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Exploring Two Fluid Cosmological Model Varying Decelerating Parameter In $f(G)$ gravity

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Abstract: This study explores Bianchi type I cosmological model within the framework of $f(G)$gravity, utilizing an interacting field as the energy source. The interacting field comprises a linear combination of electromagnetic field, massless scalar and charged perfect fluid components. We investigate the dynamics of such models within the framework of $f(G)$ gravity. These solutions provide valuable insights into the evolution of the Universe and how it is influenced by the modified gravity theory. Furthermore, we derive cosmological parameters in terms of redshift, offering a convenient way to interpret observational data and connect theoretical predictions to empirical measurements. Our findings contribute to a deeper understanding of the dynamics of Bianchi Type-I cosmological models.

Keywords: LRS Bianchi Type-I Spacetime, *f (G)* gravity, Massless Scalar Field, Statefinder Diagnostics.

1. Introduction:

Cosmological models have been vital for expanding our knowledge of how the Universe developed and is structured. Among these, Bianchi Type-I cosmological models have been particularly important. Their simplicity and ability to be applied to different cosmic situations make them a powerful tool. These models are based on the assumption that matter and radiation in the early Universe were distributed evenly but not identically in all directions, a property known as anisotropy. This makes them especially useful for studying the mechanics of how the cosmos expanded.

Numerous observational data point to this unknown energy, sometimes referred to as dark energy, being the source of unexpected changes in the cosmos with tremendous negative pressure. Acceleration and expansion of the universe is todays burning issue in cosmology Thus, to comprehend the process underlying the existence of dark energy and late- time acceleration in the universe, scientists have developed several modified theories of gravity, including f(R) gravity, f(R,T) gravity, f(G) gravity and so on. Another approach of explaining accelerated expansion is to modify the Einstein Hilbert action. One of the most popular modifications of GR is based on introduction of function of the Ricci scalar, i.e. f(R) in Einstein-Hilbert action. It is known as f(R) theories of gravitation. All the well established results of GR are preserved in the f(R) theories. However, f(R) is not the end of modifications. An interesting curvature term G called Gauss–Bonnet (GB) curvature gives us another modified theory of gravitation. When the Lagrangian density f is a function of G i.e. f(G), it is possible to construct viable cosmological model that are consistent with local constraints of General Relativity. The term G can avoid ghost contribution. Recently, the f(G) theory of gravitation is introduced. The f(G) is obtained by introducing the Gauss Bonnet curvature invariants G in the Einstein-Hilbert action. The modified Gauss-Bonnet theory of gravity, also known as the f(G) gravity theory[1]. In the f(G), f is Lagrangian density function of G. The curvature invariant G can avoid ghost contribution and useful into the regularization of the gravitational action [2]. Recently, various cosmological models have been constructed in the f(G) theory for various physical fluids. Capozziello et al. [3] have discussed Noether symmetry approach in the framework of the f(G) cosmology. Myrzakulov et al. [4] have studied cosmological solution on the Λ CDM model in the f(G) gravity. Dadhich [5] has coupled four dimensional space times with Gauss-Bonnet gravity. Bamba et al. [6] have explored bouncing cosmology in the f(G) gravity. Kang et al. [7] have obtained static spherically symmetric star in Gauss-Bonnet gravity. Katore et al. [8] have discussed string bulk viscous cosmological models in the f(G) theory of gravitation. Further, it can contribute to the regularization of the gravitational action [9]. Gauss–Bonnet gravity (GB) [10, 11] is widely studied by eminent authors as a higher curvature gravity theory. In the GB theory, the gravitational action includes functions of Gauss–Bonnet invariant. In the context of scholar history, Myrzakulov et al. [12] have solved the cosmological constant problem in f(G) gravity. Dadhich [13] has discussed the problem of extra dimension in f(G) gravity. Bamba et al. [14] have explored bouncing cosmology in f(G) theory of gravitation. Ernazarov and Ivashchuk [15] have considered a D-dimensional model with GB and Λ term. Static spherically symmetric star in f(G) gravity is explored by Kang et al. [16]. Barcelo et al. [17] have shown that solutions of the black string type are not allowed in Einstein–Gauss–Bonnet gravity. In their work, Fayaz et al. [18] found specific power-law solutions for anisotropic universes using Gauss-Bonnet gravity. Li et al. [19] investigated the Universe's accelerated expansion in its later stages. Simultaneously, Nojiri et al. [20] proposed a new idea called Gauss-Bonnet dark energy. Additionally, some workable models were shown to successfully pass the solar system test, as discussed by [21, 22]. Shekh et al. [23] examined quintessential gravity and statistically fit their findings. This theoretical framework, which includes functions of the Gauss-Bonnet invariant in the gravitational action, has been studied extensively for its ability to mimic the cosmic evolution mentioned in references [24-39].

