



ONE SHAPE PARAMETER RATIONAL QUINTIC SPLINE

¹Amit Pathak, ²Jaya Kushwah

¹ Research Scholar, Department of Mathematics,
Swami Vivekanand University Sagar M.P. India.

² Associate Professor, Department of Mathematics,
Swami Vivekanand University Sagar M.P. India.

Abstract

This paper discuss the construction of new C^3 rational quintic spline interpolation with quintic numerator and cubic denominator the idea has been extended to shape preserving interpolation for positive data using the constructed rational quintic spline interpolation. There is one parameter ν play very crucial role to change shape of the resolution interpolating curves. However, as per spline interpolating with C^3 continuously is not able to preserve the positive data notably our scheme is easy to use and does not requires knots insertion and C^3 continuity unable to achieve by solving for the second derivations $i=1, 2, \dots, n-1$. An error analysis which the function to be interpolation is .

In this method for application point view straight line blow the curve shows wastage of mixture of copper, silver , gold .iron and other in earth but if curve above straight line shows minimum wastage other than copper, silver gold and iron. After all can find pure things . This is also true for diseases in blued and applying this method after get pure blued

KEY WORDS : Rational, Interpolation, quintic, cubic, shape preserving, parameter..

1.INTRODUCTION

Parameters play an important role to change shape of curve and in computer Aided Geometric Design. Monotonicity positivity and convexity these properties beneficial to change shape of curve . In spline this facility is not available .Mangasarian and Schumaker [11,12] have been investigate discrete spline by using differences in place of derivate .Lower order spline are useful so many purpose for example design and air space satellite .some time positive value of data gives negative value of cubic spline this uncharacteristic behavior destroy data .this extra data are unusual but this not possible in the rational cubic spline because using continuity requirement and Also positivity convexity and shape preserving manage unwanted behavior of the original data and .positivity ..When obtained positive and convex curve by rational cubic spline with two shape free parameter we sub divide whole interval in subintervals so result is not accurate but in discrete rational cubic spline our focus over full interval and obtained accurate results. .Rational discrete cubic spline preserve positivity but normal spline do not preserve positivity this is useful to find shape of curve introduces one shape parameter. Controlling of interpolating curve based on functional values introduce by Duan [13] in which curves to lie above, blow or between the straight lines. .Rational cubic spline interpolation to preserve positivity and monotonicity given by Brodlie and Butt [8] and Butt and Brodlie [9]and Karim [1].. This methods did not give any extra freedom to the user in controlling the final shape of the interpolating curves because of rational spline,another advantage is that their methods require the modification of the first derivative parameters. Sarfraz [5], Sarfraz et al. [4, 6], and Abbas [7] studied the application of rational cubic interpolant for preserving the positive data. Our method useful when difference of function is given in place of derivative .In this paper shape Preserving Discrete C^2 Rational Cubic Spline (Cubic/Quadratic) of one shape parameter has been proposed. Positive preserving and error bound obtained

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Remark : When $h \rightarrow 0$ gives Rational Cubic spline and error obtained by this method is sharper than Hussain et al [4] and sarfraze [5]

2. Rational Discrete Cubic Spline Function.

Let us consider a mesh P on [0,1] defined by

$p: 0 = x_0 < x_1 < \dots < x_n = 1$ such that $x_i - x_{i-1} = h_i$ for $i = 1, 2, \dots, n$. For a given real $h > 0$, suppose a real continuous rational function $s(x, h)$ defined over [0, 1] and its restriction s_i on $[x_{i-1}, x_i]$ is a polynomial of degree 3 or less for $i = 1, 2, \dots, n$. The class $S(p, h)$ of discrete splines of degree 3, is the set of all continuous rational functions $s(x, h)$ such that $i=1, 2, \dots, n-1$, the restriction s_i of $s(x, h)$ on $[x_{i-1}, x_i]$ is a polynomial of degree 3 or less and satisfies

$$D_n^{(j)} s_i(x, h) = D_n^{(j)} s_{i+1}(x, h) \quad j = 0, 1, 2.$$

Let $D_h^{(2)} s(x)$ the second differences with respect to x and $d_i = D_h^{(1)} s(x_i)$

denote first differences value at knots x ,

Consider interpolatory conditions are

$$s(x_i) = f_i; s(x_{i+1}) = f_{i+1}$$

$$D_h^{(1)} f(x_i) = d_i, D_h^{(1)} f(x_{i+1}) = d_{i+1}$$

Now $s(x)$ is defined as

$$s(x) = s(x_i) = \frac{P_i(\theta)}{q_i(\theta)} = \frac{\sum_{i=0}^3 w_i (1-\theta)^{3-i} \theta^i}{q_i(\theta)}$$

