y_k.$$

Using the triangle inequality and the condition $|y_k| \le |x_k|$, we have:

$$|A_n(y) - \ell| \le \frac{1}{n}\sum_{k=1}^{n} |y_k - \ell| \le \frac{1}{n}\sum_{k=1}^{n} |x_k - \ell|.$$

Since $M$ is increasing and convex, applying Jensen's inequality:

$$M\left( \frac{1}{\epsilon}|A_n(y) - \ell| \right) \le M\left( \frac{1}{\epsilon}\cdot\frac{1}{n}\sum_{k=1}^{n} |x_k - \ell| \right) \le \frac{1}{n}\sum_{k=1}^{n} M\left( \frac{1}{\epsilon}|x_k - \ell| \right).$$

Therefore,

$$\sup_{n\in N,\, \epsilon>0} M\left( \frac{1}{\epsilon}|A_n(y) - \ell| \right) < \infty,$$

since the right-hand side is bounded by assumption (because $x \in C[\sigma_{n,l,M}]$).

Hence, $y \in C[\sigma_{n,l,M}]$, and so:

$$\boxed{C[\sigma_{n,l,M}] \text{ is a monotone sequence space.}}$$

## 3.5    Theorem 4 : The space $c_{\sigma,\ell,M}$ is solid.

**Proof :**

Let $x = x_k \in c[\sigma_n, M]$ and let $\alpha_k$ satisfy $|\alpha_k| \leq 1$.

Define, $y = (\alpha_k x_k)$.
we have to show that $y \in C[\sigma_{n,l,M}]$
Now, Cesaro mean of $y$ is given by,

$$\sigma_n(y) = \frac{1}{n} \sum_{k=1} \alpha_k x_k$$

.

Since, $|\alpha_k| \leq 1$, we have ,

$$\sigma_n(y) - l| \leq | \frac{1}{n}\sum_{k=1} \alpha_k x_k \quad -l| \leq \frac{1}{n}\sum_{k=1} |x_k - l| \leq |\sigma_n \quad (x) - l|$$

Also using monotonicity of M,

$$M(\frac{|\sigma_n(y) - l|}{\epsilon}) \leq M(\frac{|\sigma_n(x) - l|}{\epsilon})$$

Since,

$$\lim_{n \to \infty} M(\frac{|\sigma_n(x) - l|}{\epsilon}) = 0$$

, it follows that

$$\lim_{n \to \infty} M(\frac{|\sigma_n(y) - l|}{\epsilon}) = 0$$

Thus, the sequence $y = (\alpha_k x_k)$ satisfies the modular convergence criterion for $c[\sigma_n, M]$

Hence, the sequence space $c[\sigma_n, M]$ is solid under the coordinate wise multiplication by bounded scalars.

## 3.6    The space $C[\sigma_{n,l,M}]$ is not symmetric.

Consider a permutation $\pi$ of the natural numbers . The permuted sequence is defined as ,

$$x^\pi = (x_{\pi(1)}, x_{\pi(2)}, x_{\pi(3)}\ldots)$$

$$\sigma_n(x^\pi) = \frac{1}{n} \sum_{n} x_{\pi(k)}$$

Since $\pi$ rearranges the order of terms, the sums $\frac{\sum_{k=1} x_{\pi(k)}}{n}$ can be different from $\frac{\sum_{k=1} x_k}{n}$ , especially for small or moderate n.

As for example, consider the sequence,

$$x = (1, 0, 0, 0, \ldots)$$

$$\sigma_1(x) = 1,$$

$$\sigma_2(x) = \frac{1+0}{2} = 0.5$$

$$\sigma_3(x) = \frac{1+0+0}{3} = \frac{1}{3}$$

and so on tending to 0 as $r \to \infty$. Now, permute x , so that the lines moved to the $10^{th}$ position,

$$x^\pi = (0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, ...)$$

$$\therefore \sigma_1(x^\pi) = 0,$$

$$\sigma_2(x^\pi) = 0,$$

$$......$$

$$\sigma_{10}(x^\pi) = \frac{1}{10},$$

$$\sigma_{11}(x^\pi) = \frac{1}{11} = 0.0909$$

and so on tending to 0 as $\to \infty$,

Thus we can observed that cesaro means doiffer significantly between x and $x^\pi$.

