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On Soft Fixed Point Results in Soft $\mathcal{S}$-Metric Spaces with Altering Distance Functions

Dr. Yashpal[1], Associate Professor, Govt. P. G. College for Women, Rohtak

Dr. Parul Singh[2], Assistant Professor, Govt. P. G. College for Women, Rohtak

Dr. Sushma[3], Assistant Professor, Govt. P. G. College for Women, Rohtak

Abstract: In article we establish new type of soft contractive conditions and prove some soft fixed point theorems via soft altering distance function in soft S-metric space. Further, some consequences are also present on the basis of mains results.

Keywords: Soft fixed point, Soft altering distance function, Soft contractive conditions, Soft $\mathcal{S}$-metric space

MSC: 47H10, 54H25

1. Introduction

In 1922 Banach [2] proved the fixed point theorem which provide us existence and uniqueness of a self-mapping on a metric space. This theorem was then generalized by a large number of people in different metric spaces, which can be explored in ([7], [8], [10], [11], [18] and so on).

S. Sessa [22], M. S. Khan and M. Swalech [13] in 1984, expanded the research of metric fixed point theory to a new category by introducing a control function which they called an altering distance function.

Definition 1.1.[13] "A function $\psi : \mathbb{R}^+ \to \mathbb{R}^+$ is called an altering distance function if the following property is satisfied:

(Θ_1) $\psi(0) = 0$,

(Θ_2) ψ is monotonically non-decreasing function,

(Θ_3) ψ is a continuous function,

By Ψ we denote the set of all altering distance functions."

In 2012, the concept of $\mathcal{S}$- metric space was established by Sedghi *et al.* [21].

Definition 1.2.[21] "Let $\mathcal{X}$ be a non-empty set. An $\mathcal{S}$-metric on X is a function $\mathcal{S}:\mathcal{X}\times\mathcal{X}\times\mathcal{X}\to[0,\infty)$ satisfying the following axioms:

1. $\mathcal{S}(u,v,w)=0$ if and only if $u=v=w$,
2. $\mathcal{S}(u,v,w)\le\mathcal{S}(u,u,a)+\mathcal{S}(v,v,a)+\mathcal{S}(w,w,a)$, for all $u,v,w\in\mathcal{X}$.

An $\mathcal{S}$-metric space is a pair $(\mathcal{X},\mathcal{S})$ where $\mathcal{S}$ is a metric on $\mathcal{X}$."

In 1999, Molodtsov [19] introduced soft sets as a mathematical tool to handle the uncertainty associated with real world data based problems. It provides sufficient capabilities to cope with uncertainty in a data and to represent it in a useful way. A vast amount of mathematical activity has been carried out to obtain many remarkable results showing the applicability of soft set theory in decision making, demand analysis, forecasting, information science, mathematics, and other disciplines (see for detailed survey ([14], [15], [16], [20], and so on).

Definition1.3.[19]: "A pair (F,E) is called a soft set over a given universal set X, if and only if F is a mapping from a set of parameters E (each parameter could be a word or a sentence) into the power set of X denoted by $P(X)$. That is, $F:E\to P(X)$. Clearly, a soft set over X is a parameterized family of subsets of the given universe X."

Definition 1.4.[17]: "A soft set (F,E) over X is said to be a null soft set denoted by $\widetilde{\Phi}$, if for all $e\,\epsilon\,E, F(e)=$ null set ϕ."

Definition 1.5.[17]: "A soft set (F,E) over X is said to be an absolute soft set denoted by $\tilde{X}$ if for all $e\,\epsilon\,E, F(e)=X$."

Das and Samanta ([5]-[6]) introduced the notions of soft real set and soft real number, and discussed their properties. Based on these notions, they introduced in the concept of soft metric.

Definition 1.6.[5]: "Let $\mathbb{R}$ be the set of real numbers and $\mathcal{B}(\mathbb{R})$ the collection of all non-empty bounded subsets of $\mathbb{R}$ and E be taken as a set of parameters. Then a mapping $F:E\to\mathcal{B}(\mathbb{R})$ is called a soft real set. If a real soft set is a singleton soft set, it will be called a soft real number and denoted by $\tilde{r},\ \tilde{s},\ \tilde{t}$ etc $\tilde{0}$ and $\tilde{1}$ are the soft real numbers where $\tilde{0}(e) = 0$, $\tilde{1}(e) = 1$, for all $e \epsilon E$ respectively."

Abbas *et al.* [1] introduced the notion of soft contraction mapping based on the theory of soft elements of soft metric spaces. Wardowski [23] introduced a notion of soft mapping and obtained its fixed point in the setup of soft topological spaces. They studied fixed points of soft contraction mappings and obtained among others results, a soft Banach contraction principle.

In 2018, Aras *et al.* ([3]-[4]) introduced the concept of soft $\mathcal{S}$-metric spaces and also discussed its important properties which are as follows:

"Let $\widetilde{\mathcal{X}}$ be an absolute soft set, E be a non-empty set of parameters and $SP(\widetilde{\mathcal{X}})$ be the collection of all soft points of $\widetilde{\mathcal{X}}$. Let $\mathbb{R}(E)^*$ denotes the set of all non-negative soft real numbers."

Definition 1.7.[3] "A soft $\mathcal{S}$-metric on $\widetilde{\mathcal{X}}$ is a mapping $\mathcal{S} : SP(\widetilde{\mathcal{X}}) \times SP(\widetilde{\mathcal{X}}) \times SP(\widetilde{\mathcal{X}}) \rightarrow \mathbb{R}(E)^*$ which satisfies the following conditions:

$(\overline{\mathcal{S}_1})$ $\mathcal{S}(\hat{u}_a, \hat{v}_b, \widehat{w}_c) \geq \tilde{0}$;

$(\overline{\mathcal{S}_2})$ $\mathcal{S}(\hat{u}_a, \hat{v}_b, \widehat{w}_c) = \tilde{0}$, if and only if $\hat{u}_a = \hat{v}_b = \widehat{w}_c$;

$(\overline{\mathcal{S}_3})$ $\mathcal{S}(\hat{u}_a, \hat{v}_b, \widehat{w}_c) \leq \mathcal{S}(\hat{u}_a, \hat{u}_a, \hat{t}_d) + \mathcal{S}(\hat{v}_b, \hat{v}_b, \tilde{t}_d) + \mathcal{S}(\widehat{w}_c, \widehat{w}_c, \hat{t}_d)$.

For all $\hat{u}_a, \hat{v}_b, \widehat{w}_c, \hat{t}_d \in SP(\widetilde{\mathcal{X}})$, then the soft set $\tilde{X}$ with a soft $\mathcal{S}$-metric is called soft $\mathcal{S}$-metric space and denoted by $(\widetilde{\mathcal{X}}, \mathcal{S}, E)$."

