



Forecasting INR Exchange Rates Using ARIMA: A Comparative Study with Euro, Pound, and Singapore Dollar

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This paper presents the results of investigation into the nature of the Exchange Rate variations over the period 2012:I to 2022:XII in the economy of Europe, England and Singapore. Monthly Exchange Rate are found to be I(1) variables. This paper models the ARIMA(p,d,q) structures monthly exchange rates of EUR/IND POUND/IND and SGD/IND, exchange rates and compares the actual data with derived relevant ARIMA(p,d,q) forecasts on the basis using time series analysis over the period from 2012:I to 2022:XII. The official monthly data collected from International Financial Statistics (IMF) are used for present study. The accuracy of the forecast is also examined with relevant statistical measures.

Keywords: Stationarity, ADF Test, Ordinary Least Square ACF, PACF Box-Jenkins Methodology White Noise Forecast Errors, Mean Absolute Error, Mean Absolute Percentage Error, Root Mean Square Error, ARIMA.

JEL Classification: F17, F47, F470

Introduction: The exchange rate series of EURO/IND, POUND/IND and SGD/IND for the period from January 01, 2012 to November 30, 2022 have been exhibited in Figure 1. Monthly exchange rates were used from January 2012 to November 2022. Exchange rate can be forecasted in wider ways by using the multivariate approach in which exchange rates of these countries maintains a relationship with both macroeconomic variables like money supply, output, inflation etc.

Properties of Stationarity Series:

In case of time series analysis, unit root tests are used to detect the stationarity and non- stationarity of the time series data. A stationary time series data set has three basic properties: -

First, it has a finite mean, which implies that a stationary series fluctuates around a constant long run mean.

Second, a stationary time series has a finite variance. This implies that variance is time invariant.

Third, a stationary time series data set has finite auto-covariances.

Integrability of e_t Series ADF Tests

Stationarity of first differenced series of exchange rate (Δe_t) have been studied with the Augmented Dickey–Fuller (ADF) test. The basic ADF Test equations are

$$\Delta e_t = \alpha_1 + \gamma_1 e_{t-1} + \delta_{1t} \sum_{i=1}^k \Delta e_{t-i} + \varepsilon_{1t} \quad \dots\dots\dots(1)$$

$$\text{where } \Delta e_t = (e_t - e_{t-1}) \quad \varepsilon_{1t} \sim iidN(0, \sigma_{\varepsilon t}^2)$$

These basic equations have been estimated with some maintained alternative assumptions like

- i. $\alpha_1 \neq 0, \gamma_1 = 0$
- ii. $\alpha_1 = 0, \gamma_1 \neq 0$

AR and MA structures for any series and relevant correlogram

An autoregressive model is one where the current value of the variable can be explained in terms of the values of the variable taken in the past plus and error term. An autoregressive model of order p , $AR(p)$ is explained as

$$E_t = \alpha + \beta_1 E_{t-1} + \beta_2 E_{t-2} + \dots + \beta_p E_{t-p} + u_t \dots \dots \dots (2)$$

Where u_t is the white noise distribution term. In the lag operator form equation (2) can be written as

$$E_t = \alpha + \gamma(L)u_t$$

$$\text{where } \gamma(L) = 1 + \gamma_1 L + \gamma_2 L + \dots + \gamma_q L^q$$

A moving average processes assumes that the current value of the variable can be explained in terms of sum of a constant term plus a moving average of current and past white noise disturbance terms. A moving average of order q , $MA(q)$ is explained as

$$E_t = \alpha + u_t + \gamma_1 u_{t-1} + \gamma_2 u_{t-2} + \dots + \gamma_q u_{t-q} \dots \dots \dots (3)$$

