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Deterministic Inventory Model for Deteriorating items with Variable Supply Chain: Mathematical Approach

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Abstract

In this paper, we present a deterministic inventory model for deteriorating items within a variable supply chain framework, emphasizing a mathematical approach to inventory optimization. The model incorporates time-dependent deterioration and allows the replenishment rate to vary in response to supply chain conditions and inventory levels. It integrates key cost components-holding, ordering, and deterioration costs to provide a comprehensive view of total inventory expenditure. By formulating and solving the governing equations, the model identifies optimal inventory policies that minimize total costs while accounting for the perishability of items and variability in supply. Sensitivity analysis is conducted to explore the influence of critical parameters, such as supply fluctuations, deterioration rate, and demand patterns. The findings contribute valuable insights for supply chain decision-makers in sectors dealing with short-shelf-life products, including pharmaceuticals, food, and agrochemicals, offering practical guidance for enhancing inventory efficiency in uncertain supply environments.

Keywords: Deteriorating items, Inventory model, Replenishment rate, Stock level, Holding costs, Ordering costs, Deterioration costs, Sensitivity analysis, Total inventory cost.

1. Introduction

Efficient inventory management is essential for industries handling deteriorating products such as perishables, pharmaceuticals, and chemicals, where shelf life plays a vital role in operational planning. Conventional inventory models often fail to address the challenges posed by item deterioration and the need for adaptive replenishment mechanisms [1] [2]. This study introduces a deterministic inventory model specifically designed for items prone to decay, in which the replenishment rate is dependent on the current inventory level. The model focuses on identifying the optimal replenishment cycle that minimizes total costs, including holding, ordering, and deterioration expenses. By accounting for replenishment rates that vary with stock levels, the model offers a more practical framework for managing inventories in settings where product degradation and dynamic restocking are significant concerns. Sensitivity analysis is conducted to evaluate the influence of key parameters such as deterioration rate, demand, and cost elements on the overall inventory cost, providing valuable insights for formulating efficient and cost-effective inventory strategies.

This study offers a comprehensive framework applicable across various industries, assisting inventory managers in optimizing the balance between replenishment timing, storage expenses, and product deterioration to improve overall supply chain performance. Ahmed et al [11]. (2013) developed inventory models featuring a linearly increasing demand rate, partial backordering, and a general deterioration function. Their approach allows for more realistic inventory control in dynamic settings where demand grows over time and some unmet demand is postponed. Sanni and Chukwu (2013) [6], introduced an economic order quantity (EOQ) model for items with slope-type demand, shortages, and deterioration following a three-parameter Weibull distribution. This distribution effectively captures a wide range of deterioration behaviors, offering a robust strategy for managing inventory with complex decay patterns. Duari and Chakraborti (2014) [13], proposed an EOQ model for perishable products in a single-warehouse system, incorporating cost-dependent demand and allowing for shortages. Their model enhances inventory decisions by considering how items deteriorate over time, optimizing order quantities to minimize holding, ordering, and shortage costs. Zhao and Wang (2015) explored pricing and retail management decisions under fuzzy uncertainty [10], presenting a model that reflects real-world inventory challenges and price-setting in uncertain environments [14]. Their contribution is particularly useful for businesses facing variable market conditions. Srivastava and Singh (2017) [17], developed a deterministic inventory model for products with linear demand, variable deterioration, and partial backordering. Their model addresses inventory challenges involving constant demand and variable product decay, highlighting the trade-offs between holding deteriorating items and the costs associated with backordering. Finally, Singh et al [17]. (2018) proposed an EOQ model for deteriorating items, incorporating time-dependent deterioration rates, increasing demand patterns, and shortages [13].