Lemma 1.8.[3] "Let $(\widetilde{\mathcal{X}}, \mathcal{S}, E)$ is a soft $\mathcal{S}$-metric space. Then we have

$$\mathcal{S}(\hat{u}_a, \hat{u}_a, \hat{v}_b) = \mathcal{S}(\hat{v}_b, \hat{v}_b, \hat{u}_a)."$$

Definition 1.9.[4] "A soft sequence $\{\hat{u}^n_{a_n}\}$ in $(\widetilde{\mathcal{X}}, \mathcal{S}, E)$ converges to $\hat{v}_b$ if and only if $\mathcal{S}(\hat{u}^n_{a_n}, \hat{u}^n_{a_n}, \hat{v}_b) \rightarrow \tilde{0}\ as\ n \rightarrow \infty$ and we denote this by $\lim_{n\rightarrow\infty} \hat{u}^n_{a_n} = \hat{v}_b$."

Definition 1.10.[4] "A soft sequence $\{\hat{u}^n_{a_n}\}$ in $(\widetilde{\mathcal{X}}, \mathcal{S}, E)$ is called a Cauchy sequence if for $\tilde{\varepsilon} > \tilde{0}$, there exists $n_0 \epsilon \mathbb{N}$ such that $\mathcal{S}(\hat{u}^n_{a_n}, \hat{u}^n_{a_n}, \hat{u}^m_{a_m}) < \tilde{\varepsilon}$ for each $m, n \geq n_0$."

Definition 1.11.[4] "A soft $\mathcal{S}$-metric space $(\widetilde{\mathcal{X}}, \mathcal{S}, E)$ is said to be complete if every Cauchy sequence is convergent."

Definition 1.12.[4] "Let $(\widetilde{\mathcal{X}}, \mathcal{S}, E)$ and $(\widetilde{\Upsilon}, \mathcal{S}', E')$ be two soft $\mathcal{S}$-metric spaces. The mapping $f_\varphi: (\widetilde{\mathcal{X}}, \mathcal{S}, E) \to (\widetilde{\Upsilon}, \mathcal{S}', E')$ is a soft mapping, where $f: \widetilde{\mathcal{X}} \to \widetilde{\Upsilon}$ and $\varphi: E \to E'$ are two mappings."

Definition 1.13.[4] "Let $f_\varphi: (\widetilde{\mathcal{X}}, \mathcal{S}, E) \to (\widetilde{\Upsilon}, \mathcal{S}', E')$ be a soft mapping from soft $\mathcal{S}$-metric space $(\widetilde{\mathcal{X}}, \mathcal{S}, E)$ to a soft S-metric space $(\widetilde{\Upsilon}, \mathcal{S}', E')$. Then f_φ is soft continuous at a soft point $\hat{u}_a \epsilon\, SP(\widetilde{\mathcal{X}})$ if and only if $f_\varphi(\{\hat{u}^n_{a_n}\}) \to f_\varphi(\hat{u}_a)$."

In 2018, Elif G. *et al.* [9] establish the following definition of soft altering distance function in soft metric space.

Definition 1.14.[9] "A soft function $\psi : \mathbb{R}(E)^* \to \mathbb{R}(E)^*$ is called a soft altering distance function if ψ satisfies the following property:

(Θ_1) $\psi(\bar{0}) = \bar{0}$,

(Θ_2) ψ is monotonically non-decreasing function,

(Θ_3) ψ is a sequentially continuous function i.e., $\hat{u}^n_{a_n} \to \hat{u}_a$, then $\psi(\hat{u}^n_{a_n}) \to \psi(\hat{u}_a)$."

Theorem 1.15.[9] "Let $(\tilde{X}, d)$ be a complete metric space. Let $\psi : \mathbb{R}(E)^* \to \mathbb{R}(E)^*$ be a soft altering distance function and $T: \tilde{X} \to \tilde{X}$ be a soft mapping which satisfies the following inequality:

$$\psi\big(d(T(\hat{u}^n_{a_n}), T(\hat{v}^n_{b_n}))\big) \leq \bar{c}\, \psi\big(d(\hat{u}_a, \hat{v}_b)\big),$$

for some $\bar{0} < \bar{c} < \bar{1}$ and $\hat{u}_a, \hat{v}_b \epsilon\, SP(\tilde{X})$. Then T has a unique soft fixed point."

2. Soft fixed point theorems via new types of soft contractive conditions

Theorem 2.1: Let $\psi \in \Psi$ and $(\widetilde{\mathcal{X}}, \mathcal{S}, E)$ be a complete soft $\mathcal{S}$-metric space. Let f_φ be soft self mapping on $(\widetilde{\mathcal{X}}, \mathcal{S}, E)$ which satisfies the following condition:

$$\psi\left\{\mathcal{S}\left(f_\varphi(\hat{u}_a), f_\varphi(\hat{u}_a), f_\varphi(\hat{v}_b)\right)\right\}$$

$$\leq \alpha\, \psi \left\{\frac{\mathcal{S}(\hat{u}_a,\hat{u}_a,f_\varphi(\hat{u}_a))\mathcal{S}(\hat{v}_b,\hat{v}_b,f_\varphi(\hat{v}_b))\mathcal{S}(\hat{u}_a,\hat{u}_a,f_\varphi(\hat{v}_b))+\mathcal{S}(\hat{u}_a,\hat{u}_a,\hat{v}_b)\mathcal{S}(\hat{v}_b,\hat{v}_b,f_\varphi(\hat{u}_a))\mathcal{S}(\hat{v}_b,\hat{v}_b,f_\varphi(\hat{v}_b))}{[\mathcal{S}(\hat{u}_a,\hat{u}_a,\hat{v}_b)]^2+\mathcal{S}(\hat{u}_a,\hat{u}_a,f_\varphi(\hat{v}_b))\mathcal{S}(\hat{v}_b,\hat{v}_b,f_\varphi(\hat{v}_b))}\right\}$$

$$+\beta\, \psi \left\{\mathcal{S}\left(\hat{u}_a,\hat{u}_a,f_\varphi(\hat{u}_a)\right)+\mathcal{S}\left(\hat{v}_b,\hat{v}_b,f_\varphi(\hat{v}_b)\right)\right\}+\gamma\, \psi\{\mathcal{S}(\hat{u}_a,\hat{u}_a,f_\varphi(\hat{v}_b))+\mathcal{S}(\hat{v}_b,\hat{v}_b,f_\varphi(\hat{u}_a))\}$$

$$+\eta\, \psi \left\{\frac{\mathcal{S}(\hat{u}_a,\hat{u}_a,f_\varphi(\hat{u}_a))\mathcal{S}(\hat{v}_b,\hat{v}_b,f_\varphi(\hat{v}_b))}{1+\mathcal{S}(\hat{u}_a,\hat{u}_a,\hat{v}_b)}\right\}+\delta\, \psi\{\mathcal{S}(\hat{u}_a,\hat{u}_a,\hat{v}_b)\}\,. \qquad (2.1)$$

For all $\hat{u}_a, \hat{v}_b \,\epsilon\, SP(\tilde{X})$ with $\hat{u}_a \neq \hat{v}_b$, and for some $\alpha,\beta,\gamma,\eta,\delta > \bar{0}$ with $\alpha+2\beta+3\gamma+\eta+\delta<\bar{1}$, then, f_φ has a unique soft fixed point $\widehat{w}_c \in SP(\tilde{X})$ and moreover for each soft point $\hat{u}_a$ we have $\lim_{n\to\infty} {f_\varphi}^n\, \hat{u}_a = \widehat{w}_c$.