In the lag operator form equation (3) can be written as

$$Y_t = \alpha + \gamma(L)u_t$$

$$\text{where } \gamma(L) = 1 + \gamma_1 L + \gamma_2 L + \dots + \gamma_q L^q$$

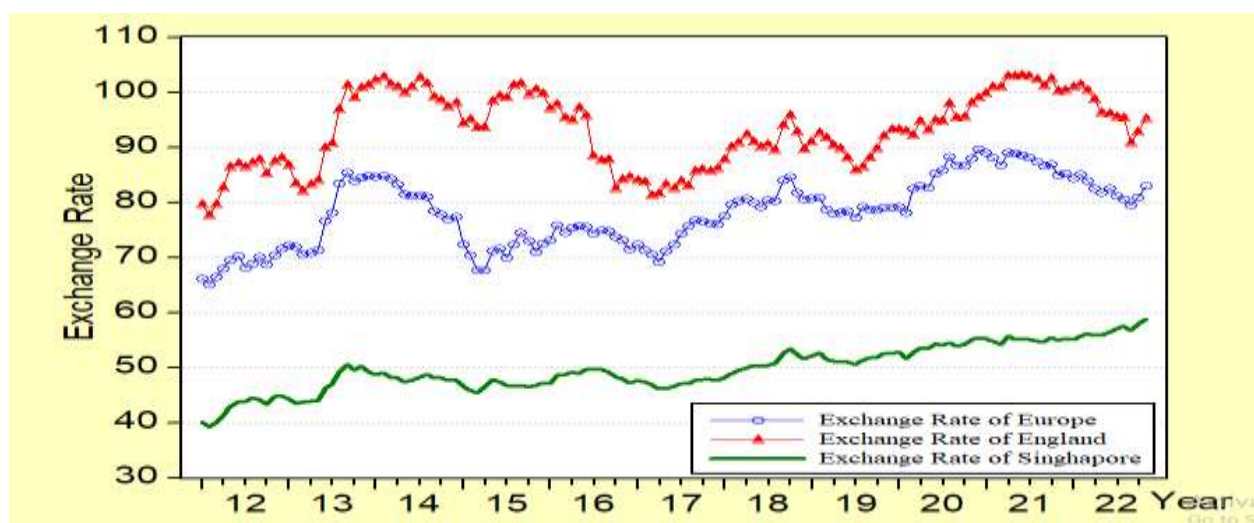
$ARIMA(p,q)$ process is the combination of $AR(p)$ and $MA(q)$ model. In the lag operator from this model is written as

$$B(L)E_t = \alpha + \gamma(L)u_t \dots \dots \dots (4)$$

$ARIMA$ process shows a combination of the characteristics of AR and MA process. AR process has a geometrically declining ACF (Autocorrelation Function) and a non-zero points a $PACF$ (partial autocorrelation function) while MA process has a number of non-zero points in ACF and a geometrically declining $PACF$. $ARIMA$ will be having both geometrically declining ACF and $PACF$. One very essential condition of the time series analysis is that underlying series must be stationary. For the stationary conversion of the series one more letter (I) is added to the $ARMA$ process, which shows the number of the times underlying series is differenced for making it stationary. on account of this transformation, $ARMA$ process is also referred to as $ARIMA$ process.

When time series data are used in econometric analyses, the preliminary statistical step is to test the stationary of each individual series. Unit root tests provide information about stationarity of the data. Non-stationarity data contain unit roots. The main objective of unit root tests is to determine the degree of integration of each individual time series. Various methods for unit root tests have been applied in the study.

Figure:1
Time Plot of Exchange Rate in Europe, England & Singapore



In the above figure we see from the graph that there is an upward trend for all three currencies with respect to India throughout eleven years.

Table:1
Results of the AUGMENTED DICKEY FULLER (Unit Root Test)
 (Automatic based on SIC, MAXLAG=12) [Sample:- 2012:I -2022:XI]

COUNTRY	Variable	ADF Test Stat.	Prob* Value	Mackinnon Critical Value			Remarks
				1%	5%	10%	
EUROPE	e_t	-2.014	0.2806	-3.481	-2.884	-2.579	Non-Stationary
	Δe_t	-9.631	0.000	-3.482	-2.884	-2.579	Stationary
SINGHAPORE	e_t	-1.177	0.683	-3.481	-2.884	-2.579	Non-Stationary
	Δe_t	-10.294	0.000	-3.482	-2.884	-2.579	Stationary
ENGLAND	e_t	-2.348	0.159	-3.481	-2.884	-2.579	Non-Stationary
	Δe_t	-10.256	0.000	-3.482	-2.884	-2.579	Stationary

where e_t stands for Exchange Rate at level and

Δe_t stands for 1st difference of Exchange Rate

It is observed from the ADF Tests that

- Exchange rate(e_t) series at level are having unit roots even at 10% level of significance.
 - the Exchange Rate (Δe_t) are free from unit roots even at 1% level of significance.
 - e_t is non-stationary and I(1) variable in all countries.
 - Δe_t is stationary in all countries and Δe_t is I(0) variable
- So we proceed by using 1st differencing datasets of exchange rate in ARIMA forecasting.