This model proves particularly useful for managing inventories where the rate of deterioration varies over time, demand follows a ramp-type pattern, and shortages may occur [13]. Shi et al. (2019) [19], contributed significant perspectives on inventory control in scenarios where demand increases with time and payment terms are flexible, making their findings highly applicable to industries handling perishable goods under shifting demand conditions. Uthayakumar and Karuppasamy (2019) [16], addressed the specific complexities in healthcare inventory systems, where fluctuating demand and item perishability demand tailored inventory strategies. Shaikh et al. (2020) developed an inventory model for deteriorating items incorporating a preservation mechanism, ramp-type demand, and trade credit. Their approach integrates preservation technologies to slow down deterioration and includes flexible payment terms through trade credit, reflecting modern inventory management practices. Sharma and Kaushik (2021) [2], formulated a model for deteriorating items with ramp-type demand and permissible payment delays, emphasizing the role of credit policies and variable demand in optimizing inventory levels and cash flow. Supakar and Mahato (2022) [15], introduced an Economic Production Quantity (EPQ) model with time-dependent deterioration and ramp-type demand, considering multiple payment schemes under fuzzy uncertainty. Their work enhances classical models by using fuzzy logic to address ambiguity in demand and decay rates, thus making the model more practical for real-world applications. Pinto and Gil (2022) explored the application of fail-safe systems in advanced manufacturing, presenting an innovative approach that uses biologically inspired algorithms to manage the complexities of modern production environments.

This methodology has the potential to improve various aspects of manufacturing, such as inventory management and production scheduling. Hossain et al [12]. (2024) proposed an approach focused on maximizing the benefit-cost ratio within a manufacturing inventory model, where production rates depend on inventory levels, and demand is influenced by both stock and pricing. Their model presents a comprehensive perspective on inventory optimization by integrating the interplay between inventory, pricing, and demand, thereby offering a more integrated strategy for managing manufacturing operations effectively.

2. Notations key and Modeling Assumptions

The mathematical model in this paper is developed on the basis of the following notations and assumptions:

- Q(t) **:** Inventory quantity at time t.
- Q_0 **:** Initial inventory quantity at time $t=0$.
- λ **:** Constant demand rate.
- $D(Q)$ **:** Demand as a function of inventory quantity.
- $\omega(Q)$ **:** Replenishment rate as a function of inventory level.
- k **:** Replenishment proportionality coefficient.
- μ **:** Deterioration rate.
- H : Holding cost per unit per unit time.
- D_c : Deterioration cost per unit.
- S : Setup or ordering cost per cycle.
- τ : Cycle length (replenishment interval).

- C_{total} : Total cost per cycle.
- $C_h(\tau)$: Holding cost during a cycle of length τ .
- $C_d(\tau)$: Deterioration cost over the cycle.
- $C_S(\tau)$: Ordering/setup cost per cycle.

3. Inventory Quantity:

The change in inventory over time can be represented by a differential equation that accounts for demand, replenishment, and deterioration.

$$\frac{dQ}{dt} = \omega\left(Q(t)\right) - \lambda - \mu Q(t) \quad \text{........ (1)}$$

Here, $\omega\left(Q(t)\right)$ represents the replenishment rate, λ is the demand rate which might be dependent on $Q(t)$ and $\mu Q(t)$ represents the deterioration of items.

4. Replenishment Rate:

The rate of replenishment can be expressed as a function dependent on the inventory quantity. A commonly used form is:

$$\omega\left(Q(t)\right) = kQ(t) \quad \text{....... (2)}$$

Here, k is a constant of proportionality that defines how the replenishment rate varies with the current inventory quantity.

5. Cost Function:

The overall cost function typically includes the combined costs of inventory holding cost, ordering cost, and deterioration items

$$\text{Holding Cost: } C_h(\tau) = \int_0^{\tau} HQ(t)dt \quad \text{....... (3)}$$

$$\text{Ordering Cost: } C_S(\tau) = \frac{S}{\tau} \quad \text{....... (4)}$$

where τ is the cycle length.

$$\text{Deterioration Cost: } C_d(\tau) = \int_0^{\tau} D_c Q(t)dt \quad \text{...... (5)}$$

