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TOTAL DOMINATOR COLOR CLASS TOTAL DOMINATING SETS IN JOIN GRAPHS

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Abstract :

Let G be a finite, undirected and connected graph with minimum degree at least one. A proper coloring $\mathcal{C}$ of G is said to be a total dominator color class total dominating set of G if each vertex properly dominates a color class in $\mathcal{C}$ and each color class in $\mathcal{C}$ is properly dominated by a vertex in V(G). A total dominator color class total dominating set D of G is a minimal total dominator color class total dominating set if no proper subset of D is a total dominator color class total dominating set of G. The total dominator color class total domination number is the minimum cardinality taken over all minimal total dominator color class total dominating sets in G and is denoted by $\gamma_\chi^{td}(G)$. Here we obtain total dominator color class total dominating sets in join graphs

Mathematics Subject Classification: **05C15, 05C69**

1. INTRODUCTION

All graphs considered in this paper are finite, undirected graphs with minimum degree at least one and we follow standard definitions of graph theory as found in [9]. Let G = (V, E) be a connected graph with no isolated vertices. The open neighborhood N(v) of a vertex v ∈ V(G) consists of the set of all vertices adjacent to v. The closed neighborhood of v is N[v] = N (v)∪{v}. For a set S ⊆ V, the open neighborhood N(S) is defined to be $\cup_{v\in S} N(v)$, and the closed neighborhood of S is N [S] = N(S)∪S. For any set H of vertices of G, the induced subgraph ⟨H⟩ is the maximal subgraph of G with vertex set H.

A subset S of V is called a dominating set if every vertex in V − S is adjacent to some vertex in S. A dominating set S is called a minimal dominating set if no proper subset of S is a dominating set of G. The domination number γ(G) is the minimum cardinality taken over all minimal dominating sets of G. A γ-set of G is any minimal dominating set with cardinality γ. A proper coloring of G is an assignment of colors to the vertices of G such that adjacent vertices have different colors. The smallest number of colors for which there exists a proper coloring of G is called chromatic number of G and is denoted by χ(G). A total dominator coloring of G is a proper coloring of G with the extra property that every vertex in G properly dominates a color class. The total dominator chromatic number is denoted by χ_{td}(G). This notion was introduced by A.Vijayalekshmi et al[10].

A color class dominating set of G is a proper coloring $\mathcal{C}$ of G with the extra property that every color classes in $\mathcal{C}$ is dominated by a vertex in G. A color class dominating set is said to be a minimal color class dominating set if no proper subset of $\mathcal{C}$ is a color class dominating set of G. The color class domination number of G is the minimum cardinality taken over all minimal color class dominating sets of G and is denoted by γ_χ(G). This notion was introduced by A.Vijayalekshmi et al[2].

A dominator color class dominating set of G is a proper coloring $\mathcal{C}$ of G with the extra property that each vertex v in G is dominated by a color class $\mathcal{C}_i$∈ $\mathcal{C}$ and each color class $\mathcal{C}_i$∈$\mathcal{C}$ is dominated by a vertex in G. The dominator color class domination number of G is the minimum cardinality taken over all dominator color class dominating sets in G and is denoted by γ_χ^d(G). This notion was introduced by A.Vijayalekshmi et al[4].The join $G_1 + G_2$ of graphs G_1 and G_2 with disjoint vertex set V_1 and V_2 and edge sets E_1 and E_2, respectively, is the graph union $G_1 \cup G_2$ together with each vertex in V_1 is adjacent to every vertices in V_2.

2.MAIN RESULTS

Definition 2.1.

Let G be a connected graph with minimum degree at least one. A proper coloring $\mathcal{C}$ of G is said to be a total dominator color class total dominating set of G if each vertex properly dominates a color class in $\mathcal{C}$ and each color class in $\mathcal{C}$ is properly dominated by a vertex in V(G). A total dominator color class total dominating set D is a minimal total dominator color class total dominating set if no proper subset of D of G is a total dominator color class total dominating set of G. The total dominator color class total domination number is the minimum cardinality taken over all minimal total dominator color class total dominating sets in G and is denoted by $\gamma_\chi^{td}(G)$.

Theorem 2.2

Let G and H be any connected graph without isolated vertices. Then $\gamma_\chi^{td}(G+H) \leq \gamma_\chi^{td}(G) + \gamma_\chi^{td}(H)$ and equality holds if and only if $G \cong K_2$ and $G \cong P_3, C_3$

Proof:

Let G and H be two connected graphs of order p_1 and p_2 respectively. Let $\mathcal{C}_1 = \{c_1, c_2, \dots \dots, c_m\}$ and $\mathcal{C}_2 = \{c_1', c_2', \dots \dots, c_n'\}$ be the total dominator color class total dominating sets of G and H respectively.

