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LOCATION IDENTIFICATION USING ANTI-MAGIC GRAPHS

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Abstract : Graphs are discrete structures consisting of vertices and edges which connect the vertices. Graph theory has now branched off into various directions - domination in graph, hypergraph, topological graph theory, algebraic graph theory, labeling of graphs to name a few. Among these, graph labeling has various applications in different fields. Different types of graph labelling techniques are available in literature. A graph labeling is called an anti- magic labeling if the edges of the graph can be labeled with 1, 2, . . . , E (where E is the number of edges in the graph) without repetition such that the sums of the labels of the edges incident to each vertex(vertex sum) are distinct. In this paper we found an application of anti- magic labeling in Location identification problems. Location identification problems refer to the process of evaluating and assessing the suitability of a particular location for a specific purpose. An application in locating warehouse is illustrated.

Index Terms - Graph Labeling, Magic Labeling, Anti- Magic Labeling, Location Problems.

I. Introduction

Graph theory is used to model real world problems. It emerged in the 18th century when the natives of Konigsberg tried to traverse through the 7 bridges over the Pregel river exactly once and come back at the starting point. Mathematician Euler modelled this puzzle as a graph with 4 vertices and edges. The systematic study of graph theory began in 1935. Graph labeling has become an interesting field due to its broad range of applications. A vital role has been played by labeled graphs in various fields. Graph labelings were first introduced in the mid 1960s by Alex Rosa. Sedlacek introduced the magic labelling concept in 1963. The notion of an anti-magic graph was introduced by Hartsfield and Ringel in 1989. Hartsfield and Ringel conjectured in 1990 that every connected graph other than K2 has an anti-magic labeling. Later Alon et al. in 2004 proved that dense graphs are anti- magic. Anti-magic graphs are used to solve real world problems in various fields. In this study we introduce a real world application of anti-magic labeling.

II. PRELIMINARIES

Definition 2.1 Graph [5]

A graph G = (V, E) consists of a nonempty set V of vertices (also called nodes) and a set E of edges that connects the vertices. Each edge has either one or two vertices associated with it, called its endpoints.

Definition 2.2 Complete Graph [5]

A complete graph on n vertices, is a simple undirected graph that contains exactly one edge between each pair of distinct vertices.

Definition 2.3 Graph Labeling [4, 6]

A graph labeling is an assignment of integers to the vertices or edges, or both, subject to certain conditions.

Definition 2.4 Magic Labeling[4, 7]

A labelling is a magic labeling if the edges of the graph can be labeled by the numbers 1 , 2 , 3 , … , q in such a way that the sum of labels of all the edges incident with any vertex is the same where q is the number of edges of the graph. A graph is said to be a **magic graph** if it has a magic labeling.

Definition 2.5 Anti- magic Labeling[4, 3]

A graph with E edges is called anti- magic if its edges can be labeled with 1, 2, . . ., E without repetition such that the sums of the labels of the edges incident to each vertex(vertex sum) are distinct.

Theorem 2.6 [1, 2]

Connected graphs except K_2 has an anti-magic labeling.

III. RESULT

Anti-magic labeling can be used to solve location problems (identifying a suitable location) in various situations by considering distance between various places. Suppose there are n places, out of these n places we need to locate a distribution centre of goods to the other n-1 places by considering distances between each places.

3.1 Algorithm for finding the distribution centre

Step1 : Let the n places are denoted by $G_1, G_2, \ldots G_n$ and identify the $\frac{n(n-1)}{2}$ shortest distance between the places.
Step 2 : Arrange the shortest distance between the places in ascending order $d_1 < d_2 < \cdots < d_{\frac{n(n-1)}{2}}$.
Step3 : Map the $\frac{n(n-1)}{2}$ distances to the set $\left\{1,2, \ldots \frac{n(n-1)}{2}\right\}$ using the bijection $f(d_i) = i$
Step4 : Model this as a complete graph G with n vertices and $\frac{n(n-1)}{2}$ edges. The vertices are the n places and the edges are the shortest route between the places.
Step5 : Find an anti- magic labeling of the graph G and identify the vertex with smallest vertex sum. The place corresponding to this vertex is the suitable place for the distribution centre.

3.2 Illustration

A national company in the field of grocery stores has decided to open outlets in six places of Kottayam district in Kerala State. It has also decided to start a warehouse facility at one of these six places. Identify a location within easy reach of five other locations.

The above data can be modelled as a complete graph with 6 vertices and 15 edges(Fig1). The vertices are six places and edges are shortest route between the vertices. Name of the places are as follows:

G_1 –Pala, G_2 –Erattupetta, G_3 – Kottayam, G_4- Ettumanoor, G_5- Vaikom, G_6- Changanassery

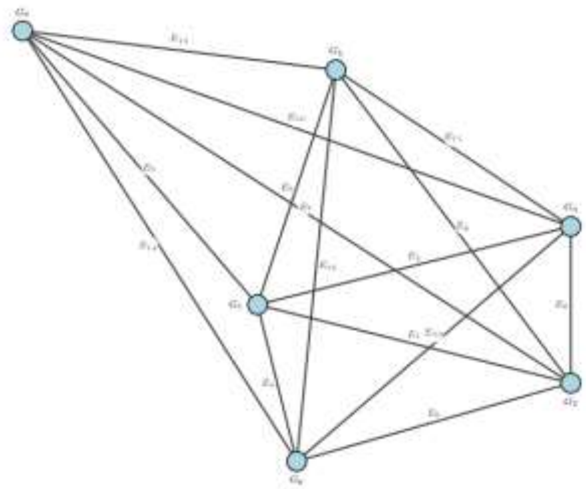

***Figure** 1*

Shortest Distance between places in kilometers are shown in Table 1.