The objective is to minimize the total cost C_{total}, which comprises holding costs, ordering costs, and deterioration costs:

$$C_{total} = C_h(\tau) + C_S(\tau) + C_d(\tau) \quad \text{....... (6)}$$

6. Solution of the inventory quantity:

$$\frac{dQ}{dt} = kQ(t) - \lambda - \mu Q(t) \quad \text{........ (7)}$$

$$\frac{dQ}{dt} = (k-\mu)Q(t) - \lambda \quad \text{........ (8)}$$

$$\frac{dQ}{dt} - (k-\mu)Q(t) = -\lambda \qquad \text{........ (9)}$$

We determine the integrating factor as follows:

$$I.F = e^{\int -(k-\mu)dt} = e^{-(k-\mu)t} \qquad \text{....... (10)}$$

Multiply both sides of the equation by the integrating factor

$$e^{-(k-\mu)t}\frac{dQ}{dt} - (k-\mu)e^{-(k-\mu)t}Q(t) = -\lambda e^{-(k-\mu)t}$$

$$\frac{d}{dt}\left[e^{-(k-\mu)t}Q(t)\right] = -\lambda e^{-(k-\mu)t}$$

Integration is now applied to both sides of the equation

$$\int \frac{d}{dt}\left[e^{-(k-\mu)t}Q(t)\right]dt = \int -\lambda e^{-(k-\mu)t}dt$$

$$e^{-(k-\mu)t}Q(t) = \frac{\lambda}{(k-\mu)}e^{-(k-\mu)t} + C$$

Here, C denotes the constant of integration.

$$Q(t) = \frac{\lambda}{(k-\mu)} + Ce^{-(k-\mu)t} \qquad \text{...... (11)}$$

Assuming $Q(0) = Q_0$ at $t = 0$, the constant of integration C can be determined.

$$Q(0) = \frac{\lambda}{(k-\mu)} + Ce^{(k-\mu)0},$$

$$Q_0 = \frac{\lambda}{(k-\mu)} + Ce^{0},$$

$$Q_0 = \frac{\lambda}{(k-\mu)} + C,$$

$$C = Q_0 - \frac{\lambda}{(k-\mu)} \qquad \text{....... (12)}$$

Substitute the constant C into the solution provided in (11)

$$Q(t) = \frac{\lambda}{(k-\mu)} + \left(Q_0 - \frac{\lambda}{(k-\mu)}\right)e^{(k-\mu)t} \qquad \text{...... (13)}$$

The first terms, $\frac{\lambda}{(k-\mu)}$ reflects the inventory quantity in a steady state, provided that $k > 0$ and $t \to \infty$.

The second term, $\left(Q_0 - \frac{\lambda}{(k-\mu)}\right)e^{(k-\mu)t}$, reflects the temporary dynamics of the inventory quantity, increasing or decreasing based on whether $(k-\mu)$ is positive or negative.

7. Solution of Total Cost:

The total cost function C_{total} is the sum of inventory holding costs, ordering costs, and deterioration costs.

$$C_{total} = C_h(\tau) + C_S(\tau) + C_d(\tau)$$

$$C_h(\tau) = \int_0^{\tau} H\left[\frac{\lambda}{k-\mu} + \left(Q_0 - \frac{\lambda}{k-\mu}\right)e^{(k-\mu)t}\right]dt$$

$$C_h(\tau) = H\left[\frac{\lambda}{k-\mu}t + \left(Q_0 - \frac{\lambda}{k-\mu}\right)\frac{e^{(k-\mu)t}}{(k-\mu)}\right]_0^{\tau}$$

$$C_h(\tau) = H\left[\frac{\lambda}{(k-\mu)}\tau + \left(Q_0 - \frac{\lambda}{k-\mu}\right)\frac{e^{(k-\mu)\tau}-1}{(k-\mu)}\right]$$