Hence we can conclude that the cesaro mean $\sigma_n(x)$ depends on the order of the sequence elements. If permute the sequence, the partial sums changes. Thus the sequence of cesaro means changes.

The Cesaro-orlicz modular(and the norm) depends on the space and the norm are not preserved under arbitrarily.

Hence the sequence space $C[\sigma_{n,l,M}]$ is not symmetric.

## 3.7   Duals of the Sequence Space $C[\sigma_{n,l,M}]$

Consider the sequence space:

$$C[\sigma_{n,l,M}] = \left\{ (x_k) \in \omega(X) : \lim_{n \to \infty} M\left( \frac{\frac{1}{n} \sum_{k=1}^{n} x_k - \ell}{\epsilon} \right) = 0 \text{ for some } \ell \in \mathbb{R}, \epsilon > 0 \right\},$$

where $M$ is an Orlicz function.

Let us now determine the **duals** of the space $C[\sigma_{n,l,M}]$.

## 1. $\alpha$-Dual

The $\alpha$-dual of a sequence space $E$ is defined as:

$$E^\alpha = \left( (a_k) \in \omega : \sum_{k=1}^{\infty} |a_k x_k| < \infty \text{ for all } (x_k) \in E \right).$$

Since every sequence $x \in C[\sigma_{n,l,M}]$ is controlled via Cesàro-type convergence under an Orlicz function, the sequences are bounded in average. Hence, for absolute convergence, $(a_k)$ must be absolutely summable.

$$C[\sigma_{n,l,M}]^\alpha = \ell_1.$$

## 2. $\beta$-Dual

The $\beta$-dual is:

$$E^\beta = \left( (a_k) \in \omega : \sum_{k=1}^{\infty} a_k x_k \text{ converges for all } (x_k) \in E \right).$$

Since $x \in C[\sigma_{n,l,M}]$ implies $\frac{1}{n}\sum_{k=1}^{n} x_k \to \ell$, it behaves similarly to Cesàro-convergent sequences. Thus, for $a_k x_k$ to converge, the sequence $(a_k)$ must have bounded partial sums.

$$\Rightarrow C[\sigma_{n,l,M}]^\beta = \left( (a_k) \in \omega : \sup_n \left| \sum_{k=1}^{n} a_k \right| < \infty \right) = bs.$$

## 3. $\gamma$-Dual

The $\gamma$-dual is:

$$E^\gamma = \left( (a_k) \in \omega : \sup_n \left| \sum_{k=1}^{n} a_k x_k \right| < \infty \text{ for all } (x_k) \in E \right).$$

This means the partial sums of the series $\sum a_k x_k$ are uniformly bounded for all $x \in E$. As before, for Cesàro-type convergence, this holds when $(a_k)$ has bounded partial sums.

$$\Rightarrow C[\sigma_{n,l,M}]^\gamma = bs.$$

### Summary of Duals

| Dual Type | Dual Space |
|---|---|
| $\alpha$-dual | $\ell_1$ |
| $\beta$-dual | $bs$ (bounded partial sums) |
| $\gamma$-dual | $bs$ (bounded partial sums) |

# 4 Conclusion

The paper systematically constructs the sequence space $C[\sigma_n, \ell, M]$, defined via Cesàro means and an Orlicz function $M$, and establishes its foundational properties:

**Linearity and Completeness:** The space is proven to be a complete paranormed linear space under the paranorm

$$p(x) = \sup_{n,\epsilon} M\left( \frac{1}{\epsilon} \left| \frac{1}{n} \sum_{k=1}^{n} x_k - \ell \right| \right),$$

with closure under addition and scalar multiplication.

**Topological Structure:** The space is monotone and solid but not symmetric, as permutations of sequences can alter Cesàro means.

**Dual Spaces:** The $\alpha$-, $\beta$-, and $\gamma$-duals are characterized as $\ell_1$ (absolutely summable sequences) and $\ell_\infty$ (sequences with bounded partial sums), respectively.

These results extend the framework of Orlicz-type sequence spaces and contribute to summability theory by integrating modular convergence with Cesàro averaging.

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