In recent years, The deceleration parameter was widely extended by researchers to investigate the accelerating universe's dynamic behavior. Cosmological models with deceleration parameters (q) are of great interest to researchers. For depending on its sign, the deceleration parameter, which is a geometric parameter, represents the dynamics of acceleration or deceleration of the cosmos. Tiwari et al. [40-42] used time-varying deceleration parameter to explore a Bianchi type-I cosmological model in f (R, T) theory in the context of a different law of variation. The time-dependent deceleration parameter considered by Khade [43]. Pawar [44-46] investigated the LRS Bianchi type -I cosmological model and variable deceleration parameter. Additionally, the variable deceleration parameter shown in the literature [47], provides the transitional behavior of anisotropic Bianchi type-I cosmological models from the early deceleration to the late time acceleration in the presence of a magnetic field. They also showed the model bounds from the decelerating phase to the de-sitter expansion or the exponential expansion phase of the universe. The fourth section describes the Jerk parameter and

It is found that interacting field has not been considered in the framework of Gauss Bonnet gravity. In summary, this research presents a comprehensive exploration of Bianchi Type-I cosmological models in Gauss Bonet Gravity, incorporating a massless scalar and charged perfect fluid. The paper is organized as follows: the second section discusses metric and field equations. The third section analyzes the solutions for time varying Deceleration parameter. The fourth section describes the jerk parameter and statefinder parameters discuss in section five. Finally, Section six is devoted to discussion and conclusion.

2. Metric and Field Equation:

The action of $f(G)$ gravity is given by the following equation

$$S = \frac{1}{2K}\int [R + f(G)]\sqrt{-g}d^4x + s_\varphi(g^{ij}, \emptyset) \tag{1}$$

where g is the determinant of the metric tensor g_{ij} , K is the coupling constant. s_φ is the action of matter. The matter is minimally coupled to the metric tensor gij which means $f(G)$ is a purely metric theory of gravity. ϕ represents the matter field. The $f(G)$ is an arbitrary function of G which is given by

$$G = R^2 - 4R_{ij}R^{ij} + R_{ij\mu\vartheta}R^{ij\mu\vartheta} \tag{2}$$

where R is the Ricci scalar, R_{ij} stands for Ricci tensor and $R_{ij\mu\vartheta}$ denotes Riemannian tensors. Varying action (1) with respect to metric g_{ij} , we obtain the field equations as,

$$R_{ij} - \frac{1}{2}Rg_{ij} + \delta[R_{i\mu j\vartheta} + R_{\mu j}g_{\vartheta i} - R_{\mu\vartheta}g_{ji} - R_{ij}g_{\vartheta\mu} + R_{i\vartheta}g_{j\mu}$$
$$+\frac{1}{2}R(R_{ij}g_{\mu\vartheta} - g_{i\vartheta}g_{j\mu})]\nabla^\mu\nabla^\vartheta + (Gf_G - f)g_{ij} = kT_{ij} \tag{3}$$

Here ∇^μ denotes the covariant derivative and f_G stand for the derivative of $f(G)$ with respect to G.

The line element for a flat, homogeneous and anisotropic LRS Bianchi type-I space time is

$$ds^2 = dt^2 - A^2dx^2 - B^2(dy^2 + dz^2) \tag{4}$$

Here, t represents time, x is one spatial coordinate, and y and z are the other two spatial coordinates. The functions A(t) and B(t) are scale factors that describe the expansion or contraction of the space in the x and y-z directions, respectively.

The Ricci scalar R and Gauss-Bonnet (GB) invariant for Bianchi type I is found to be

$$R = -2\left[\frac{\ddot{A}}{A} + 2\frac{\ddot{B}}{B} + 2\frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\dot{B}^2}{B^2}\right] \tag{5}$$

$$G = 8\left[\frac{\ddot{A}\dot{B}^2}{AB^2} + 2\frac{\dot{A}\dot{B}\ddot{B}}{AB^2}\right] \tag{6}$$

where over dot denotes differentiation with respect to t.