(.....2.1)

with $h_i = x_{i+1} - x_i$

$$\theta = \frac{(x - x_i)}{h_i}, \quad i = 1, 2, \dots, n-1$$

where $w_0 = l_i f_i$

$$w_1 = [h_i d_i (l_i^2 h_i^2 - (1-l_i)h^2) - l_i f_i \{2(h^2 - h_i^2)l_i - (h_i^2 + 3h^2)\}] + [(-d_{i+1} h_i \{(l_i h + h_i)^2 - h_i^2\} - l_i h_i h^2 h_i + h f_{i+1} \{l_i h_i^2 + 3h_i^2 h l_i\}) (2h^2 + 3l_i h_i^2 h l_i + 2h_i^2) h h_i - l_i h_{i+1} h^2 [2h^2 h_i l_i - h^2 h_i - h_i^3] / l_i (h_i^2 + h^2) + 2h^2 [2h^2 h_i l_i - h^2 h_i - h_i^3]]$$

and

Type equation here.

$$w_2 = [-d_{i+1} h_i \{(l_i h + h_i)^2 - h_i^2\} - l_i h_i h^2 h_i - f_{i+1} \{l_i h_i^2 + 3h_i^2 h l_i\} \frac{2h^2 + 3l_i h_i^2 h l_i + 2h_i^2}{[2h^2 h_i l_i - h^2 h_i - h_i^3]}]$$

$$w_3 = v_i f_{i+1} + \text{and}$$

$$q_i(\theta) = l_i(1 - \theta)^2 + (l_i + 1)\theta(1 - \theta) + \theta^2$$

After simple calculation $s(x) = f_i\theta + f_{i+1}(1 - \theta)$

Where shape parameters $l_i \rightarrow 0$ gives straight line .

Using second difference in (2.2) i.e.

$$D_h^{(2)}(x_i +) = D_n^{(2)}(x_i -), i = 1, 2, \dots, n-1 \tag{.....2.3}$$

Where $D_h^{(1)}(x) = \{f(x+h) - f(x-h)\}/2h$, $D_h^{(2)}f(x) = [f(x+h) - 2f(x) + f(x-h)]/h^2$.

Produce, on the first difference parameters $d_2, d_3, \dots, d_{(n-1)}$ the following system of linear equation.

$$\begin{aligned} & \frac{h_i l_i d_{i-1} [h^2(l_{i-1}-1) + h_{i-1}^2]}{\{-(l_{i-1}-1)^2 h^2 + h_{i-1}^2\}} \\ & + \frac{d_i \{ [h_i(h^2(l_{i-1}-1) + h_{i-1}^2 l_{i-1} + h_{i-1}^2)] \}}{[-(l_{i-1}-1)^2 h^2 + h_{i-1}^2]} \\ & + h_{i-1} l_i d_i \{ -2l_i(l_i - 1)^2 h^2 + l_i(h_i^2 + h^2) \} / [-l_i(l_i - 1)^2 h^2 + l_i^2 h^2] \\ & + \frac{h_{i-1} d_{i+1} [h^2 l_i(l_i-1) + l_i^2 h_i^2]}{[-l_i(l_i-1)^2 h^2 + l_i^2 h^2]} \\ & = F_i * i = 1, 2, \dots, n - 1. \text{ Where} \\ & F_i^* \\ & = h_{i-1} l_i (l_i + 2) \\ & [(l_i - 1)(h^2 + h_i^2) + 2h_i^2 l_i^2 + 2h_i^2 l_i^2] \Delta_i / [-l_i(l_i - 1)^2 h^2 + l_i^2 h^2] \\ & + h_i(1 + 2l_{i-1}) \\ & [(h_{i-1}^2 + h^2)(l_{i-1} - 1) + 2h_{i-1}^2] \Delta_{i-1} / \\ & [(l_{i-1} - 1)^2 h^2 + h_{i-1}^2] \tag{.....2.4} \end{aligned}$$

Since the system of linear equation is diagonally dominant for all $l_i, > 0$, it has a unique solution for the first difference d_{is} . We can find solution of above equation.