Proof: Let $\hat{u}^0_{a_0} \epsilon\, SP(\tilde{X})$ be an arbitrary point and let $\{\hat{u}^n_{a_n}\}$ be a sequence defined as follows

$\hat{u}^{n+1}_{a_{n+1}} = f_\varphi(\hat{u}^n_{a_n}) = {f_\varphi}^{n+1}(\hat{u}_a)$, for each $n > 0$. Then form (2.1) we have

$$\psi\left\{\mathcal{S}(\hat{u}^n_{a_n},\hat{u}^n_{a_n},\hat{u}^{n+1}_{a_{n+1}})\right\}$$

$$= \psi\left\{\mathcal{S}\left(f_\varphi(\hat{u}^{n-1}_{a_{n-1}}), f_\varphi(\hat{u}^{n-1}_{a_{n-1}}), f_\varphi(\hat{u}^n_{a_n})\right)\right\}$$

$$\leq \alpha\, \psi \frac{1}{[\mathcal{S}(\hat{u}^{n-1}_{a_{n-1}},\hat{u}^{n-1}_{a_{n-1}},\hat{u}^n_{a_n})]^2+\mathcal{S}(\hat{u}^{n-1}_{a_{n-1}},\hat{u}^{n-1}_{a_{n-1}},f_\varphi(\hat{u}^n_{a_n}))\mathcal{S}(\hat{u}^n_{a_n},\hat{u}^n_{a_n},f_\varphi(\hat{u}^n_{a_n}))}\times$$

$$[\mathcal{S}(\hat{u}^{n-1}_{a_{n-1}},\hat{u}^{n-1}_{a_{n-1}},f_\varphi(\hat{u}^{n-1}_{a_{n-1}}))\mathcal{S}(\hat{u}^n_{a_n},\hat{u}^n_{a_n},f_\varphi(\hat{u}^n_{a_n}))\mathcal{S}(\hat{u}^{n-1}_{a_{n-1}},\hat{u}^{n-1}_{a_{n-1}},f_\varphi(\hat{u}^n_{a_n}))$$

$$+\mathcal{S}(\hat{u}^{n-1}_{a_{n-1}},\hat{u}^{n-1}_{a_{n-1}},\hat{u}^n_{a_n})\mathcal{S}(\hat{u}^n_{a_n},\hat{u}^n_{a_n},f_\varphi(\hat{u}^{n-1}_{a_{n-1}}))\mathcal{S}(\hat{u}^n_{a_n},\hat{u}^n_{a_n},f_\varphi(\hat{u}^n_{a_n}))]$$

$$+\beta\, \psi\left\{\mathcal{S}\left(\hat{u}^{n-1}_{a_{n-1}},\hat{u}^{n-1}_{a_{n-1}},f_\varphi(\hat{u}^{n-1}_{a_{n-1}})\right)+\mathcal{S}\left(\hat{u}^n_{a_n},\hat{u}^n_{a_n},f_\varphi(\hat{u}^n_{a_n})\right)\right\}$$

$$+\gamma\, \psi\{\mathcal{S}(\hat{u}^{n-1}_{a_{n-1}},\hat{u}^{n-1}_{a_{n-1}},f_\varphi(\hat{u}^n_{a_n}))+\mathcal{S}(\hat{u}^n_{a_n},\hat{u}^n_{a_n},f_\varphi(\hat{u}^{n-1}_{a_{n-1}}))\}$$

$$+\eta\, \psi\left\{\frac{\mathcal{S}(\hat{u}^{n-1}_{a_{n-1}},\hat{u}^{n-1}_{a_{n-1}},f_\varphi(\hat{u}^{n-1}_{a_{n-1}}))\mathcal{S}(\hat{u}^n_{a_n},\hat{u}^n_{a_n},f_\varphi(\hat{u}^n_{a_n}))}{1+\mathcal{S}(\hat{u}^{n-1}_{a_{n-1}},\hat{u}^{n-1}_{a_{n-1}},\hat{u}^n_{a_n})}\right\}+\delta\, \psi\{\mathcal{S}(\hat{u}^{n-1}_{a_{n-1}},\hat{u}^{n-1}_{a_{n-1}},\hat{u}^n_{a_n})\}$$

$$\leq \alpha\, \psi\left\{\frac{\mathcal{S}(\hat{u}^{n-1}_{a_{n-1}},\hat{u}^{n-1}_{a_{n-1}},\hat{u}^n_{a_n})\mathcal{S}(\hat{u}^n_{a_n},\hat{u}^n_{a_n},\hat{u}^{n+1}_{a_{n+1}})\mathcal{S}(\hat{u}^{n-1}_{a_{n-1}},\hat{u}^{n-1}_{a_{n-1}},\hat{u}^{n+1}_{a_{n+1}})}{[\mathcal{S}(\hat{u}^{n-1}_{a_{n-1}},\hat{u}^{n-1}_{a_{n-1}},\hat{u}^n_{a_n})]^2+\mathcal{S}(\hat{u}^{n-1}_{a_{n-1}},\hat{u}^{n-1}_{a_{n-1}},\hat{u}^{n+1}_{a_{n+1}})\mathcal{S}(\hat{u}^n_{a_n},\hat{u}^n_{a_n},\hat{u}^{n+1}_{a_{n+1}})}\right\}$$

$$+\beta\, \psi\{\mathcal{S}(\hat{u}^{n-1}_{a_{n-1}},\hat{u}^{n-1}_{a_{n-1}},\hat{u}^n_{a_n})+\mathcal{S}(\hat{u}^n_{a_n},\hat{u}^n_{a_n},\hat{u}^{n+1}_{a_{n+1}})\}$$

$$+\gamma\, \psi\{\mathcal{S}(\hat{u}^{n-1}_{a_{n-1}},\hat{u}^{n-1}_{a_{n-1}},\hat{u}^{n+1}_{a_{n+1}})+\mathcal{S}(\hat{u}^n_{a_n},\hat{u}^n_{a_n},\hat{u}^n_{a_n})\}$$

$$+\eta\, \psi\left\{\frac{\mathcal{S}(\hat{u}^{n-1}_{a_{n-1}},\hat{u}^{n-1}_{a_{n-1}},\hat{u}^n_{a_n})\mathcal{S}(\hat{u}^n_{a_n},\hat{u}^n_{a_n},\hat{u}^{n+1}_{a_{n+1}})}{1+\mathcal{S}(\hat{u}^{n-1}_{a_{n-1}},\hat{u}^{n-1}_{a_{n-1}},\hat{u}^n_{a_n})}\right\}+\delta\, \psi\{\mathcal{S}(\hat{u}^{n-1}_{a_{n-1}},\hat{u}^{n-1}_{a_{n-1}},\hat{u}^n_{a_n})\}$$

$$\leq \alpha\, \psi\{\mathcal{S}(\hat{u}^{n}_{a_n}, \hat{u}^{n}_{a_n}, \hat{u}^{n+1}_{a_{n+1}})\} + \beta\, \psi\{\mathcal{S}(\hat{u}^{n-1}_{a_{n-1}}, \hat{u}^{n-1}_{a_{n-1}}, \hat{u}^{n}_{a_n}) + \mathcal{S}(\hat{u}^{n}_{a_n}, \hat{u}^{n}_{a_n}, \hat{u}^{n+1}_{a_{n+1}})\}$$

$$+\gamma\, \psi\{2\mathcal{S}(\hat{u}^{n-1}_{a_{n-1}}, \hat{u}^{n-1}_{a_{n-1}}, \hat{u}^{n}_{a_n}) + \mathcal{S}(\hat{u}^{n}_{a_n}, \hat{u}^{n}_{a_n}, \hat{u}^{n+1}_{a_{n+1}})\} + \eta\, \psi\{\mathcal{S}(\hat{u}^{n}_{a_n}, \hat{u}^{n}_{a_n}, \hat{u}^{n+1}_{a_{n+1}})\}$$

$$+\delta\, \psi\{\mathcal{S}(\hat{u}^{n-1}_{a_{n-1}}, \hat{u}^{n-1}_{a_{n-1}}, \hat{u}^{n}_{a_n})\}.$$