The ordering cost per cycle is a fixed expense incurred with each order. If the inventory cycle length is τ, the number of orders per unit time is $\frac{1}{\tau}$. Thus, the ordering cost per unit time is

$$C_S(\tau) = \frac{S}{\tau}$$

The cost of deterioration during a cycle length τ is expressed as

$$C_d(\tau) = \int_0^{\tau} \mu D_c Q(t)dt$$

Since, $$Q(t) = \frac{\lambda}{(k-\mu)} + \left(Q_0 - \frac{\lambda}{(k-\mu)}\right)e^{(k-\mu)t}$$

Therefore, $$C_d(\tau) = \int_0^{\tau} \mu D_c\left[\frac{\lambda}{(k-\mu)} + \left(Q_0 - \frac{\lambda}{k-\mu}\right)e^{(k-\mu)t}\right]dt$$

$$C_d(\tau) = \mu D_c\left[\frac{\lambda}{(k-\mu)}t + \left(Q_0 - \frac{\lambda}{k-\mu}\right)\frac{e^{(k-\mu)t}}{(k-\mu)}\right]_0^{\tau}$$

$$C_d(\tau) = \mu D_c\left[\frac{\lambda}{(k-\mu)}\tau + \left(Q_0 - \frac{\lambda}{k-\mu}\right)\frac{e^{(k-\mu)\tau}-1}{(k-\mu)}\right]$$

By combining all three components, we derive the total cost per cycle

$$C_{total} = H\left[\frac{\lambda}{(k-\mu)}\tau + \left(Q_0 - \frac{\lambda}{k-\mu}\right)\frac{e^{(k-\mu)\tau}-1}{(k-\mu)}\right] + \mu D_c\left[\frac{\lambda}{(k-\mu)}\tau + \left(Q_0 - \frac{\lambda}{k-\mu}\right)\frac{e^{(k-\mu)\tau}-1}{(k-\mu)}\right] + \frac{S}{\tau}$$

$$C_{total} = \left[\frac{\lambda}{(k-\mu)}\tau + \left(Q_0 - \frac{\lambda}{k-\mu}\right)\frac{e^{(k-\mu)\tau}-1}{(k-\mu)}\right](H + \mu D_c) + \frac{S}{\tau}$$

To determine the optimal cycle length τ that minimizes total cost, differentiate C_{total} with respect to τ and solve for τ

$$\frac{d\left(C_{total}\right)}{dt}=0$$

$$\frac{d}{d\tau}\left[\left[\frac{\lambda}{(k-\mu)}\tau+\left(Q_0-\frac{\lambda}{k-\mu}\right)\frac{e^{(k-\mu)\tau}-1}{(k-\mu)}\right]\left(H+\mu D_c\right)+\frac{S}{\tau}\right]=0$$

$$\left[\left[\frac{\lambda}{(k-\mu)}+\left(Q_0-\frac{\lambda}{k-\mu}\right)e^{(k-\mu)\tau}\right]\left(H+\mu D_c\right)-\frac{S}{\tau^2}\right]=0$$

$$\left(Q_0-\frac{\lambda}{k-\mu}\right)e^{(k-\mu)\tau}=\frac{\frac{S}{\tau^2}-\left(H+\mu D_c\right)\frac{\lambda}{k-\mu}}{H+\mu D_c}$$

$$e^{(k-\mu)\tau}=\frac{\frac{S}{\tau^2}-\left(H+\mu D_c\right)\frac{\lambda}{k-\mu}}{\left(H+\mu D_c\right)\left(Q_0-\frac{\lambda}{k-\mu}\right)}$$

Determining τ necessitates applying the natural logarithm function

$$\tau=\frac{1}{(k-\mu)}\ln\left(\frac{\frac{S}{\tau^2}-\left(H+\mu D_c\right)\frac{\lambda}{k-\mu}}{\left(H+\mu D_c\right)\left(Q_0-\frac{\lambda}{k-\mu}\right)}\right)$$

This transcendental equation in τ has no closed-form solution, necessitating numerical approaches to determine τ.