Remark 2.1 :When $h \rightarrow 0$ the Rational Discrete cubic spline reduce to rational cubic spline.

Remark 2.2 :Without shape parameter when $l_i = 1$ so the rational discrete cubic spline reduce to discrete cubic spline.

3. C² POSITIVE RATIONAL DISCRETE CUBIC SPLINE FUNCTION FOR THE POSITIVE SET OF DATA

$\{x_i, f_i\}, i = 1, 2, \dots, n, x_1 < x_2 < \dots < x_n$ and $f_1 > 0, f_2 > 0, \dots, f_n > 0$ it is required to construct a positive interpolant $s(x)$. The rational discrete cubic function (1) is positive if $p_i(\theta)$ and $q_i(\theta)$ both positive, since $q_i(\theta) > 0, \forall l_i, > 0$.

Now $p_i(\theta)$ can be rewritten as follows :

$$p_i(\theta) = a_i\theta^3 + b_i\theta^2 + c_i\theta + d_i.$$

Where

$$a_i = w_1 - w_0 - w_2 + w_3$$

$$b_i = 3w_0 - 2w_1$$

$$c_i = -3w_0 + w_1 + w_2$$

$$d_i = w_0.$$

$$\text{So } a_i + b_i + c_i + d_i \tag{...3.1}$$

$$= w_3 = f_{i+1} > 0. \tag{...3.4}$$

Theorem 3.1 :

The C^2 Rational Discrete Cubic function (2.1) is positive in each interval $[x_i, x_{i+1}]$ if the shape parameter l_i , satisfy following constraint's

$$\text{and } w_3 = f_{i+1} > 0.$$

4. ERROR BOUNDS :

For a given $h \rightarrow 0$, we introduce the set

$R_{ha} = \{a + jh : j \text{ is an integer}\}$ and define a discrete interval as follows :

$$[a, b]_h = [a, b] \cap R_{ha}$$

For a function f and three disjoint points x_1, x_2, x_3 in its domain. The first and second divided differences are defined by

$$[x_1, x_2]_f = \frac{\{f(x_1) - f(x_2)\}}{(x_1 - x_2)}$$

$$\text{and } \{x_1, x_2, x_3\}_f = \frac{\{[x_1, x_2]_f - [x_2, x_3]_f\}}{(x_1 - x_3)} \text{ respectively.}$$

For convenience, we write $f^{(2)}$ for $D_h^{(2)} f$ and $w(f, p)$ for the modulus of continuity of f . The discrete norms of a function f over the interval $[a, b]_h$ is defined by

$$\|f\| = \max_{x \in [a, b]_h} |f(x)|$$

Without assuming any smoothness condition on the data f , we shall obtain in the following the bounds for the error function over the discrete interval $[a, b]_h$.

Theorem 4.1 :

Let $s(x, h)$ be the unique periodic discrete cubic spline interpolant f under the assumption 2.1. Then over the discrete interval $[a, b]_h$.

$$\|e(x)\| \leq p k(h) w(D_h^{(1)} f, p) \tag{...4.1}$$

Where $k(h)$ is some function h defined earlier and $w(f, p)$ is the discrete modules of the continuity of f .

In order to show the convergence of the discrete rational spline. We shall need the following Lemma due to Lyche [14].

Lemma 4.1 : Let $\{a_j\}_{j=1}^m$ and $\{b_j\}_{j=1}^n$ be given sequence of non negative real number such $\sum_{j=1}^m a_j = \sum_{j=1}^n b_j$. Then for any real valued function f , defined on a discrete interval $[0, 1]$ we have

$$\left| \sum_{j=1}^m a_j [x_j, x_{j,1} - x_{j,k}]_f - \sum_{j=1}^n b_j [y_{j,0}, y_{j,1}, \dots, y_{j,n}]_f \right| \leq w\{D_h^{(k)} f, |1 - kh|\} \sum a_j / k! \tag{.....4.2}$$

Where $y_{jk} = f[0, 1]_h$ for rational values of j, k .