Identification of the model:

AR(P) structure Identification(EUROPE):

The ACF and PACF both are significant spikes at lag one which indicate that the exchange rate (e_t) series defines are AR(1) structure. Consequently, the estimable AR(1) model is

$$\Delta e_t = \alpha + \alpha_1 \Delta e_{t-1} + u_t \dots \dots \dots (5)$$

Results of estimation

The estimated equation (5) is as follows

$$\Delta e_t = 0.122 + 0.152 \Delta e_{t-1} \dots \dots \dots (6)$$

t-stat.	0.885	1.727
Prob.	0.378	0.086
S.E.	0.138	0.088

$R^2 = 0.023$ Adj $R^2 = 0.015$ DW = 1.979 F. Stat. = 2.983

The equation (6) shows that

- $\hat{\alpha}_1$ is found be significant at 5% level.
- The equation is free from autocorrelation since DW = 1.978755.

MA(q) structure for identification

$$\Delta e_t = \alpha_1 + \alpha_2 \mu_t \dots \dots \dots (7)$$

Estimated equation (7) indicates

$$\Delta e_t = 0.127 + 0.999 \mu_{t-20} \dots \dots \dots (8)$$

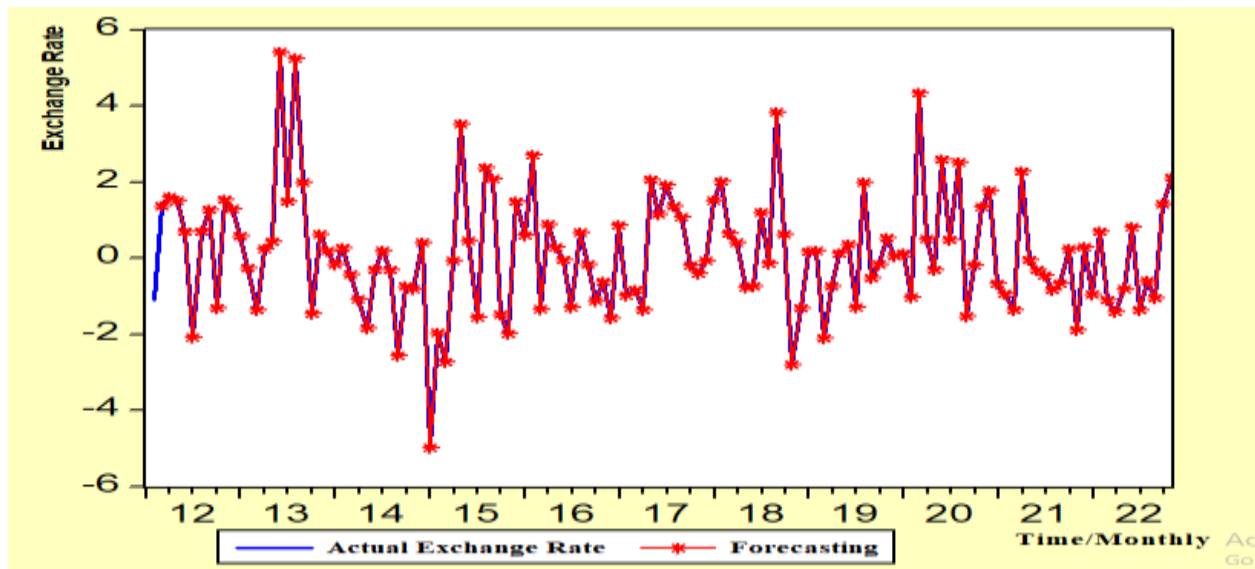
t-stat.	5.729	68.459
Prob.	0.000	0.000
S.E.	0.022	0.014

$R^2 = 0.977$ Adj. $R^2 = 0.977$ D.W. = 1.702 F. Stat. = 4686.670

ARIMA(1,1,20) forecasts for e_t . The estimated model becomes

$$e_t = \alpha_1 + \beta_1 e_{t-1} + \gamma_1 \delta_t + v_t$$

ARIMA (1,1,20) model as given by the equation 1 has been used for generating one period ahead forecast for e_t . The time plots of Exchange Rate (e_t) and the corresponding forecast (e_t) are being presented through the figure 1. e_t is found to be coincident with forecast value over the period concerned.