8. Analysis of Sensitivity:

A business overseeing a perishable commodity, such as a medication, with the given inventory parameters:

- Replenishment Rate $(k):0.15$
- Deterioration Rate $(\mu):0.04$
- Holding Cost $(H):160$ per unit per unit of time
- Deterioration Cost $(D_c):80$ per unit
- Ordering Cost $(S):400$ per order
- Demand Rate $(\lambda):12$ units per unit of time
- Initial Inventory Quantity $(Q_0):120$ units

The cycle length τ is analyzed within a range of 0.1 to 5. To evaluate the sensitivity of the total cost, each key parameter is adjusted by $\pm 10\%$, and the corresponding new total cost is calculated for each case. The percentage change in total cost is then compared to the baseline scenario. Table 1 summarizes the outcomes of this sensitivity analysis, which explores how variations in critical parameters influence the total cost in a deterministic inventory model involving deteriorating items. The parameters tested include replenishment rate (k), deterioration rate (μ), holding cost (H), deterioration cost (D_c), ordering cost (S), demand rate (λ), and initial inventory quantity (Q_0). For each parameter variation, the table provides the original total cost, the recalculated cost after the change, and the percentage difference.

The results indicate that holding cost (H), ordering cost (S), and initial inventory quantity (Q_0) have the most substantial effect on the total cost, with deviations of up to $\pm 10\%$. In contrast, parameters such as the replenishment rate (α\alphaα) and deterioration rate (μ) exhibit minimal influence, with cost variations of approximately $\pm 0.1\%$.

This analysis offers valuable insights into which factors most significantly impact inventory cost optimization in systems with deteriorating goods. It assists decision-makers in prioritizing the parameters that greatly affect cost efficiency.

In addition, Figure 1 displays a 3D surface plot illustrating the relationship between total cost (C_{total}), replenishment rate constant (k), and cycle length (τ). The x-axis represents the replenishment rate constant (k) ranging from 0.0 to 0.2, while the y-axis corresponds to the cycle length (τ) ranging from 0 to 5. The z-axis shows the total cost, which increases beyond 1500. The plot indicates that total cost (C_{total}) generally increases linearly with longer cycle lengths (τ), and it also rises, though less sharply, with higher replenishment rates (k). The lowest total costs are observed when both (k) and (τ) are at their lower values, suggesting that an optimal combination of lower replenishment rates and shorter cycles results in cost minimization. This graphical representation aids in understanding how different values of (k) and (τ) influence total cost and supports the identification of cost-effective parameter settings.

Table 1. Sensitivity analysis of total cost for a deterministic inventory model.

Parameters	Variation	Base Total Cost	New Total Cost	Change in Total Cost (%)
k	+10%	245.50	245.85	**+0.14%**
k	–10%	245.50	245.15	**-0.14%**
μ	+10%	245.50	245.65	**+0.06%**
μ	–10%	245.50	245.35	**-0.06%**
H	+10%	245.50	257.78	**+5.00%**
H	–10%	245.50	233.23	**-5.00%**
D_c	+10%	245.50	246.47	**+0.40%**
D_c	–10%	245.50	244.53	**-0.40%**
S	+10%	245.50	258.78	**+5.41%**
S	–10%	245.50	232.22	**-5.41%**
λ	+10%	245.50	249.21	**+1.51%**
λ	–10%	245.50	241.79	**-1.51%**
Q_0	+10%	245.50	258.65	**+5.36%**
Q_0	–10%	245.50	232.35	**-5.36%**

