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A Scientific Technical Application of Fixed Point Theorems in Metric Space

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Abstract:

In this paper, we study the scientific and technical application of fixed-point theorems in metric spaces using contraction mapping. When integrated with contraction mapping, a framework tool designed to handle uncertainty and imprecision, these theorems extend their applicability to a broader range of scientific and technical fields. Our main aim is to discuss fixed-point theorems in metric spaces of interactive functions using contraction mapping, along with a collection of applications and a comparative analysis of scientific and technical terms.

Keywords: Fixed point, metric space, distance function, contraction mapping,

1. Introduction:

The technical word of science has fixed-point theorems in metric spaces using Banach contraction mapping theorems and revealed itself as a very important powerful tool in the study of nonlinear equation phenomena. This particular technique of fixed points has been applied in many fields such as engineering, Hamming distance, astronomy, geometry,computer science , game theory , image processing, Levenshtein distance, physics, economics, chemistry, mechanics, testing of algorithms, and biology, etc. The fixed point equation $f(x^*) = x^*$, where, $x^* \in f(x^*)$. The fixed point results for (α, β)-(ϕ, ψ)-contractive map are based on the work done by Alizadeh, S. [1]. The fixed-point theorem was introduced and
established by Banach, S. [2] in 1922. The contraction mapping for set-valued and graphs is from the work of Bega, Butt.,A.R.,and Radiojevic, S.[3]. Some new results related to the fixed point theorem are based on the work of Chatterjea, S.K.[4] in 1972 , the extension of fixed point theorems involving two metric spaces in iterated functions was work done of Charls , and Stinson, P.[5]. The generalised contraction and fixed point theorem was introduced by Ciric, J.[6] and is considerable in this work. The results on fixed point theorems in complete metric space were presented by Dolhare, U. and Johar, G. D.[7].The fixed point theorem in R-trees with application was introduced by Espinola, R. and Kirk, W. A. [8]. The graphical rectangular b-metric spaces with an application to the vibrations of a heavy hanging cable was introduced by Goyal, A. and Younis, M. and Singh D. [9] in 2019. The recent concept of graphically fixed point theorems on a metric Space introduced by Hemavathy, R. and

Gayathri, R.O. [10] in 2022. The fixed point by the extension work done of Kannan, R. [11] in 1968. The multivalued contraction mapping result on fixed point theorems in complete metric space by the introduced of Nadaler, S. B. [12] in 1968.The Sequence of contractions and fixed point results by the study of Nadaler, S. B. [13] in 1968. The iterated functions consist of F-contraction map, fixed point theory by the extension work of Secelean, N. A. [14] in 2013. Some fixed point results via R-functions by the study of Nastasi, A. and Vectro, P. [15]. A common fixed point theorem in metric space, by the work done of Sedge, S., Shobe, N. and Zhou, H. Y. [16]. The fixed theorem generalized in metric space and mapping induced by Dhage, B. C. [17] in 1992.The fixed point theorems on weakly contractive mapping are from the work done by Rhoades, B. E. [18] The fixed point theory of cont. mathematics has extended this work of Reich [19]. The design of fuzzy logic controller using a genetic algorithm. Proceedings of the 4th International Conference on Genetic Algorithms,by the work done of Karr,C.L.[20] in 1991 and also study of [21, 22, 23, 24, 25, 26, 27, 28, 29]. Our main purpose is to show the development of different results of unique fixed (UF) points and their collection in metric space of Interactive functions using contraction mapping, along with a collection of applications and a comparative analysis of scientific and technical terms.

2. **Preliminaries**

Basic Definition

[2.1] Fixed point: Suppose that (X, d) is a complex(C or R) metric space and a mapping $f: X \to X$, and $x^* \in X$.Then a point x^* belonging to X is called a fixed point of f if X is mapped into itself.That is $f(x^*) = x^*$ for some of x^*.

[2.2] Metric Space:-Suppose X is a non-empty set. A distance function d on X associates elements $x^*, y^* \in X$.Then we write a complex number $d(x^*, y^*)$

Such that,

I. $d(x^*, y^*) \geq 0$ and $d(x^*, y^*) = 0, x^* = y^*$ (positivity)

II. $d(x^*, y^*) = d(y^*, x^*)$ (symmetric)

III. $d(x^*, z^*) = d(x^*, y^*) + d(y^*, z^*)$ (Triangle inequality),since $\forall, x^*, y^*, z^* \in X$.

Then d is said to be a metric on X, and (X, d) is called a metric space. Then (X, d) is referred to as the distance between x^* and y^*.

EX: Real and complex numbers are both metric spaces endowed with the distance function.