We considered the source of energy of the gravitational field is an interacting field with dark energy and observed the behavior of the cosmological model in the presence of linearly coupled perfect fluid distribution, mass-less scalar field, and source of a free electromagnetic field. That is,

$$\widetilde{T_{ij}} = S_{ij} + T_{ij} \tag{7}$$

where, S_{ij} is the energy-momentum tensor for perfect fluid distribution and it is given by,

$$S_{ij} = (p + \rho)u_iu_j - pg_{ij} \tag{8}$$

with $g^{ij}u_iu_j = 1$

Where, p, ρ, u_i are internal pressure, rest mass density and four-velocity vectors of the distribution respectively. T_{ij} is the energy-momentum tensor for mass-less scalar field and it is given by,

$$T_{ij} = U_{,i}U_{,j} - \frac{1}{2}g_{ij}U_{,s}U'^{s} \tag{9}$$

Mass-less scalar field U also satisfy

$$g^{ij}U_{;ij} = \rho_c \tag{10}$$

Where, ρ_c is the charge density, semicolon (;) and comma (,) denotes covariant derivative and partial derivative respectively.

T_{ij} is the energy-momentum tensor for mass-less scalar field and it is given by,

$$T_{ij} = U_{,i}U_{,j} - \frac{1}{2}g_{ij}U_sU'^s \tag{11}$$

Mass-less scalar field U also satisfy

$$g^{ij}U_{;ij} = \rho_c \tag{12}$$

Where, ρ_c is the charge density, semicolon (;) and comma (,) denotes covariant derivative and partial derivative respectively. E_{ij} is the electromagnetic energy-momentum tensor given by

$$E_{ij} = \frac{1}{4\pi}\left[F_{i\alpha}F_j^{\alpha} - \frac{1}{4}g_{ij}F_{\alpha\beta}F^{\alpha\beta}\right] \tag{13}$$

Here F_{ij} is the electromagnetic field tensor obtained from the four potential ϕ_i,

$$F_{ij} = \phi_{i,j} - \phi_{j,i} \tag{14}$$

$$F^{ij}_{;j} = -4\pi\rho_c u^i \tag{15}$$

In the co-moving transformation system the magnetic field is considered along z −axis only, therefore non-vanishing components of electromagnetic fields F_{ij} are only

F_{12} and F_{21}. Also, we have electromagnetic field tensor is anti-symmetric.

The first set of Maxwell equation are,

$$F_{ij,k} + F_{jk,i} + F_{ki,j} = 0 \tag{16}$$

leads to

$$F_{23} = constant = M \tag{17}$$

Now from Eqn.(8), (9), (13) for the metric (4), we have

$$\bar{T} = -\dot{U}^2 - 3p + \rho \tag{18}$$

Now, by using $F_{23} = const. = M$ and $u^4 \neq 0$, from Eqn.(15) we have, charge density is zero ($\rho_c = 0$).

The field equation corresponding to metric (4) are obtained by

$$\frac{\dot{B}^2}{B^2} + 2\frac{\ddot{B}}{B} - 16\frac{\dot{B}\ddot{B}}{B^2}\dot{f}_G - 8\frac{\dot{B}^2}{B^2}\ddot{f}_G + Gf_G - f = -k^2\left[p + \frac{\dot{U}^2}{2} - \frac{M^2}{8\pi B^4}\right] \tag{19}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - 8\left[\frac{\dot{A}\ddot{B}}{AB} + \frac{\ddot{A}\dot{B}}{AB}\right]\dot{f}_G - 8\frac{\dot{A}\dot{B}}{AB}\ddot{f}_G + Gf_G - f = -k^2\left[p + \frac{\dot{U}^2}{2} + \frac{M^2}{8\pi B^4}\right] \tag{20}$$

$$\frac{\dot{B}^2}{B^2} + 2\frac{\dot{A}\dot{B}}{AB} - 24\frac{\dot{A}\dot{B}^2}{AB^2}\dot{f}_G + Gf_G - f = k^2\left[\rho + \frac{\dot{U}^2}{2} + \frac{M^2}{8\pi B^4}\right] \tag{21}$$

The crucial parameters in cosmological observations include the mean scale factor a, mean Hubble parameter H, scalar expansion θ, deceleration parameter q, shear scalar σ , and mean anisotropic parameter Am. These quantities, derived from metric (4), are expressed as:

$$a = (AB^2)^{\frac{1}{3}} \tag{22}$$

$$H = \frac{1}{3}\left[\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right] \tag{23}$$

$$\theta = \frac{\dot{A}}{A} + 2\frac{\dot{B}}{B} \tag{24}$$

$$q = -1 + \frac{d}{dt}\left(\frac{1}{H}\right) \tag{25}$$

$$\sigma = \frac{1}{\sqrt{3}}\left[\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right] \tag{26}$$

$$A_m = \frac{2}{9H^2}\left[\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right] \tag{27}$$