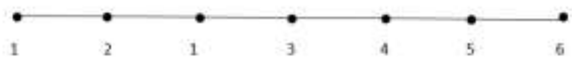
Let $V(G)= \{u_1, u_2, \dots, u_{p_1}\}$ and $V(H)= \{v_1, v_2, \dots \dots, v_{p_2}\}$. We assume that $p_1, p_2 \geq 4$. Then $\gamma_\chi^{td}(G) + \gamma_\chi^{td}(H) = m + n$. We consider 3 cases.

Case 1: When $p_1 < p_2$

$G + H$ is obtained by $G \cup H$ and each vertex in G is adjacent to every vertices in H. Since $N_{G+H}(u_i) = V(H) \cup N_G(u_i)$ and each u_i is adjacent to every v_j $\forall j = 1,2,\dots, p_2$. So we assign new colors say m+1 to the vertex $v_1, \dots\dots$ $m + \chi(G)$ colors are needed to get the minimal total dominator color class total dominating sets of $G+H$. At least $\gamma_\chi^{td}(G+H)$ colors are needed to get total dominator color class total dominating sets of $G + H$.

$$\gamma_\chi^{td}(G+H) \leq \gamma_\chi^{td}(G) + \chi(H)$$
$$\leq \gamma_\chi^{td}(G) + \gamma_\chi^{td}(H)$$

$\gamma_\chi^{td}(G) = 4$



$\gamma_\chi^{td}(H) = 6$

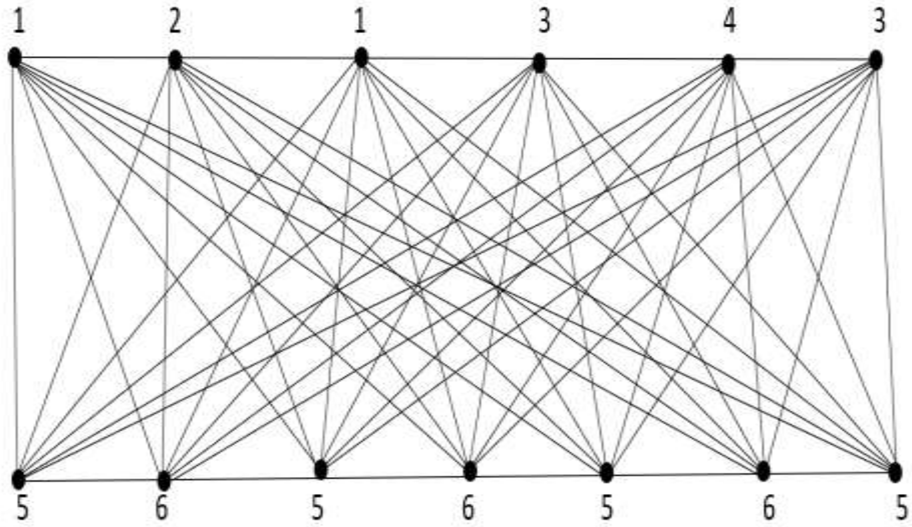


Figure 1. $\gamma_\chi^{td}(G+H)=6$

Case 2: When $p_1>p_2$

Since $N_{G+H}(v_j)= V(G) \cup N_H(v_j)$ and each v_j is adjacent to every u_i $\forall$ i=1,2,…, p_1 and assume case 1,

$$\gamma_\chi^{td}(G+H) < n+\chi(G)$$
$$= \gamma_\chi^{td}(H)+\chi(G)$$
$$\le \gamma_\chi^{td}(H)+\gamma_\chi^{td}(G)$$

$\gamma_\chi^{td}(G)=4$ $\gamma_\chi^{td}(H)=4$

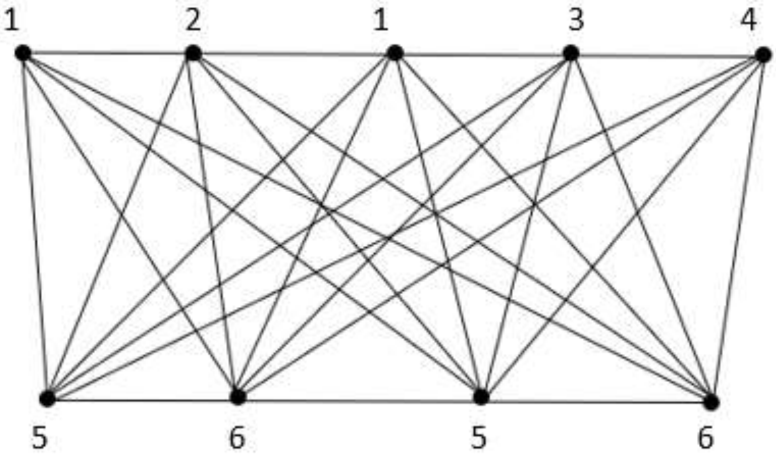


Figure 2. $\gamma_\chi^{td}(G+H)=6$

Case 3: When $p_1=p_2$

Assign $\min\{\gamma_\chi^{td}(G)+\chi(H),\chi(G)+\gamma_\chi^{td}(H)\}$ colors to obtain a total dominator color class total dominating sets of $G+H$. So

$$\gamma_\chi^{td}(G+H)=\min\{\gamma_\chi^{td}(G)+\chi(H),\chi(G)+\gamma_\chi^{td}(H)\}$$
$$\le \gamma_\chi^{td}(G)+\gamma_\chi^{td}(H)$$