Replacing m_i by $D_h^{(1)}e(x_i)$ in Equation (2.4).

We have

$$\begin{aligned}
 e(x) &= h_{i-1}e_{i-1} \frac{[h_i^2(l_{i-1}-1)+h_{i-1}^2]}{[-(l_{i-1}-1)^2h^2+h_{i-1}^2]} \\
 &+ \{e_i[\{h_i\{h^2(l_{i-1}-1) + h_{i-1}^2(l_{i-1}+1) \\
 &+ h_{i-1}\{-(l_i-v_i)l_i(h^2+h^2)+2h_i^2l_i^2\} \\
 &\frac{+h_{i-1}[-(l_i-v_i)l_i(h^2+h^2)+2h_i^2l_i^2]}{\{(l_{i-1})^2h^2+l_i^2h_i^2\}} \\
 &+ e_{i+1}[l_i h_{i-1}[(h^2l_i)(l_{i-1})+l_i^2h_i^2] \\
 &\frac{+h_{i-1}l_i(l_i+2)[-l_i(l_{i-1})(h^2+h_i^2)+2h_i^2l_i]}{[-(l_{i-1})^2h^2+l_i^2h_i^2]} \\
 &\dots\} \quad (\dots 4.3)
 \end{aligned}$$

$$\begin{aligned}
 &\frac{\Delta_i+h_i(1+2l_{i-1})[(h_{i-1}^2+h^2)(l_{i-1}-1)v_{i-1}+2h_{i-1}^2]}{[-(l_{i-1}-1)^2h^2+h_{i-1}^2]} \Delta_{i-1} \\
 &\frac{-h_i l_{i-1} h_i D_j^1 f_{i-1} [h^2(l_{i-1}-1) + h_{i-1}^2]}{[-(\mu_{i-1}-1)^2h^2 + h_{i-1}^2]}
 \end{aligned}$$

$$\begin{aligned}
 &D_j^1 h_i f_i \{h_i(h^2(l_{i-1}-1) + h_{i-1}^2(l_{i-1}+1))\} \\
 &\frac{+h_{i-1}\{-(l_{i-1})l_i h^2+2l_i^2 h_i^2\}}{[-(l_{i-1})^2h^2+l_i^2 h_i^2]} - \frac{l_i h_{i-1} D_i^1 f_{i+1}[-h^2 l_i(l_{i-1})+2h_i^2]}{\{(l_{i-1})^2h^2+l_i^2 h_i^2\}} +
 \end{aligned}$$

$$\frac{l_i h_i D_i^{(1)} f_{i+1}[-h^2 l_i(l_{i-1})+2h_i^2 l_i^2]}{\{(l_{i-1})^2h^2+l_i^2 h_i^2\}}$$

After using from Lyche [12] formula we get,

$$\frac{\sum a_i = h_{i-1} l_i(l_i+2) \frac{h^2(h^2+l_i^2)(h_{i-1}^2+h_i^2)+2h_i^2 l_i^2}{[-(l_{i-1})^2h^2+l_i^2 h_i^2]}}{\mu_i v_i h_{i-1} h_i}$$

$$\frac{+h_i(1+2l_{i-1}) \frac{h_i^{2-v_i}(h_{i-1}^2+h^2)(l_{i-1}-1)+2h_{i-1}^2}{[-(l_{i-1})^2h^2+h_{i-1}^2]}}{\mu_i v_i h_{i-1} h_i}$$

$$= \sum b_j$$

Where $x_{10} = x_i + h$

$$x_{20} = x_i$$

$$x_{30} = x_i - h$$

Where $x_{10} = y_{30} = x_{i+1} + h$

$$x_{11} = y_{31} = x_i - h$$

$$y_{20} = x_{20} = x_i + h$$

$$y_{21} = x_{21} = x_i - h$$

$$x_{30} = y_{1-0} = x_{i-1} + h$$

$$x_{31} = y_{11} = x_{i-1} - h$$

and $a_1 = \frac{h_{i-1} l_i(l_i+2)(-l_i(l_{i-1})(h^2+h_i^2)+2h_i^2 l_i)}{[-(l_{i-1})^2h^2+l_i^2 h_i^2]}$