Figure: 2**The Time Plot of the Exchange Rate and the corresponding ARIMA(1,1,20) Forecasts****Findings from the Figure:2**

From the figure :2 shows that The ARIMA(1,1,20) forecasts almost coincide with the exchange rate level data and exchange rate dataset is marked by the absence of unusual variability over the period 2012:1-2022.:11.

Exchange Rate Forecast in England**Identification of the model:**

AR(P) structure Identification(England): The ACF and PACF both are significant spikes at lag eleven which indicate that the exchange rate (Δe_t) series defines as AR(11) structure. Consequently, the estimable AR(11) model is

$$\Delta e_t = \alpha + \alpha_1 \Delta e_{t-11} + u_t \dots \dots \dots (9)$$

Results of estimation

The estimated equation (9) is as follows

$$\Delta e_t = 0.056 + 0.154 \Delta e_{t-11} \dots \dots \dots (10)$$

t-stat.	0.308	1.698
Prob.	0.759	0.092
S.E.	0.181	0.091
R^2	0.023	Adj R^2 = 0.015
DW	1.844	F. Stat. = 2.882

Findings from the equation (10) shows that

- I. $\hat{\alpha}_1$ is found be significant at 5% level.
- II. The equation is free from autocorrelation since DW = 1.844

MA(0) structure of identification

$$\Delta e_t = \alpha + \alpha_1 \epsilon_t \dots \dots \dots (11)$$

Estimated model of equation (11) indicates

$$\Delta e_t = 0.067 + 0.999 v_t$$

t-stat.	2.390	71.513
prob.	0.018	0.000
S.E	0.028	0.014
R^2	0.977	Adj R^2 = 0.977
D.W.	1.839	F. Stat. = 5114.134

- i. α is found to be significant at 1% level.
- ii. $\hat{\alpha}_1$ is found be significant at 1% level.

The ACF and PACF of the residuals of the equation (11) are shows that the ACF contains no significant spikes and these observations testify for MA(0) structure for Δe_t .

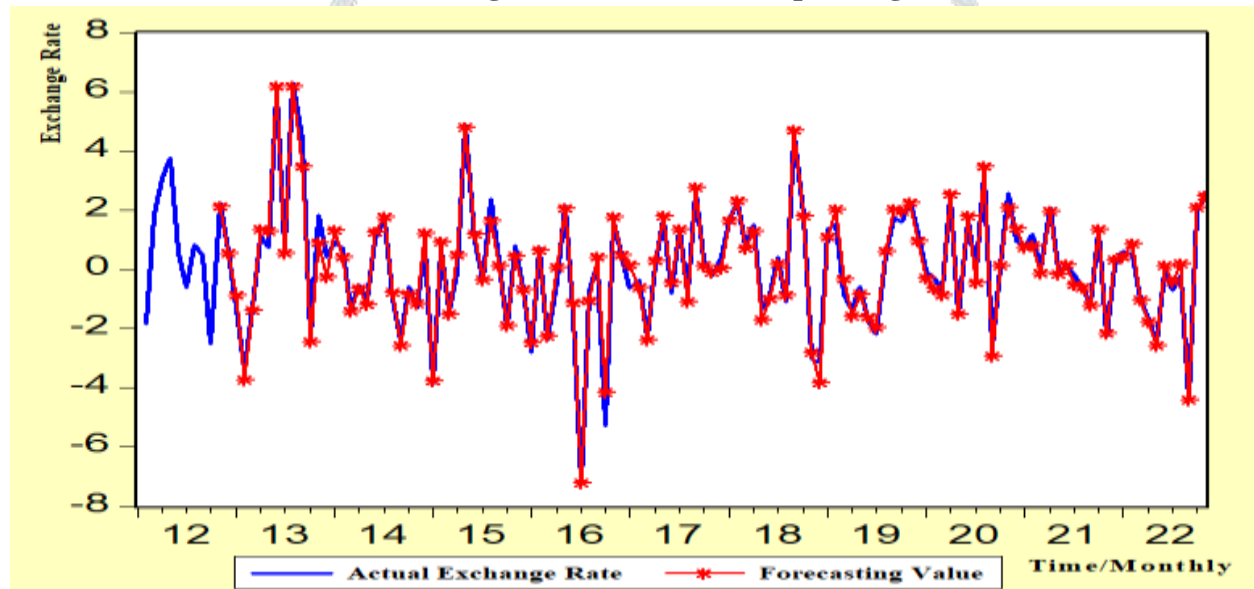
ARIMA (11,1,0) forecasts for e_t . The estimated model becomes