Thus, we get

$$(1-\alpha-\beta-\gamma-\eta)\psi\{\mathcal{S}(\hat{u}^{n}_{a_n}, \hat{u}^{n}_{a_n}, \hat{u}^{n+1}_{a_{n+1}})\} \leq (\beta+2\gamma+\delta)\psi\{\mathcal{S}(\hat{u}^{n-1}_{a_{n-1}}, \hat{u}^{n-1}_{a_{n-1}}, \hat{u}^{n}_{a_n})\}$$

$$\psi\{\mathcal{S}(\hat{u}^{n}_{a_n}, \hat{u}^{n}_{a_n}, \hat{u}^{n+1}_{a_{n+1}})\} \leq \left(\frac{\beta+2\gamma+\delta}{1-\alpha-\beta-\gamma-\eta}\right) \psi\{\mathcal{S}(\hat{u}^{n-1}_{a_{n-1}}, \hat{u}^{n-1}_{a_{n-1}}, \hat{u}^{n}_{a_n})\}.$$

Therefore,

$$\psi\{\mathcal{S}(\hat{u}^{n}_{a_n}, \hat{u}^{n}_{a_n}, \hat{u}^{n+1}_{a_{n+1}})\} \leq \tilde{k}\, \psi\{\mathcal{S}(\hat{u}^{n-1}_{a_{n-1}}, \hat{u}^{n-1}_{a_{n-1}}, \hat{u}^{n}_{a_n})\} \text{ where } \tilde{k} = \frac{\beta+2\gamma+\delta}{1-\alpha-\beta-\gamma-\eta}$$

$$\leq \tilde{k}^2\, \psi\{\mathcal{S}(\hat{u}^{n-2}_{a_{n-2}}, \hat{u}^{n-2}_{a_{n-2}}, \hat{u}^{n-1}_{a_{n-1}})\}$$

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$$\psi\{\mathcal{S}(\hat{u}^{n}_{a_n}, \hat{u}^{n}_{a_n}, \hat{u}^{n+1}_{a_{n+1}})\} \leq \tilde{k}^n\, \psi\{\mathcal{S}(\hat{u}^{0}_{a_0}, \hat{u}^{0}_{a_0}, \hat{u}^{1}_{a_1})\} \quad (2.2)$$

Since $\bar{0} \leq \tilde{k} < \bar{1}$, from (2.2) we obtain $\lim_{n\to\infty} \psi\{\mathcal{S}(\hat{u}^{n}_{a_n}, \hat{u}^{n}_{a_n}, \hat{u}^{n+1}_{a_{n+1}})\} = \bar{0}$.

From the fact that $\psi \in \Psi$, we have $\lim_{n\to\infty} \mathcal{S}(\hat{u}^{n}_{a_n}, \hat{u}^{n}_{a_n}, \hat{u}^{n+1}_{a_{n+1}}) = \bar{0}$. (2.3)

Now, we will prove that $\{\hat{u}^{n}_{a_n}\}$ is a Cauchy sequence in $(\tilde{\mathcal{X}}, \mathcal{S}, E)$. Suppose that $\{\hat{u}^{n}_{a_n}\}$ is not a Cauchy sequence which means that there is a constant $\bar{\epsilon_0} > \bar{0}$ and two subsequence $\left\{\hat{u}^{n_k}_{a_{n_k}}\right\}$ and $\left\{\hat{u}^{m_k}_{a_{m_k}}\right\}$ of $\{\hat{u}^{n}_{a_n}\}$ such that for every $n \in \mathbb{N} \cup \{0\}$, we find that $n_k > m_k > n$, $\mathcal{S}\left(\hat{u}^{n_k}_{a_{n_k}}, \hat{u}^{n_k}_{a_{n_k}}, \hat{u}^{m_k}_{a_{m_k}}\right) \geq \bar{\epsilon_0}$ and $\mathcal{S}\left(\hat{u}^{n_k-1}_{a_{n_k-1}}, \hat{u}^{n_k-1}_{a_{n_k-1}}, \hat{u}^{m_k}_{a_{m_k}}\right) < \bar{\epsilon_0}$. For each $n > 0$, we have

$$\bar{\epsilon_0} \leq \mathcal{S}\left(\hat{u}^{n_k}_{a_{n_k}}, \hat{u}^{n_k}_{a_{n_k}}, \hat{u}^{m_k}_{a_{m_k}}\right) \leq 2\, \mathcal{S}\left(\hat{u}^{n_k}_{a_{n_k}}, \hat{u}^{n_k}_{a_{n_k}}, \hat{u}^{n_k-1}_{a_{n_k-1}}\right) + \mathcal{S}\left(\hat{u}^{n_k-1}_{a_{n_k-1}}, \hat{u}^{n_k-1}_{a_{n_k-1}}, \hat{u}^{m_k}_{a_{m_k}}\right)$$

Taking limit as $n \to \infty$, from (2.3) we obtain

$$\bar{\epsilon_0} \leq \mathcal{S}\left(\hat{u}^{n_k}_{a_{n_k}}, \hat{u}^{n_k}_{a_{n_k}}, \hat{u}^{m_k}_{a_{m_k}}\right) < \bar{\epsilon_0},$$

which implies that

$$\lim_{n\to\infty,} \mathcal{S}\left(\hat{u}^{n_k}_{a_{n_k}}, \hat{u}^{n_k}_{a_{n_k}}, \hat{u}^{m_k}_{a_{m_k}}\right) = \bar{\epsilon_0} \tag{2.4}$$

Similarly, we can show that

$$\lim_{n\to\infty} \text{ that } \mathcal{S}\left(\hat{u}^{n_k+1}_{a_{n_k+1}}, \hat{u}^{n_k+1}_{a_{n_k+1}}, \hat{u}^{m_k+1}_{a_{m_k+1}}\right) = \bar{\epsilon_0} \text{ and } \lim_{n\to\infty,} \mathcal{S}\left(\hat{u}^{n_k}_{a_{n_k}}, \hat{u}^{n_k}_{a_{n_k}}, \hat{u}^{m_k+1}_{a_{m_k+1}}\right) = \bar{\epsilon_0} \tag{2.5}$$