3. Solution of Field Equation:

To construct a physically realistic cosmological model that aligns with observational data, we introduce the following reasonable physicalrelation.
First we have considered

$$f(G) = \lambda G^{\mu}, \tag{28}$$

where λ & μ are arbitrary constants. This framework is referred to as the power-law model of f(G), which has been shown to be cosmologically consistent. Its significance lies in the fact that it avoids the Big-Rip singularity while remaining compatible with observational evidence. Similar to the f(R) model, it accounts for both early-universe inflation and late-time cosmic acceleration [48]. Since no universal method exists for solving nonlinear differential equations, one must either rely on established analytical approaches or devise new strategies. Nevertheless, any additional assumptions introduced must retain physical plausibility. For this reason, we have also undertaken a careful review of the relevant literature.

Secondly, we have assumed time varying Deceleration parameter

$$q = b - \frac{n}{H}, \tag{29}$$

where, b, n are constants.
Finally, we have employed the linear relationship between the directional Hubble parameters H_1 and H_2 as

$$H_1 = \alpha H_2, \tag{30}$$

By comparing equations (25) and (29), and selecting $c = -\frac{(b+1)}{n}$ leads to a point type singularity at $t = 0$.we can derive the following expression for H and the scale factor $a(t)$

$$H = \frac{ne^{nt}}{(b+1)(e^{nt}-1)} \tag{31}$$

$$a = \eta(e^{nt} - 1)^{\frac{1}{b+1}}, \tag{32}$$

where $\eta = \delta(b+1)^{\frac{1}{b+1}}$

The deceleration parameter q in terms of cosmic time t is expressed as

$$q = \frac{(b+1)}{e^{nt}} - 1 \tag{33}$$

In our model, when $t = \frac{1}{n}\log[\eta(1+z)^{-(b+1)} + 1]$, the deceleration parameter's sign changes. Furthermore, we may determine the relationship between the cosmic time (t) and redshift (z) and the universe's scale factor

$$a(t) = (1+z)^{-1} \tag{34}$$

The Hubble parameter (H) is expressed in terms of redshift (z) as

$$H(z) = \frac{n}{b+1}\left[\eta(1+z)^{(b+1)} + 1\right] \tag{35}$$

Finally, the Hubble rate function takes the form

$$H(z=0) = H_0 = \frac{n}{b+1}(1+\eta) \tag{36}$$

Here, $H_0 = 100h$ denotes the Hubble parameter at the present epoch $(z = 0)$, whereas δ and b represent free parameters that must be constrained through observations.

The Hubble parameter is related to the rate of expansion of the universe. Using (35), the Hubble parameter calculated as,

$$H(z) = \frac{H_0}{1+\eta}\left[\eta(1+z)^{(b+1)} + 1\right] \tag{37}$$

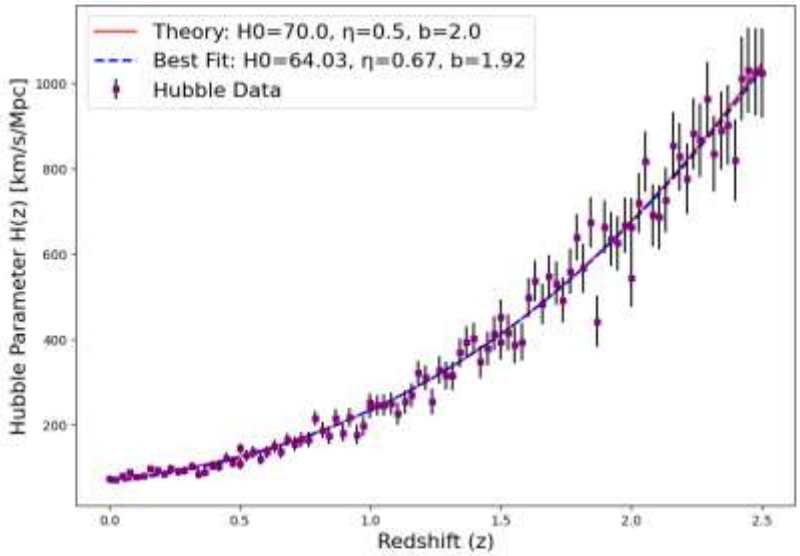

Figure 1. The plot of Hubble parameter (H) vs Redshift (z) along with Hubble Data-Set