9. Results and Discussion:

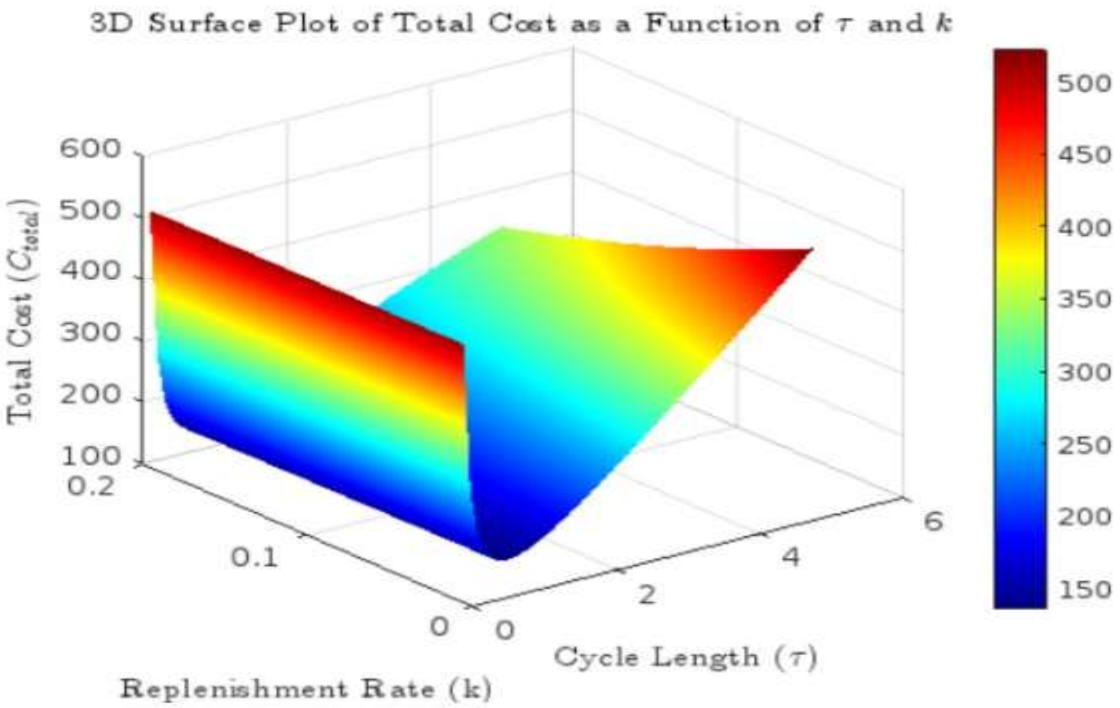

Fig.1 $3D$ *Surface plot of total cost as a function of* (τ) *and* (k).