From the hypothesis, we deduce

$$\psi\left(\mathcal{S}(f_\varphi(\hat{u}^{n_k}_{a_{n_k}}), f_\varphi(\hat{u}^{n_k}_{a_{n_k}}), f_\varphi(\hat{u}^{m_k}_{a_{m_k}})\right)$$

$$\leq \alpha\, \psi \frac{1}{\left[\mathcal{S}\left(\hat{u}^{n_k}_{a_{n_k}}, \hat{u}^{n_k}_{a_{n_k}}, \hat{u}^{m_k}_{a_{m_k}}\right)\right]^2 + \mathcal{S}\left(\hat{u}^{n_k}_{a_{n_k}}, \hat{u}^{n_k}_{a_{n_k}}, f_\varphi(\hat{u}^{m_k}_{a_{m_k}})\right)\mathcal{S}\left(\hat{u}^{m_k}_{a_{m_k}}, \hat{u}^{m_k}_{a_{m_k}}, f_\varphi(\hat{u}^{m_k}_{a_{m_k}})\right)} \times$$

$$[\mathcal{S}\left(\hat{u}^{n_k}_{a_{n_k}}, \hat{u}^{n_k}_{a_{n_k}}, f_\varphi(\hat{u}^{n_k}_{a_{n_k}})\right)\mathcal{S}\left(\hat{u}^{m_k}_{a_{m_k}}, \hat{u}^{m_k}_{a_{m_k}}, f_\varphi(\hat{u}^{m_k}_{a_{m_k}})\right)\mathcal{S}\left(\hat{u}^{n_k}_{a_{n_k}}, \hat{u}^{n_k}_{a_{n_k}}, f_\varphi(\hat{u}^{m_k}_{a_{m_k}})\right)$$

$$+\mathcal{S}\left(\hat{u}^{n_k}_{a_{n_k}}, \hat{u}^{n_k}_{a_{n_k}}, \hat{u}^{m_k}_{a_{m_k}}\right)\mathcal{S}\left(\hat{u}^{m_k}_{a_{m_k}}, \hat{u}^{m_k}_{a_{m_k}}, f_\varphi(\hat{u}^{n_k}_{a_{n_k}})\right)\mathcal{S}\left(\hat{u}^{m_k}_{a_{m_k}}, \hat{u}^{m_k}_{a_{m_k}}, f_\varphi(\hat{u}^{m_k}_{a_{m_k}})\right)]$$

$$+\beta\, \psi\left\{\mathcal{S}\left(\hat{u}^{n_k}_{a_{n_k}}, \hat{u}^{n_k}_{a_{n_k}}, f_\varphi(\hat{u}^{n_k}_{a_{n_k}})\right) + \mathcal{S}\left(\hat{u}^{m_k}_{a_{m_k}}, \hat{u}^{m_k}_{a_{m_k}}, f_\varphi(\hat{u}^{m_k}_{a_{m_k}})\right)\right\}$$

$$+\gamma\, \psi\left\{\mathcal{S}\left(\hat{u}^{n_k}_{a_{n_k}}, \hat{u}^{n_k}_{a_{n_k}}, f_\varphi(\hat{u}^{m_k}_{a_{m_k}})\right) + \mathcal{S}\left(\hat{u}^{m_k}_{a_{m_k}}, \hat{u}^{m_k}_{a_{m_k}}, f_\varphi(\hat{u}^{n_k}_{a_{n_k}})\right)\right\}$$

$$+\eta\, \psi\left\{\frac{\mathcal{S}\left(\hat{u}^{n_k}_{a_{n_k}}, \hat{u}^{n_k}_{a_{n_k}}, f_\varphi(\hat{u}^{n_k}_{a_{n_k}})\right)\mathcal{S}\left(\hat{u}^{m_k}_{a_{m_k}}, \hat{u}^{m_k}_{a_{m_k}}, f_\varphi(\hat{u}^{m_k}_{a_{m_k}})\right)}{1+\mathcal{S}\left(\hat{u}^{n_k}_{a_{n_k}}, \hat{u}^{n_k}_{a_{n_k}}, \hat{u}^{m_k}_{a_{m_k}}\right)}\right\} + \delta\, \psi\left\{\mathcal{S}\left(\hat{u}^{n_k}_{a_{n_k}}, \hat{u}^{n_k}_{a_{n_k}}, \hat{u}^{m_k}_{a_{m_k}}\right)\right\},$$

which implies that

$$\psi\left(\mathcal{S}(\hat{u}^{n_k+1}_{a_{n_k+1}}, \hat{u}^{n_k+1}_{a_{n_k+1}}, \hat{u}^{m_k+1}_{a_{m_k+1}})\right) \leq$$

$$\leq \alpha\, \psi \frac{1}{\left[\mathcal{S}\left(\hat{u}^{n_k}_{a_{n_k}}, \hat{u}^{n_k}_{a_{n_k}}, \hat{u}^{m_k}_{a_{m_k}}\right)\right]^2 + \mathcal{S}\left(\hat{u}^{n_k}_{a_{n_k}}, \hat{u}^{n_k}_{a_{n_k}}, \hat{u}^{m_k+1}_{a_{m_k+1}}\right)\mathcal{S}\left(\hat{u}^{m_k}_{a_{m_k}}, \hat{u}^{m_k}_{a_{m_k}}, \hat{u}^{m_k+1}_{a_{m_k+1}}\right)} \times$$

$$[\mathcal{S}\left(\hat{u}^{n_k}_{a_{n_k}}, \hat{u}^{n_k}_{a_{n_k}}, \hat{u}^{n_k+1}_{a_{n_k+1}}\right)\mathcal{S}\left(\hat{u}^{m_k}_{a_{m_k}}, \hat{u}^{m_k}_{a_{m_k}}, \hat{u}^{m_k+1}_{a_{m_k+1}}\right)\mathcal{S}\left(\hat{u}^{n_k}_{a_{n_k}}, \hat{u}^{n_k}_{a_{n_k}}, \hat{u}^{m_k+1}_{a_{m_k+1}}\right)$$

$$+\mathcal{S}\left(\hat{u}^{n_k}_{a_{n_k}}, \hat{u}^{n_k}_{a_{n_k}}, \hat{u}^{m_k}_{a_{m_k}}\right)\mathcal{S}\left(\hat{u}^{m_k}_{a_{m_k}}, \hat{u}^{m_k}_{a_{m_k}}, \hat{u}^{n_k+1}_{a_{n_k+1}}\right)\mathcal{S}\left(\hat{u}^{m_k}_{a_{m_k}}, \hat{u}^{m_k}_{a_{m_k}}, \hat{u}^{m_k+1}_{a_{m_k+1}}\right)]$$

$$+\beta\ \psi\left\{\mathcal{S}\left(\hat{u}_{a_{n_k}}^{n_k},\hat{u}_{a_{n_k}}^{n_k},\hat{u}_{a_{n_k+1}}^{n_k+1}\right)+\mathcal{S}\left(\hat{u}_{a_{m_k}}^{m_k},\hat{u}_{a_{m_k}}^{m_k},\hat{u}_{a_{m_k+1}}^{m_k+1}\right)\right\}$$