[3] Main Result

Suppose that $x_0 \in X$ is an arbitrary complex number and a fixed point of

$f(x_0) = x_0$ is the iterative sequence $\{x_o, fx_0, f^2x_0, f^3x_0,\ldots\}$.

Where $f^n = f \circ f \circ f\ldots \circ f$, $n \geq 1$.

[3.1] Theorem: Suppose that (X, d) be a complex metric space and $f: X \to X$ be a mapping of itself. If the following condition

$$d(fx^*, fy^*) < \max\{ d(x^*, fy^*), d(y^*, fy^*) \}, \ldots [1]$$

is satisfied by f for every $x^* \in X$ and $y^* \in X$, then X is the fixed point of f.

Proof: Suppose that the complex number X be a fixed point of f.

If we take $x^*, y^* \in X$ and $x^* \neq y^*$, from the equation [1], we obtain

$d(x^*, y^*) = d(fx^*, fy^*) < \max\{d(y^*, fx^*), d(y^*, fy^*)\} = d(x^*, y^*)$

That is a contradiction, then we have $x^* = y^*$ for every $x^*, y^* \in X$.

Then X has only one fixed point of f.

[3.2] Theorem: Suppose that (X, d) is a complete complex metric space and the mapping f be from $X \to X$ itself with fixed point of C.The self mapping f satisfied the condition

$$d(x^*, fx^*) \leq \partial\,[\max\{ d(x^*, yx^*), d(x_0, fx^*)\} - d(x_0, fx^*)\} \ldots [2]$$

for every $x^* \in X$ and some $\partial \in (0, 1)$, iff when $f = \mathbf{I}_x$.

Proof: Suppose we take $x^* \in X$ with $fx^* \neq x^*$. By equation [2], if $d(x^*, fx^*) \geq d(x_0, fx^*)$,then we obtain $d(x^*, fx^*) \leq \partial\,[d(x^*,fx^*) - d(x_0, fx^*) \leq \partial\, d(x^*, fx^*)$,whish is a contradiction due to the fact that $\partial \in (0, 1)$. If $d(x^*,fx^*) \leq d(x_0, fx^*)$.

Then we obtain

$$d(x^*, fx^*) \leq \partial\,[d(x_0,fx^*) - d(x_0, fx^*)] = 0$$

So, we have $fx^* = x^*$ and that is $f = \mathbf{I}_x$, since x is an arbitrary point in X.

Conversely, $\mathbf{I}_x$ satisfy the condition [2]

[3.3] Theorem: Suppose that the complete complex metric space(X, d**)** and f be an F_c- interactive contraction mapping with $x_0 \in X$.Define by number μ

$$\mu = \inf\{ d(x^*, fx^*) : x^* \neq fx^*, \ x^* \in X \}$$

Then, X has a fixed point of f.

Proof: If $\mu = 0$, then clearly $X = \{x_0\}$, and by using the interactive contraction mapping we observe that X is a fixed point of f . Let $\mu > 0$ and again let $x^* = X$, If $x^* \neq x^*$, then by the definition μ, we have $d(x^*, fx^*) \geq \mu$, Since F is increasing, using the F_c- contraction mapping of f , we obtain

$$\begin{aligned} F(\mu) &\leq F(d(x^*, fx^*)) \\ &\leq F(d(x_0, x^*)) - t \\ &< F(d(x_0, x^*)) = F(\mu) \end{aligned}$$

Which is a contradiction and then, we have $d(x^*, fx^*) = 0$, that is, $fx^* = x^*$.

Hence, X has a fixed point of f.

Now, we prove that f is also a fixed point of any arbitrary X with contractive function.

Take $x^* \in X$ and suppose that $d(x^*, fx^*) > 0$. Again using the F_c- iterative contraction mapping of the itself mapping, we obtain

$$F(d(x^*, fx^*)) \leq F(d(x_0, x^*)) - t < F(r).$$

Since F is increasing, we find

$$d(x^*, fx^*) < r < \mu$$

However, $\mu = \inf\{d(x^*, fx^*)$: for every $x^* \neq fx^*\}$, which is a contradiction.

Hence, $d(x^*, fx^*) = 0$, that is $fx^* = x^*$. According X is a complex number of f.

Then the complex number X has a fixed point of f.

[3.4] Theorem: Suppose that the complex metric space (X, d) and if $f: X \rightarrow X$ is is an F_c- surjective expanding mapping with $x_0 \in X$, then X has a fixed point of f.