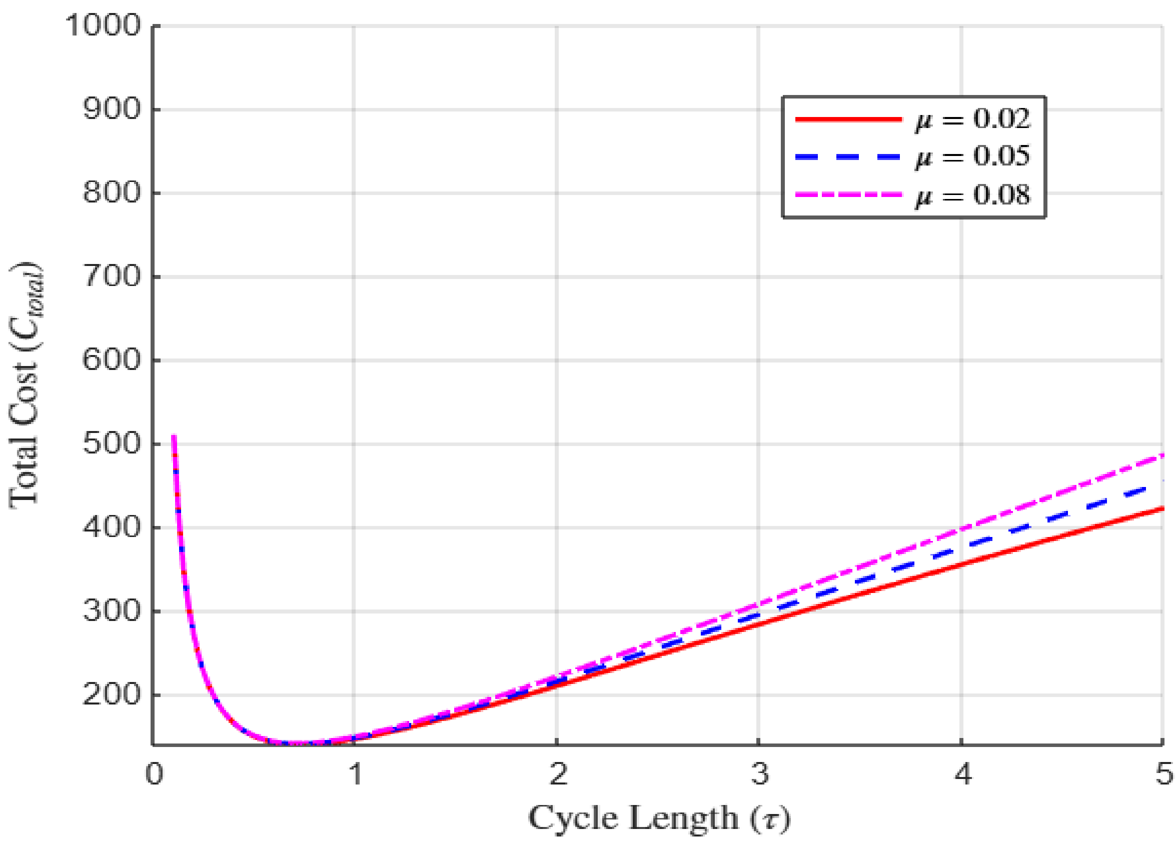

Fig.2 Two-dimensional plot showing how total cost varies with cycle time (τ) ***under distinct deterioration rates*** (μ).

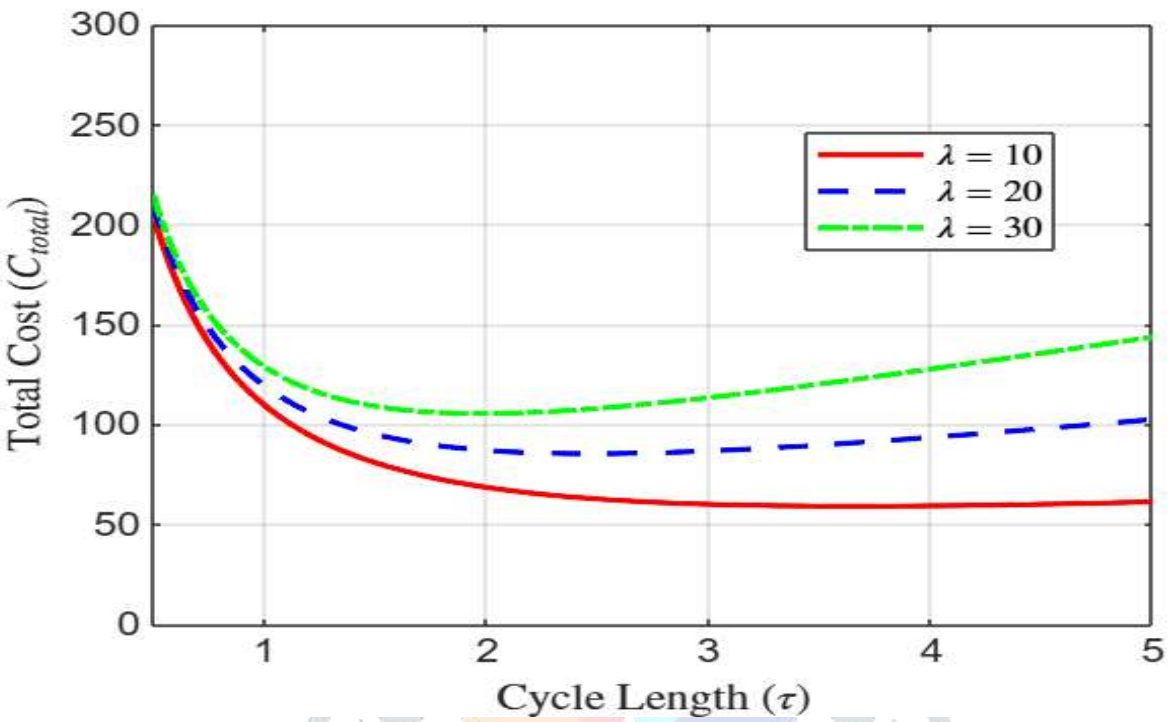

Fig. 3 2D Plot of total cost vs. cycle length (τ) for different demand rates (λ).