	Kottayam	Changanassery	Pala	Vaikom	Erattupetta	Ettumanoor
Kottayam	0	18.3	27.3	29	42.7	11.6
Changanassery	18.3	0	43	44.6	54.3	29.3
Pala	27.3	43	0	43.4	11.9	17.8
Vaikom	29	44.6	43.4	0	55.3	25.2
Erattupetta	42.7	54.3	11.9	55.3	0	27.9
Ettumanoor	11.6	29.3	17.8	25.2	27.9	0

Table: 1

The distances between the places are arranged in ascending order and label them as shown in Table 2.

Distance between	Distance	Labels of edges
Kottayam-Ettumanoor	11.6	1
Pala-Erattupetta	11.9	2
Pala-Ettumanoor	17.8	3
Kottayam -Changanassery	18.3	4
Ettumanoor -Vaikom	25.2	5
Pala-Kottayam	27.3	6
Ettumanoor- Erattupetta	27.9	7
Kottayam- Vaikom	29	8
Ettumanoor- Changanassery	29.3	9
Kottayam- Erattupetta	42.7	10
Pala- Changanassery	43	11
Pala- Vaikom	43.4	12
Changanassery-Vaikom	44.6	13
Erattupetta- Changanassery	54.3	14
Erattupetta- Vaikom	55.3	15

Table :2

The graph possessed an anti- magic labeling shown in the figure 2. The vertex sum of each vertices is shown in Table 3.

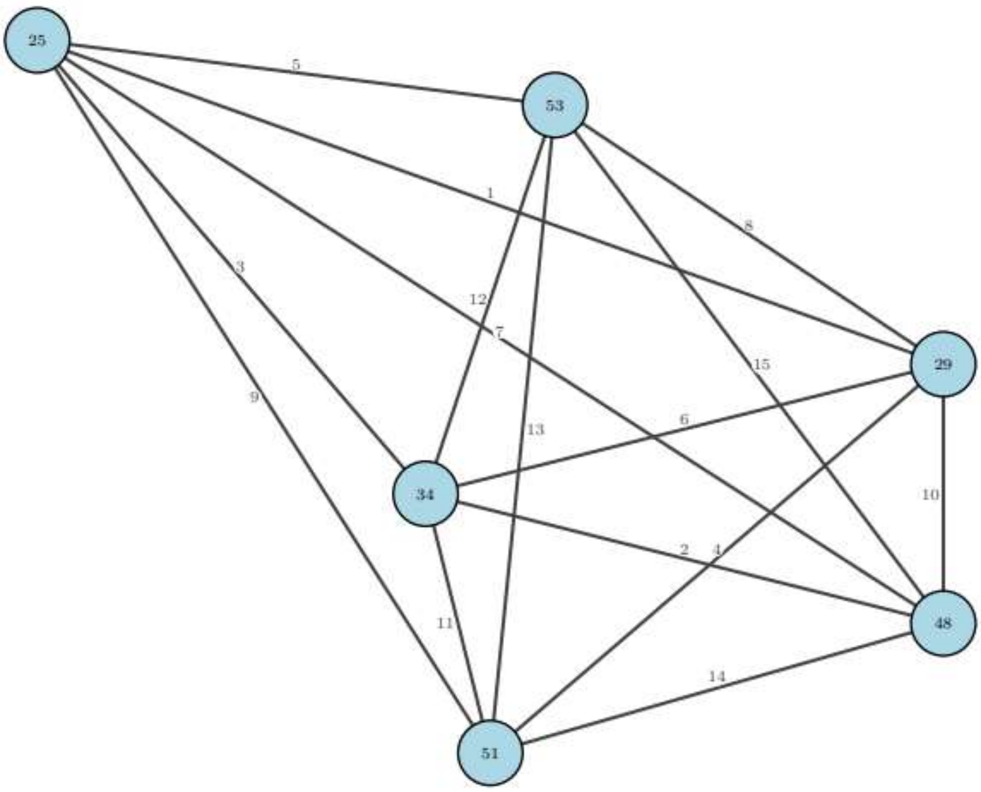

Figure *2*

Vertex	Vertex Sum
G_1	34
G_2	48

G_3	29
G_4	25
G_5	53
G_6	51

Table : 3

From this anti-magic labeling, the least vertex sum possesses the vertex G_4 (Ettumanoor) and so the suitable location to start the warehouse is the **Ettumanoor** town.

IV. CONCLUSION

Here we introduced an application of anti- magic labelling and developed an algorithm for solving the location problems. Even though various graph labelling techniques are available, anti- magic labeling is more convenient for solving location identification problems by considering the distance between various places. The information given in the table are taken from the google map.

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