In Figure 1, we observe that the Hubble parameter increases with redshift, indicating that the expansion rate of the universe was faster in the past (high z) compared to today $(z = 0)$. The red (theoretical) curve captures the general trend of the data but deviates somewhat. The blue dashed (best-fit) curve matches the observational data much more closely, showing that by adjusting the parameters, the model can accurately describe cosmic expansion. The close match of the best-fit curve with the Hubble data supports the viability of the chosen cosmological model, this indicates that the model can consistently explain the observed expansion of the universe, bridging between early-time fast expansion and present-day slower expansion.

Without loss of generality, we take $\mu = 1$, the field equations (19)-(21) are obtained as

$$\frac{\dot{B}^2}{B^2} + 2\frac{\ddot{B}}{B} = -k^2\left[p + \frac{\dot{U}^2}{2} - \frac{M^2}{8\pi B^4}\right] \tag{38}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = -k^2\left[p + \frac{\dot{U}^2}{2} - \frac{M^2}{8\pi B^4}\right] \tag{39}$$

$$\frac{\dot{B}^2}{B^2} + 2\frac{\dot{A}\dot{B}}{AB} = k^2\left[\rho - \frac{\dot{U}^2}{2} + \frac{M^2}{8\pi B^4}\right] \tag{40}$$

Solving field equations (38),(39) and (40) we obtain

$$A = D_2\eta(e^{nt} - 1)^{\frac{1}{b+1}}\, exp\left[x_2\,\frac{(b+1)(e^{nt}-1)^{\frac{b-2}{b+1}}}{\eta^3 n(b-2)e^{nt}}\right] \tag{41}$$

$$B = D_1\eta(e^{nt} - 1)^{\frac{1}{b+1}}\, exp\left[x_1\,\frac{(b+1)(e^{nt}-1)^{\frac{b-2}{b+1}}}{\eta^3 n(b-2)e^{nt}}\right] \tag{42}$$

where $D_1, D_2,\ x_1, x_2$ are constants.

Using equations (41) and (42), we have obtained the energy density in terms of redshift for perfect fluid.

$$k^2\rho = \frac{9(2\alpha+1)H_0^2}{(\alpha+2)^2(\eta+2)^2}[\eta(1+z)^{b+1} - 1]^2 + \frac{k^2k_1^2(1+z)^6}{2} - \frac{M^2}{8\pi(D_1\eta)^4(e^{nt}-1)^{\frac{4}{b+1}}\, exp\left[4x_1\frac{(b+1)(e^{nt}-1)^{\frac{b-2}{b+1}}}{\eta^3 n(b-2)e^{nt}}\right]} \tag{43}$$

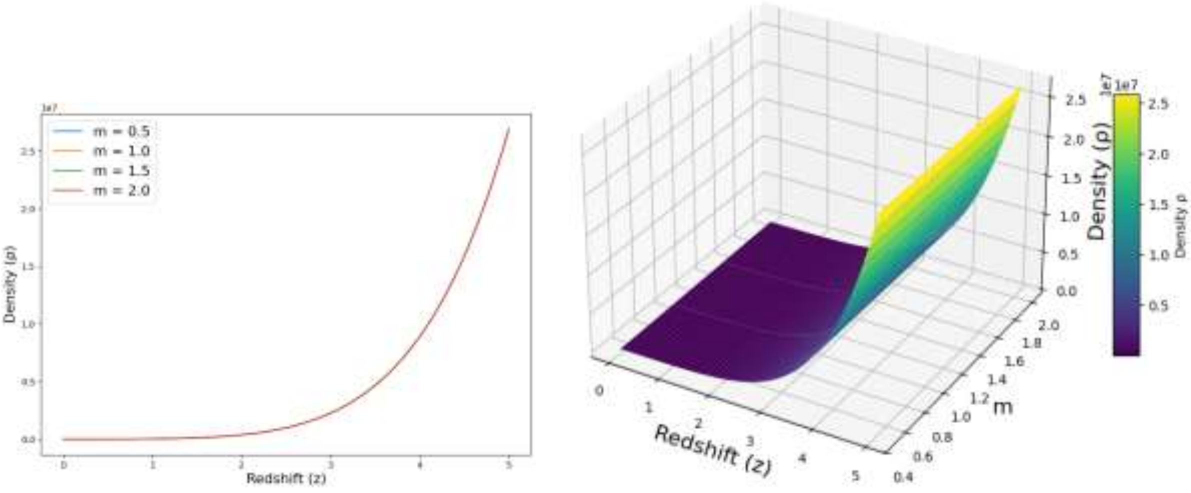

Figure 2. Density(ρ) vs. Redshift (z)