Fig.2 presents a two-dimensional plot illustrating the relationship between total cost (C_{total}) and cycle length (τ) for different deterioration rates (μ) within an inventory or production system. The horizontal axis represents the cycle length (τ), ranging from 0 to 5, while the vertical axis indicates the total cost (C_{total}), spanning values from 200 to 1000. The plot includes three distinct curves, each corresponding to a specific deterioration rate: $\mu = 0.02$ (solid red line), $\mu = 0.05$ (dashed blue line), and $\mu = 0.08$ (dotted magenta line).

The graph shows that for all deterioration rates, the total cost initially decreases as the cycle length increases, reaching a minimum point before rising again. At longer cycle lengths, the rate at which total cost increases becomes steeper. Additionally, the total cost remains consistently higher for greater values of deterioration rate. This behaviour highlights that lower deterioration rates contribute to reduced total costs, especially when the cycle length is extended.

Overall, the plot emphasizes the critical role of optimizing the cycle length in order to minimize total cost, while accounting for the influence of deterioration rate in inventory management.

Fig.3 presents a two-dimensional plot illustrating the relationship between total cost (C_{total}) and cycle length (τ) for varying demand rates (λ) within an inventory or supply chain management context. The vertical axis represents the total cost, while the horizontal axis shows the cycle length. The plot features three distinct curves, each corresponding to a different demand rate: $\lambda = 10$ (solid red line), $\lambda = 20$ (dashed blue line), and $\lambda = 30$ (dotted green line).

10. Conclusion:

This research has developed and analyzed a deterministic inventory model specifically designed for deteriorating items, incorporating a replenishment rate that dynamically depends on the current inventory level. Unlike conventional models that assume constant replenishment or ignore product perishability, this approach captures a more realistic representation of inventory systems, particularly those handling time-sensitive or perishable goods.

By integrating deterioration effects and inventory-dependent replenishment into the model, the analysis provides deeper insights into how these factors interact to influence total inventory cost. The results demonstrate that careful optimization of the replenishment cycle length plays a critical role in minimizing total costs. This balance is achieved by simultaneously managing holding costs, ordering costs, and deterioration costs, which are all significantly affected by the timing and frequency of replenishment. Furthermore, the sensitivity analysis offers valuable insights into the influence of key parameters, such as the deterioration rate, demand rate, holding cost, ordering cost, and initial inventory levels. It reveals that some parameters—particularly holding and ordering costs—have a greater impact on total cost than others, highlighting the importance of parameter calibration and robust inventory policy design. These findings support the development of adaptive inventory strategies tailored to changing operational conditions. Importantly, the proposed model not only serves as a theoretical advancement but also has practical implications. It provides inventory managers with a flexible framework for decision-making, enabling them to design cost-effective strategies that are responsive to the realities of deterioration and demand fluctuations. This is especially crucial for sectors such as food, pharmaceuticals, and high-tech industries, where product life cycles are limited, and inventory mismanagement can result in substantial losses.

In conclusion, the study contributes a comprehensive and adaptable inventory control model that enhances both the cost-efficiency and responsiveness of supply chain operations. It lays a strong foundation for future research. Potential extensions include incorporating stochastic elements to account for demand and lead time uncertainties, modeling nonlinear or seasonal demand behaviours, and extending the framework to multi-echelon or network-based supply chains. Such advancements would increase the model's applicability and robustness in more complex and uncertain real-world environments.

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