$$a_2 = \frac{h_i l_{i-1}(l_{i-1}+2l_{i-1})[(h_{i-1}^2+h^2)(l_{i-1}-1)+2h_{i-1}^2]}{[-(l_{i-1}-1)^2h^2+h_{i-1}^2]}$$

$$b_1 = \frac{h_i l_{i-1}(h^2(l_{i-1}-1)h_{i-1}^2)}{[-(l_{i-1}-1)^2h^2+h_{i-1}^2]}$$

$$b_2 = \frac{\{h_i(h^2(l_{i-1}-1)+h_{i-1}^2(l_{i-1}+1)+h_{i-1}\{-l_i-v_i\}l_i h^2+2l_i^2 h_i^2)\}}{[-(l_{i-1}-1)^2 h^2+h_{i-1}^2]}$$

$$b_3 = \frac{li h_{i-1}\{-h^2 l_1(l_i-1)+2h_i^2 l_i^2\}}{\{-l_{i-1}\}^2 h^2+l_i^2 h_i^2}$$

Clearly $k(h) = \sum a_j = \sum b_j$. (.....4.4)

This theorem (4.1) is completed.

Conclusion :In this paper we have obtained method of Discrete Rational Spline Interpolations to obtain construction , Shape ,error bound and convergence.

Table 1. A positive data.For real $h \rightarrow 0$

i	1	2	3	4
x_i	0.0	1.2	1.7	3
y_i	0.4	1.2	9	18

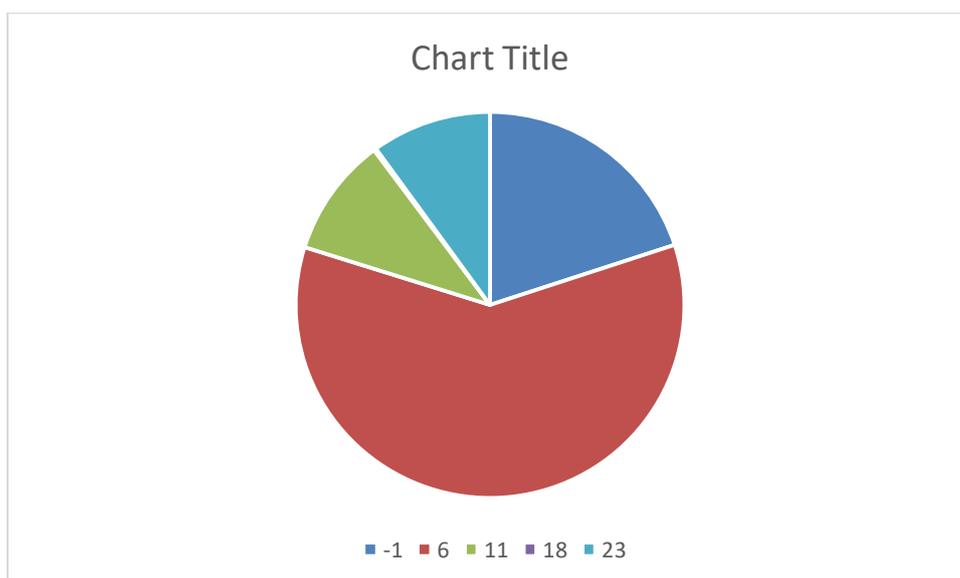
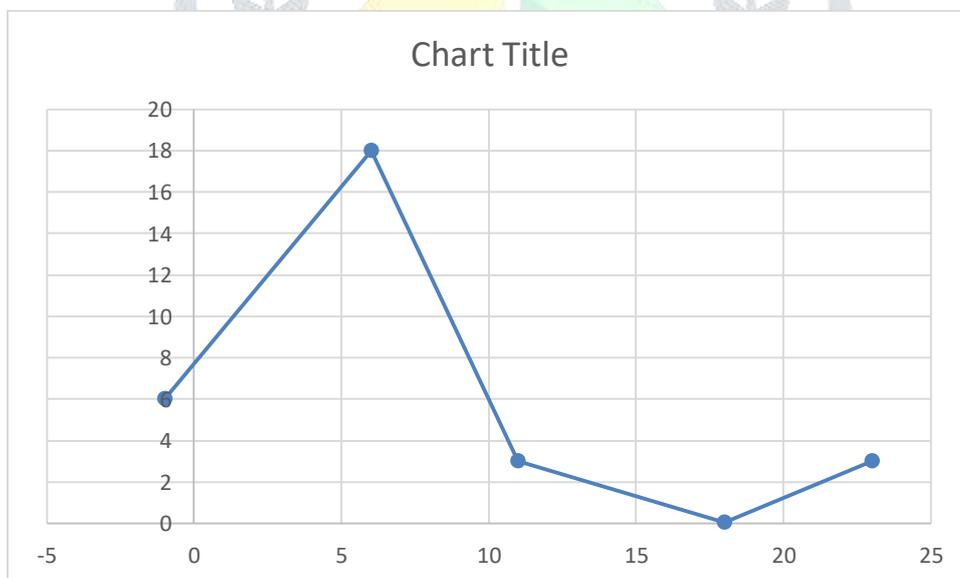