Proof: Suppose that f is a surjective mappig and there exist a $f^*: X \rightarrow X$, such that the itself mapping $(f \circ f^*)$ is the identity mapping for X .Taking $x^* \in X$ be such that $d(x^*, f^*x^*) > 0$ and $y^* = f^*x^*$, then that is

$$fy^* = f(f^*x^*) = (f \circ f^*)x^* = x^*.$$

Since

$$(dy^*, fy^*) = d(fy^*, y^*) > 0,$$

Here using F_c-expanding mapping of f, we get

$$F(d(y^*, fy^*)) \leq F(d(x_0, fy^*)) + t$$

and

$$F(d(f^*x^*, x^*)) \leq F(d(x_0, x^*)) + t$$

Therefore, we get

$$-t + F(d(f^*x^*, x^*)) \leq f(d(x_0, x^*))$$

Hence, f* is an F_c- contraction mapping of X, with x_0 as – t > 0.Then, using theorem [3],then f* has a fixed point in X. Suppose y ∈ X be any point using fy*= f (f*y*)= y*.We deduce that fy* = y*, then y* is a fixed point of f. which implies f is also a member of X. since the number is complete and complex. Hence X has a fixed point of f.

[3.5] Theorem: Suppose (X, d) is a complete complex metric space. Then the contraction mapping d(fx*, fy*) ≤ ∂ d (x*, y*) for all x*, y* ∈ X . Then X has a fixed point of f.

Proof: Since $x_0 \in X$ is an arbitrary number, the $f(x_0)$ at x_0 is the sequence $\{x_o, fx_0, f^2x_0, f^3x_0, \ldots\}$ of Iterates, where $f^n = f \circ f \circ f \ldots of$, n ≥ 1.

Then show that

$d(f^{n+1}x_0, f^nx_0) \leq \partial^n d(fx_0, x_0)$, for all n≥ 1,

and

$(f^mx_0, f^nx_0) \leq \partial^n d(fx_0, x_0), \;/\; 1- \partial$, for all m ≥ 1 , n ≥ 1.Then for any $\varepsilon > 0$, choosing a natural number n_0 such that $\partial^n < \varepsilon(1- \partial)/d(fx_0, x_0)$ it follows that $d(f^mx_0, f^nx_0) < \varepsilon$,for all $m \geq n_0$, $n \geq n_0$.Thus $[f^nx_0]_{n=1}^{\infty}$ is a Cauchy sequence is $fx_0 = (x_0) \subset X$ .Since X is complete and complex number.

$f^nx_0 \to X$, for some $x_0 \in X$. Hence X has a fixed point of f.

[4.] Main Points of Scientific and Technical Application

[4.1] Communication and collaboration

This dimension of metrics is crucial to the aspect of how well the team can collaborate and there is an optimal flow of information amongst the team members. A team that communicates better and has a culture of transparency is bound to be more productive, as team members know about the priorities and what others are working on, which makes it much easier to coordinate dependencies.

[4.2] Include the work of potential comparative topics :

- Iteration process in fixed-point theory and common fixed-point theory
- Uniqueness and nonuniqueness fixed-point theory.
- Geometric properties of non-uniqueness fixed points in difference space.
- The application of fixed-point theorems for multi-valued mapping in difference spaces.
- Process of computing fixed points.
- Nonlinear spectral theory and nonlinear eigenvalue problems.
- Functional differential equation and integral equation and inclusion.
- Different abstract spaces on fixed-point theory in metric spaces.
- The functional stability equation and inclusion to fixed-point theory.
- Differential equation of fractional and inclusion by fixed-point theory.
- The membership of genetic algorithms to the powerful class of optimisation processes.
- Their specific algorithms and the process used for a clustering technique
- **Engineering**: In control systems, fuzzy fixed-point theorems assist in designing controllers that can handle uncertainties inherent in dynamic environments.
- **Computer Science**: Algorithms in machine learning and data clustering utilise fuzzy-fixed-point concepts to manage ambiguous data and improve classification accuracy, etc.
- **Economics**: Modelling market behaviours under uncertainty benefits from fuzzy fixed-point theorems, aiding in predicting equilibrium states in volatile markets.
- **Biology**: Epidemiological models incorporating fuzzy fixed points can better represent the spread of diseases with uncertain transmission rates.

[5.] Hybridisation:

The DNA molecule is composed of two strands. Hydrogen bonds connect these two strands, which then disappear and drift apart.

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Conflicts of interest: The author declares that he has no conflicts of interest.

CONCLUSION

In this paper, we conducted a comparative study and discussed different techniques of fixed-point theorems in metric space using contraction mapping. The integration of fixed-point theorems with contractive mappings has significantly enhanced the mathematical tools available for addressing problems characterised by uncertainty and imprecision.

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