$$+\gamma\ \psi\left\{\mathcal{S}\left(\hat{u}_{a_{n_k}}^{n_k},\hat{u}_{a_{n_k}}^{n_k},\hat{u}_{a_{m_k+1}}^{m_k+1}\right)+\mathcal{S}\left(\hat{u}_{a_{m_k}}^{m_k},\hat{u}_{a_{m_k}}^{m_k},\hat{u}_{a_{n_k+1}}^{n_k+1}\right)\right\}$$

$$+\eta\ \psi\left\{\frac{\mathcal{S}\left(\hat{u}_{a_{n_k}}^{n_k},\hat{u}_{a_{n_k}}^{n_k},f_\varphi(\hat{u}_{a_{n_k}}^{n_k})\right)\mathcal{S}\left(\hat{u}_{a_{m_k}}^{m_k},\hat{u}_{a_{m_k}}^{m_k},\hat{u}_{a_{m_k+1}}^{m_k+1}\right)}{1+\mathcal{S}\left(\hat{u}_{a_{n_k}}^{n_k},\hat{u}_{a_{n_k}}^{n_k},\hat{u}_{a_{m_k}}^{m_k}\right)}\right\}+\delta\ \psi\left\{\mathcal{S}\left(\hat{u}_{a_{n_k}}^{n_k},\hat{u}_{a_{n_k}}^{n_k},\hat{u}_{a_{m_k}}^{m_k}\right)\right\}.$$

Letting $k\to\infty$, and from (2.3) – (2.5), we obtain that

$$\psi(\bar{\epsilon_0})\le\gamma\ \psi(2\ \bar{\epsilon_0})+\delta\ \psi(\bar{\epsilon_0})$$

$$\psi(\bar{\epsilon_0})\le(2\gamma+\delta)\ \psi(\bar{\epsilon_0}),$$

which is contradiction as $2\gamma+\delta<\bar{1}$. Hence, $\{\hat{u}_{a_n}^n\}$ is a Cauchy sequence. By completeness of $(\tilde{\mathcal{X}},\mathcal{S},E)$, $\{\hat{u}_{a_n}^n\}$ converges to some soft point $\hat{w}_c$.

Again taking $\hat{u}_a=\hat{u}_{a_n}^n$ and $\hat{v}_b=\hat{w}_c$ in (2.1) we get

$$\psi\left\{\mathcal{S}\left(f_\varphi(\hat{u}_{a_n}^n),f_\varphi(\hat{u}_{a_n}^n),f_\varphi(\hat{w}_c)\right)\right\}\le$$

$$\alpha\ \psi\left\{\frac{\mathcal{S}(\hat{u}_{a_n}^n,\hat{u}_{a_n}^n,f_\varphi(\hat{u}_{a_n}^n))\mathcal{S}(\hat{w}_c,\hat{w}_c,f_\varphi(\hat{w}_c))\mathcal{S}(\hat{u}_{a_n}^n,\hat{u}_{a_n}^n,f_\varphi(\hat{w}_c))+\mathcal{S}(\hat{u}_{a_n}^n,\hat{u}_{a_n}^n,\hat{w}_c)\mathcal{S}(\hat{w}_c,\hat{w}_c,f_\varphi(\hat{u}_{a_n}^n))\mathcal{S}(\hat{w}_c,\hat{w}_c,f_\varphi(\hat{w}_c))}{[\mathcal{S}(\hat{u}_{a_n}^n,\hat{u}_{a_n}^n,\hat{w}_c)]^2+\mathcal{S}(\hat{u}_{a_n}^n,\hat{u}_{a_n}^n,f_\varphi(\hat{w}_c))\mathcal{S}(\hat{w}_c,\hat{w}_c,f_\varphi(\hat{w}_c))}\right\}$$

$$+\beta\ \psi\{\mathcal{S}(\hat{u}_{a_n}^n,\hat{u}_{a_n}^n,f_\varphi(\hat{u}_{a_n}^n))+\mathcal{S}(\hat{w}_c,\hat{w}_c,f_\varphi(\hat{w}_c))\}$$

$$+\gamma\ \psi\{\mathcal{S}(\hat{u}_{a_n}^n,\hat{u}_{a_n}^n,f_\varphi(\hat{w}_c))+\mathcal{S}(\hat{w}_c,\hat{w}_c,f_\varphi(\hat{u}_{a_n}^n))\}$$

$$+\eta\ \psi\left\{\frac{\mathcal{S}(\hat{u}_{a_n}^n,\hat{u}_{a_n}^n,f_\varphi(\hat{u}_{a_n}^n))\mathcal{S}(\hat{w}_c,\hat{w}_c,f_\varphi(\hat{w}_c))}{1+\mathcal{S}(\hat{u}_{a_n}^n,\hat{u}_{a_n}^n,\hat{w}_c)}\right\}+\delta\ \psi\{\mathcal{S}(\hat{u}_{a_n}^n,\hat{u}_{a_n}^n,\hat{w}_c)\}$$

$$\le\alpha\ \psi\left\{\frac{\mathcal{S}(\hat{u}_{a_n}^n,\hat{u}_{a_n}^n,\hat{u}_{a_{n+1}}^{n+1})\mathcal{S}(\hat{w}_c,\hat{w}_c,f_\varphi(\hat{w}_c))\mathcal{S}(\hat{u}_{a_n}^n,\hat{u}_{a_n}^n,f_\varphi(\hat{w}_c))+\mathcal{S}(\hat{u}_{a_n}^n,\hat{u}_{a_n}^n,\hat{w}_c)\mathcal{S}(\hat{w}_c,\hat{w}_c,\hat{u}_{a_{n+1}}^{n+1})\mathcal{S}(\hat{w}_c,\hat{w}_c,f_\varphi(\hat{w}_c))}{[\mathcal{S}(\hat{u}_{a_n}^n,\hat{u}_{a_n}^n,\hat{w}_c)]^2+\mathcal{S}(\hat{u}_{a_n}^n,\hat{u}_{a_n}^n,f_\varphi(\hat{w}_c))\mathcal{S}(\hat{w}_c,\hat{w}_c,f_\varphi(\hat{w}_c))}\right\}$$

$$+\beta\ \psi\{\mathcal{S}(\hat{u}_{a_n}^n,\hat{u}_{a_n}^n,\hat{u}_{a_{n+1}}^{n+1})+\mathcal{S}(\hat{w}_c,\hat{w}_c,f_\varphi(\hat{w}_c))\}$$

$$+\gamma\ \psi\{\mathcal{S}(\hat{u}_{a_n}^n,\hat{u}_{a_n}^n,f_\varphi(\hat{w}_c))+\mathcal{S}(\hat{w}_c,\hat{w}_c,\hat{u}_{a_{n+1}}^{n+1})\}$$

$$+\eta\ \psi\left\{\frac{\mathcal{S}(\hat{u}_{a_n}^n,\hat{u}_{a_n}^n,\hat{u}_{a_{n+1}}^{n+1})\mathcal{S}(\hat{w}_c,\hat{w}_c,f_\varphi(\hat{w}_c))}{1+\mathcal{S}(\hat{u}_{a_n}^n,\hat{u}_{a_n}^n,\hat{w}_c)}\right\}+\delta\ \psi\{\mathcal{S}(\hat{u}_{a_n}^n,\hat{u}_{a_n}^n,\hat{w}_c)\}\ .$$