Fig .1 Rational Cubic Hermitt polynomial

Table 2. Given Data

<i>i</i>	1	2	3	4	5	6	7	8	9	10	11
x_i	-1	56	11	18	23	24	31	36	39	42	45
y_i	18	3	0.06	3	7	10	18	29	35	36	43

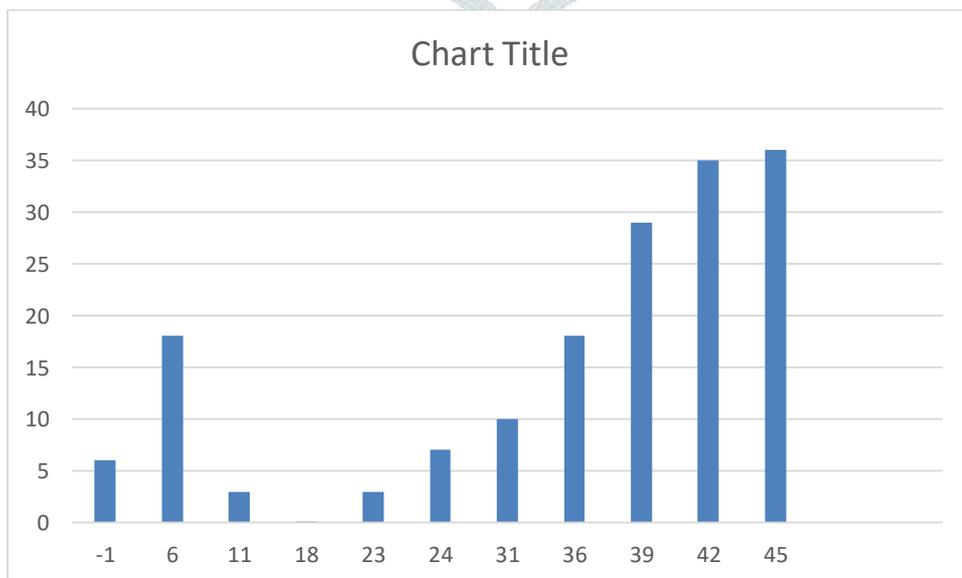
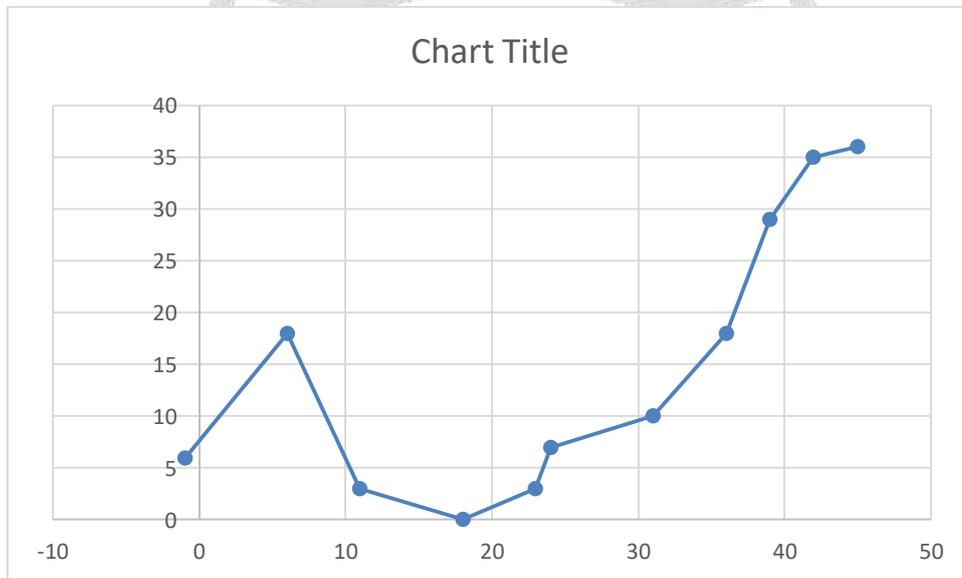