$$e_t = \alpha_1 + \beta_3 e_{t-11} + \gamma_3 \delta_t + \mu_t \dots \dots \dots (12)$$

ARIMA(11,1,0) model as given by the equation (12) has been used for generating one period ahead forecast for e_t . The time plots of Exchange Rate(e_t) and the corresponding forecast(e_t) are being presented through the figure 2. e_t is found to be coincident with δ_t over the period concerned.

Figure: 3

The Time Plot of the Exchange Rate and the corresponding ARIMA(11,1,0) Forecasts



Exchange Rate Forecast in Singapore

Identification of the model:

AR(P) structure Identification(Singapore):

The ACF and PACF both are significant spikes at lag six which indicate that the first differenced of exchange rate (Δe_t) series defines as AR(6) structure. Consequently, the estimable AR(6) model is

$$\Delta e_t = \alpha_1 + \alpha_2 \Delta e_{t-6} + u_t \dots \dots \dots (13)$$

Results of estimation

The estimated equation (11) is as follows

$$\Delta e_t = 0.141 - 0.167 \Delta e_{t-6} \dots \dots \dots (14)$$

t-stat. 2.244 -1.923

Prob. 0.027 0.057

S.E. 0.063 0.087

$R^2 = 0.029$ Adj $R^2 = 0.021$ DW = 1.861 F.Stat. = 3.698

Findings from the equation (14) shows that

- I. $\hat{\alpha}_1$ is found be significant at 1% level.
- II. $\hat{\alpha}_2$ is found be significant at 5% level.
- III. The equation is free from autocorrelation since DW = 1.861 .

MA(q) structure Identification

$$\Delta e_t = \alpha + \alpha_1 \varepsilon_{t-17} + \delta_t \dots \dots \dots (15)$$

Estimated model of the equation (15) indicates

$$\Delta e_t = 0.122 + 0.993 \omega_{t-17}$$

t-stat. 10.907 55.028

prob. 0.000 0.000

S.E. 0.011 0.018

$R^2 = 0.966$ Adj $R^2 = 0.965$ D.W. = 1.868 F.Stat. = 3028.140

The ACF and PACF of the residuals of the equation (15) are shows that the ACF contains no significant spikes and these observations testify for MA(17) structure for Δe_t .

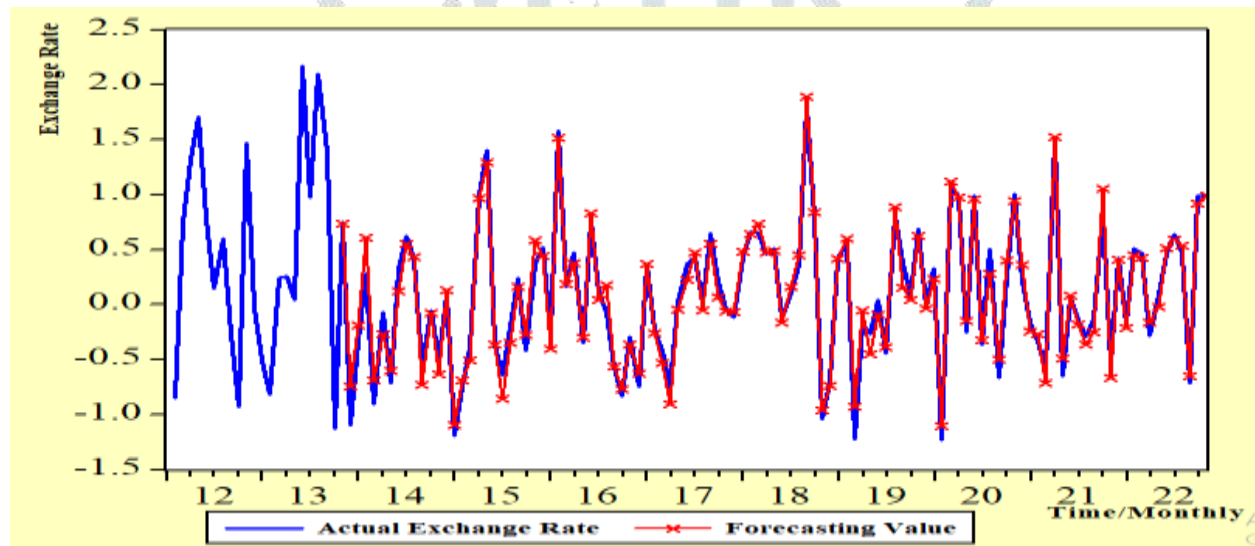
ARIMA (6,1,17) forecasts for e_t . The estimated model becomes