Therefore,

$$\lim_{n\to\infty}\psi\left\{\mathcal{S}\left(\hat{u}_{a_{n+1}}^{n+1},\hat{u}_{a_{n+1}}^{n+1},f_{\varphi}(\widehat{w}_c)\right)\right\}=\lim_{n\to\infty}\psi\,\mathcal{S}\left(f_{\varphi}(\hat{u}_{a_n}^{n}),f_{\varphi}(\hat{u}_{a_n}^{n}),f_{\varphi}(\widehat{w}_c)\right)$$

$$\le\beta\psi\left\{\mathcal{S}\left(\widehat{w}_c,\widehat{w}_c,f_{\varphi}(\widehat{w}_c)\right)\right\}+\gamma\,\psi\left\{\mathcal{S}\left(\widehat{w}_c,\widehat{w}_c,f_{\varphi}(\widehat{w}_c)\right)\right\},$$

which implies that

$$\psi\left\{\mathcal{S}\left(\widehat{w}_c,\widehat{w}_c,f_{\varphi}(\widehat{w}_c)\right)\right\}\le(\beta+\gamma)\psi\left\{\mathcal{S}\left(\widehat{w}_c,\widehat{w}_c,f_{\varphi}(\widehat{w}_c)\right)\right\}$$

Since $\beta+\gamma<\bar{1}$, then $\psi\left\{\mathcal{S}\left(\widehat{w}_c,\widehat{w}_c,f_{\varphi}(\widehat{w}_c)\right)\right\}=0\Rightarrow\mathcal{S}\left(\widehat{w}_c,\widehat{w}_c,f_{\varphi}(\widehat{w}_c)\right)=0$.

Thus, $f_{\varphi}(\widehat{w}_c)=\widehat{w}_c$. Therefore, $\widehat{w}_c$ is a fixed soft point of f_{φ}.

Now we are going to establish the soft fixed point is unique.

For that let us suppose that $\widehat{w}_c$ and $\hat{t}_d$ be two soft fixed point of f_{φ} with $\widehat{w}_c\neq\hat{t}_d$.

Taking $\hat{u}_a=\widehat{w}_c$ and $\hat{v}_b=\hat{t}_d$ in (2.1) we get

$\psi\left\{\mathcal{S}(\widehat{w}_c,\widehat{w}_c,\hat{t}_d)\right\}$

$=\psi\left\{\mathcal{S}\left(f_{\varphi}(\widehat{w}_c),f_{\varphi}(\widehat{w}_c),f_{\varphi}(\hat{t}_d)\right)\right\}$

$\le\alpha\,\psi\left\{\frac{\mathcal{S}(\widehat{w}_c,\widehat{w}_c,f_{\varphi}(\widehat{w}_c))\mathcal{S}(\hat{t}_d,\hat{t}_d,f_{\varphi}(\hat{t}_d))\mathcal{S}(\widehat{w}_c,\widehat{w}_c,f_{\varphi}(\hat{t}_d))+\mathcal{S}(\widehat{w}_c,\widehat{w}_c,\hat{t}_d)\mathcal{S}(\hat{t}_d,\hat{t}_d,f_{\varphi}(\widehat{w}_c))\mathcal{S}(\hat{t}_d,\hat{t}_d,f_{\varphi}(\hat{t}_d))}{[\mathcal{S}(\widehat{w}_c,\widehat{w}_c,\hat{t}_d)]^2+\mathcal{S}(\widehat{w}_c,\widehat{w}_c,f_{\varphi}(\hat{t}_d))\mathcal{S}(\hat{t}_d,\hat{t}_d,f_{\varphi}(\hat{t}_d))}\right\}$

$+\beta\,\psi\left\{\mathcal{S}\left(\widehat{w}_c,\widehat{w}_c,f_{\varphi}(\widehat{w}_c)\right)+\mathcal{S}\left(\hat{t}_d,\hat{t}_d,f_{\varphi}(\hat{t}_d)\right)\right\}$

$+\gamma\,\psi\left\{\mathcal{S}\left(\widehat{w}_c,\widehat{w}_c,f_{\varphi}(\hat{t}_d)\right)+\mathcal{S}\left(\hat{t}_d,\hat{t}_d,f_{\varphi}(\widehat{w}_c)\right)\right\}$

$+\eta\,\psi\left\{\frac{\mathcal{S}(\widehat{w}_c,\widehat{w}_c,f_{\varphi}(\widehat{w}_c))\mathcal{S}(\hat{t}_d,\hat{t}_d,f_{\varphi}(\hat{t}_d))}{1+\mathcal{S}(\widehat{w}_c,\widehat{w}_c,\hat{t}_d)}\right\}+\delta\,\psi\left\{\mathcal{S}(\widehat{w}_c,\widehat{w}_c,\hat{t}_d)\right\}$

$\le\alpha\,\psi\left\{\frac{\mathcal{S}(\widehat{w}_c,\widehat{w}_c,\widehat{w}_c)\mathcal{S}(\hat{t}_d,\hat{t}_d,\hat{t}_d)\mathcal{S}(\widehat{w}_c,\widehat{w}_c,\hat{t}_d)+\mathcal{S}(\widehat{w}_c,\widehat{w}_c,\hat{t}_d)\mathcal{S}(\hat{t}_d,\hat{t}_d,\widehat{w}_c)\mathcal{S}(\hat{t}_d,\hat{t}_d,\hat{t}_d)}{[\mathcal{S}(\widehat{w}_c,\widehat{w}_c,\hat{t}_d)]^2+\mathcal{S}(\widehat{w}_c,\widehat{w}_c,\hat{t}_d)\mathcal{S}(\hat{t}_d,\hat{t}_d,\hat{t}_d)}\right\}$

$+\beta\,\psi\left\{\mathcal{S}(\widehat{w}_c,\widehat{w}_c,\widehat{w}_c)+\mathcal{S}(\hat{t}_d,\hat{t}_d,\hat{t}_d)\right\}+\gamma\,\psi\left\{\mathcal{S}(\widehat{w}_c,\widehat{w}_c,\hat{t}_d)+\mathcal{S}(\hat{t}_d,\hat{t}_d,\widehat{w}_c)\right\}$

$+\eta\,\psi\left\{\frac{\mathcal{S}(\widehat{w}_c,\widehat{w}_c,\widehat{w}_c)\mathcal{S}(\hat{t}_d,\hat{t}_d,\hat{t}_d)}{1+\mathcal{S}(\widehat{w}_c,\widehat{w}_c,\hat{t}_d)}\right\}+\delta\,\psi\left\{\mathcal{S}(\widehat{w}_c,\widehat{w}_c,\hat{t}_d)\right\}$.

we obtain

$\psi\{\mathcal{S}(\widehat{w}_c, \widehat{w}_c, \hat{t}_d)\} \leq (2\gamma + \delta)\psi\{\mathcal{S}(\widehat{w}_c, \widehat{w}_c, \hat{t}_d)\},$

as $2\gamma + \delta < \bar{1}$, we get a contradiction. Thus, we obtain

$\psi\{\mathcal{S}(\widehat{w}_c, \widehat{w}_c, \hat{t}_d)\} = 0 \Rightarrow \mathcal{S}(\widehat{w}_c, \widehat{w}_c, \hat{t}_d) = 0$ which further implies that $\widehat{w}_c = \hat{t}_d$.