Fig .2 Rational Cubic Spline

Table 3. If change Datas of Butt and Brodly [9]

i	1	2	3	4	5	6	7
x_i	0	2	4	10	28	30	38
y_i	21	9	4	.8	4	6	10

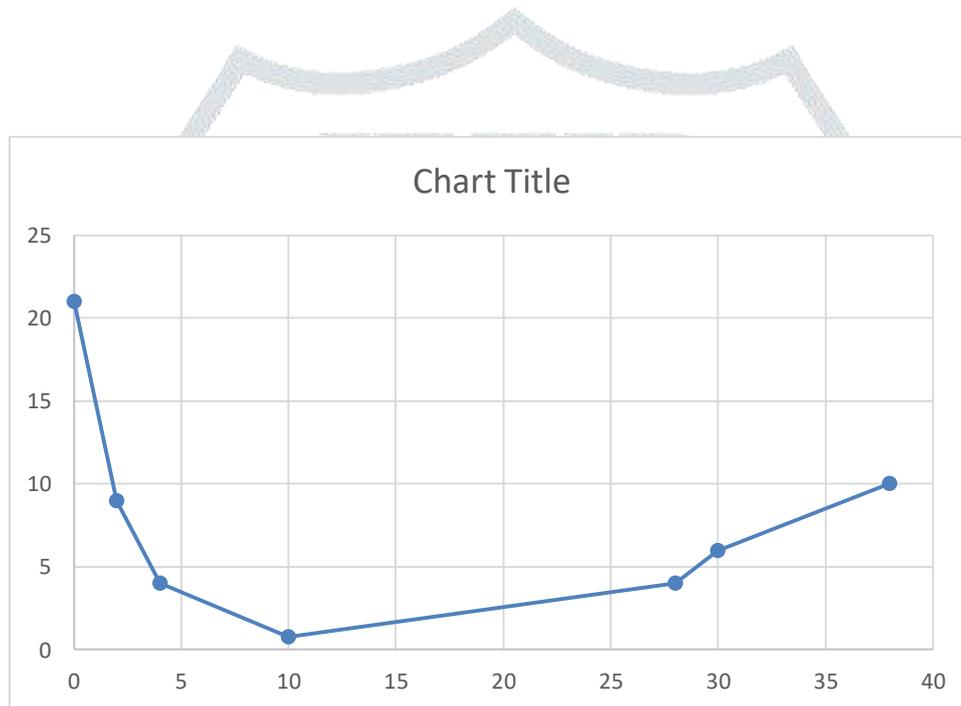


Fig3 : Rational Cubic spline above straight line (joining 0 to 1)

Ethics Statement : Author follow Journal ethics guideline

Declaration of competing interest : Author declared that I have no fanatical competing interest or personal relationship that could have appeared to influence the work reported in this paper

Data availability: Data will be made available on request of reader

Conclusion :: In this paper ,shape Preserving Discrete C^2 Rational Cubic Spline of one shape parameter has been proposed. Positive preserving , error bound and convergence are obtained. Parameters play role of change of shape of curve .

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