$$e_t = \alpha_1 + \beta_6 e_{t-6} + \gamma_6 \delta_{t-17} + \mu_t \dots \dots \dots (16)$$

ARIMA(6,1,17) model as given by the equation (16) has been used for generating one period ahead forecast for e_t . The time plots of Exchange Rate (e_t) and the corresponding forecast(e_t) are being presented through the figure 3. e_t is found to be coincident with δ_t over the period concerned.

Figure: 4

The Time Plot of the Exchange Rate and the corresponding ARIMA(6,1,17) Forecasts



Not only that we can use the forecast performance by applying the non-statistical method like as MAE, MAPE and RMSE. These performance metrics were calculating the forecast results for the above countries.

$$MSE = \frac{1}{k} \sum_{t=1}^k |\hat{X}_t - X_t|$$

$$MAPE = \frac{100}{k} \sum_{t=1}^k \left| \frac{\hat{X}_t - X_t}{X_t} \right|$$

$$RMSE = \sqrt{\frac{1}{k} \sum_{t=1}^k (\hat{X}_t - X_t)^2}$$

Where k is the no of values, X_t is the actual exchange rate, \hat{X}_t is the forecast exchange rate and t is the time. By calculating the performance matrix, the evaluation of the forecast value shows by time series ARIMA model.

Table: 2
Performance Matrix

Currency	Performance Metrics		
	MAE	MAPE	RMSE
EUR	1.548	7.516	1.897
SGD	1.032	1.753	1.576
POUND	2.523	1.421	1.491

MAE= Mean Absolute Error, **MAPE**= Mean Absolute Percentage Error

RMSE= Root Mean Square Error

In the above Table we can explain that all values of the calculated performance metrics are relatively close to the actual data, with minimum error as 1.032 for Singapore dollar and the error of Euro was maximum i.e 7.516. The error values for euro were more than the values for Singapore dollar. This can be explained by the fact that exchange rates for euro were more fluctuating than the Singapore dollar from 2012 to 2022, which we can be observed from Figure 1. According to the performance metrics, the evaluation of ARIMA model via MAE shows the minimum error values for all three currencies. For example, the MAE was 1.54 per cent for EURO dollar, 1.03 per cent for SGD, and 2.52 per cent for England Pound.

The seasonal ARIMA models were implemented to forecast short term inflation. To sum up, the ARIMA model-based forecasts of exchange rates for England, Europe and Singapore dollar with respect to India Rupee were evaluated and showed relatively adequate results in comparison with the actual data.

Conclusions: ARIMA model is the appropriate technique for predicting the magnitude of the variable. This technique can be used for forecasting long time series data.

In our study the exchange rate was found to be generated by the ARIMA (1,1,20) model in Europe, ARIMA (11,1,0) for England and ARIMA (6,1,17) for Singapore and all these models were used for forecasting exchange rate. The all ARIMA forecasts were found to be efficient since there were no heteroskedasticity of forecast residuals. All ARIMA forecasts have been found to be efficient and therefore all these forecasts are appropriate model for forecasting exchange rate in the economies of Europe, England and Singapore.

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