Therefore, the fixed soft point we get is unique.

This completes the proof.

Remark: In Theorem 2.1 if $\alpha = \beta = \gamma = \eta = 0$ and $\bar{0} \leq \delta < \bar{1}$ with $\psi(\bar{t}) = \bar{t}$, we get the result of Banach [2].

Corollary 2.2: Let $(\tilde{\mathcal{X}}, \mathcal{S}, E)$ be a complete soft $\mathcal{S}$-metric space and let f_φ be soft self mapping on $(\tilde{\mathcal{X}}, \mathcal{S}, E)$ which satisfies the following condition:

$$\mathcal{S}\left(f_\varphi(\hat{u}_a), f_\varphi(\hat{u}_a), f_\varphi(\hat{v}_b)\right)$$

$$\leq \alpha\left\{\frac{\mathcal{S}(\hat{u}_a,\hat{u}_a,f_\varphi(\hat{u}_a))\mathcal{S}(\hat{v}_b,\hat{v}_b,f_\varphi(\hat{v}_b))\mathcal{S}(\hat{u}_a,\hat{u}_a,f_\varphi(\hat{v}_b))+\mathcal{S}(\hat{u}_a,\hat{u}_a,\hat{v}_b)\mathcal{S}(\hat{v}_b,\hat{v}_b,f_\varphi(\hat{u}_a))\mathcal{S}(\hat{v}_b,\hat{v}_b,f_\varphi(\hat{v}_b))}{[\mathcal{S}(\hat{u}_a,\hat{u}_a,\hat{v}_b)]^2+\mathcal{S}(\hat{u}_a,\hat{u}_a,f_\varphi(\hat{v}_b))\mathcal{S}(\hat{v}_b,\hat{v}_b,f_\varphi(\hat{v}_b))}\right\}$$

$$+\beta\left\{\mathcal{S}\left(\hat{u}_a, \hat{u}_a, f_\varphi(\hat{u}_a)\right) + \mathcal{S}\left(\hat{v}_b, \hat{v}_b, f_\varphi(\hat{v}_b)\right)\right\} + \gamma\left\{\mathcal{S}\left(\hat{u}_a, \hat{u}_a, f_\varphi(\hat{v}_b)\right) + \mathcal{S}\left(\hat{v}_b, \hat{v}_b, f_\varphi(\hat{u}_a)\right)\right\}$$

$$+\eta\left\{\frac{\mathcal{S}(\hat{u}_a,\hat{u}_a,f_\varphi(\hat{u}_a))\mathcal{S}(\hat{v}_b,\hat{v}_b,f_\varphi(\hat{v}_b))}{1+\mathcal{S}(\hat{u}_a,\hat{u}_a,\hat{v}_b)}\right\} + \delta\{\mathcal{S}(\hat{u}_a, \hat{u}_a, \hat{v}_b)\}. \tag{2.6}$$

For all $\hat{u}_a, \hat{v}_b \epsilon\, SP(\tilde{X})$ with $\hat{u}_a \neq \hat{v}_b$, and for some $\alpha, \beta, \gamma, \eta, \delta > \bar{0}$ with $\alpha + 2\beta + 3\gamma + \eta + \delta < \bar{1}$ then, f_φ has a unique soft fixed point $\widehat{w}_c \in SP(\tilde{X})$ and moreover for each soft point $\hat{u}_a$ we have $\lim_{n\to\infty} f_\varphi{}^n\, \hat{u}_a = \widehat{w}_c$.

Proof: It is enough, if we consider $\psi(\bar{t}) = \bar{t}$ in Theorem 2.1.

Corollary 2.3: Let $(\tilde{\mathcal{X}}, \mathcal{S}, E)$ be a complete soft $\mathcal{S}$-metric space and let f_φ be soft self mapping on $(\tilde{\mathcal{X}}, \mathcal{S}, E)$ which satisfies the following condition:

$$\int_0^{\mathcal{S}\left(f_\varphi(\hat{u}_a), f_\varphi(\hat{u}_a), f_\varphi(\hat{v}_b)\right)} \xi(t)dt$$

$$\leq \alpha \int_0^{\frac{\mathcal{S}(\hat{u}_a,\hat{u}_a,f_\varphi(\hat{u}_a))\mathcal{S}(\hat{v}_b,\hat{v}_b,f_\varphi(\hat{v}_b))\mathcal{S}(\hat{u}_a,\hat{u}_a,f_\varphi(\hat{v}_b))+\mathcal{S}(\hat{u}_a,\hat{u}_a,\hat{v}_b)\mathcal{S}(\hat{v}_b,\hat{v}_b,f_\varphi(\hat{u}_a))\mathcal{S}(\hat{v}_b,\hat{v}_b,f_\varphi(\hat{v}_b))}{[\mathcal{S}(\hat{u}_a,\hat{u}_a,\hat{v}_b)]^2+\mathcal{S}(\hat{u}_a,\hat{u}_a,f_\varphi(\hat{v}_b))\mathcal{S}(\hat{v}_b,\hat{v}_b,f_\varphi(\hat{v}_b))}} \xi(t)dt$$

$$+\beta \int_0^{\mathcal{S}(\hat{u}_a,\hat{u}_a,f_\varphi(\hat{u}_a))+\mathcal{S}(\hat{v}_b,\hat{v}_b,f_\varphi(\hat{v}_b))} \xi(t)dt + \gamma \int_0^{\mathcal{S}(\hat{u}_a,\hat{u}_a,f_\varphi(\hat{v}_b))+\mathcal{S}(\hat{v}_b,\hat{v}_b,f_\varphi(\hat{u}_a))} \xi(t)dt +$$

$$+\eta \int_0^{\frac{\mathcal{S}(\hat{u}_a,\hat{u}_a,f_\varphi(\hat{u}_a))\mathcal{S}(\hat{v}_b,\hat{v}_b,f_\varphi(\hat{v}_b))}{1+\mathcal{S}(\hat{u}_a,\hat{u}_a,\hat{v}_b)}} \xi(t)dt + \delta \int_0^{\mathcal{S}(\hat{u}_a,\hat{u}_a,\hat{v}_b)} \xi(t)dt. \quad (2.7)$$

For all $\hat{u}_a, \hat{v}_b \epsilon\, SP(\tilde{X})$ with $\hat{u}_a \neq \hat{v}_b$, and for some $\alpha, \beta, \gamma, \eta, \delta > \bar{0}$ with $\alpha + 2\beta + 3\gamma + \eta + \delta < \bar{1}$, where $\xi: R^+ \to R^+$ is a Lesbesgue-integrable mapping which is summable on compact subset of R^+, non-negative and such that for each $\in > 0$, $\int_0^{\in} \xi(t)dt > 0$ then, f_φ has a unique soft fixed point $\hat{w}_c \in SP(\tilde{X})$ and moreover for each soft point $\hat{u}_a$, $\lim_{n\to\infty} f_\varphi{}^n\, \hat{u}_a = \hat{w}_c$.

Proof: If we take $\psi(t) = \int_0^t \xi(t)dt$, in Theorem 2.1, we get desired result.

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