$\gamma_\chi^{td}(G)=6$ $\gamma_\chi^{td}(H)=6$

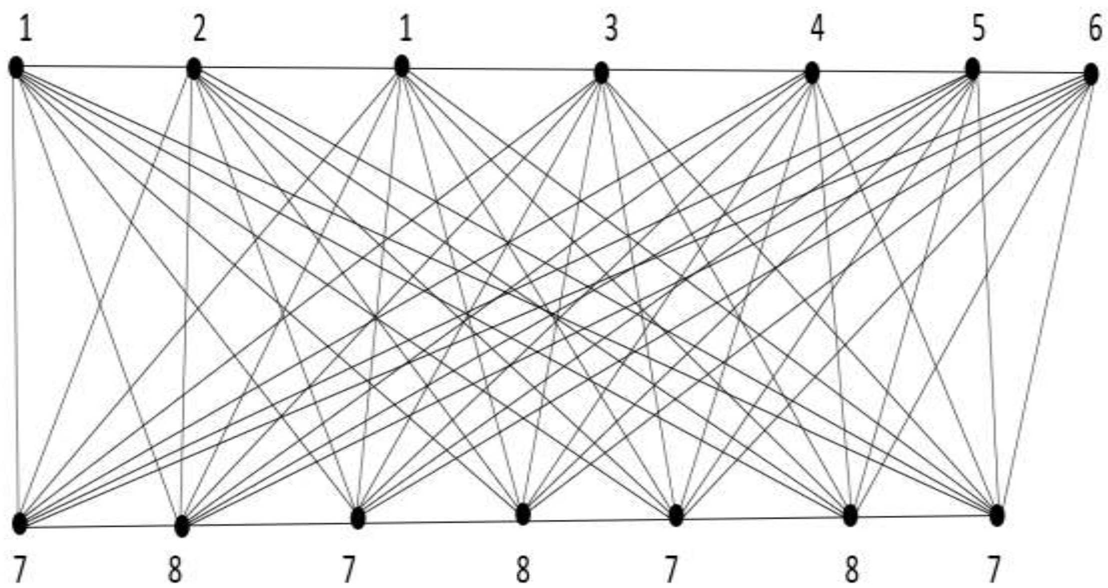
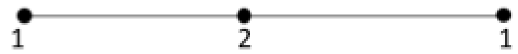


Figure 3. $\gamma_{\chi}^{td}(G+H)=6$

To prove the equality:

When p_1= 2 and p_2 = 3 then $\gamma_{\chi}^{td}(G)$ = $\chi(G)$and $\gamma_{\chi}^{td}(H)$ = $\chi(H)$. Thus the required result.



$\gamma_{\chi}^{td}(G)=2$ $\gamma_{\chi}^{td}(H)=2$

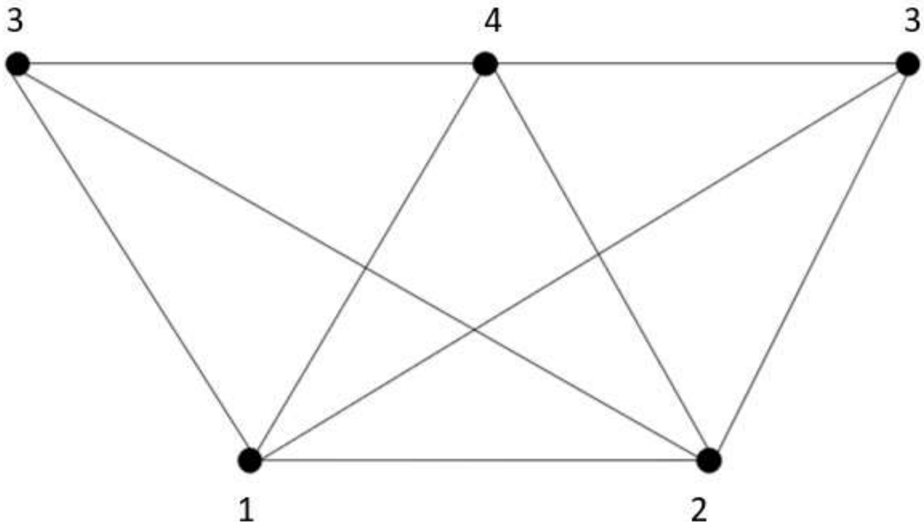


Figure 4.$\gamma_{\chi}^{td}(G+H)=4$

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