Energy density remains positive throughout the evolution of the Universe as depicted in figure 2. Interestingly, all curves almost overlap, meaning that the parameter mmm does not strongly influence the density evolution compared to the pressure case. Density increases rapidly with redshift. At low redshift $(z \approx 0)$, the density is small, consistent with the present-day low-energy universe. At high redshift $(z > 3)$, the density grows steeply, reflecting the dominance of matter in the early universe. The steep rise of $\rho(z)$ shows that the early universe was extremely dense. This is consistent with the standard cosmological picture where matter and radiation densities scale as positive powers of $(1 + z)$.

$$k^2 p = \frac{6H_0^2(b+1)(1+z)^{b+1}}{\eta^2(1+\eta)^2(\alpha+2)}\left[(1+z)^{(b+1)} + \eta\right] - \frac{27H_0^2}{(\alpha+2)^2(1+\eta)^2}\left[\eta(1+z)^{(b+1)} + 1\right] + \frac{k^2 k_1^2 (1+z)^6}{2} + \frac{M^2}{8\pi(D_1\eta)^4(e^{nt}-1)^{\frac{4}{b+1}} exp\left[4x_1 \frac{(b+1)(e^{nt}-1)^{\frac{b-2}{b+1}}}{\eta^3 n(b-2)e^{nt}}\right]} \quad (44)$$

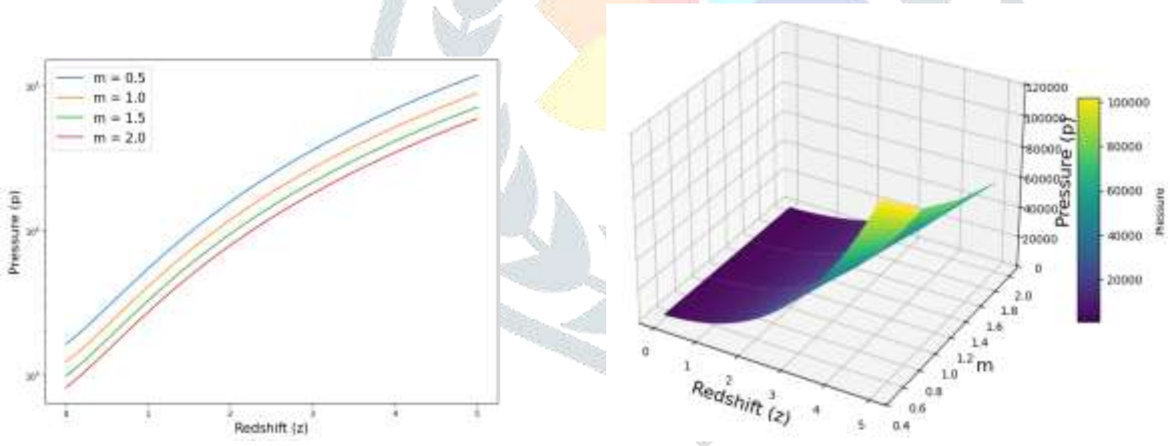

Figure 3. Pressure (p) vs. Redshift (z)

Pressure increases with redshift for all values of m as depicted in figure 3. This means that in the early universe $(z \gg 0)$, the pressure was much higher, while in the present universe $(z \approx 0)$, it is relatively small. For larger values of m, the pressure is lower at a given redshift. This suggests that increasing mmm reduces the effective pressure contribution of the cosmic fluid at all epochs. The parameter m controls the strength of pressure evolution with redshift. The rise of pressure with redshift reflects the dominance of high-energy conditions in the early universe, consistent with radiation/matter-dominated epochs. Smaller mmm leads to higher pressure, indicating a stiffer equation of state or stronger interaction. Larger mmm leads to reduced pressure, pointing toward a softer equation of state and possibly accelerated expansion at late times. The behavior suggests that tuning mmm allows the model to interpolate between stronger and weaker pressure contributions, potentially fitting observational data.

The formulas for the scalar expansion θ, shear scalar σ, and the mean anisotropic parameter Am are obtained as follows:

$$\theta = \frac{3H_0}{(1+\eta)}\left[\eta(1+z)^{b+1} + 1\right. \quad (45)$$

$$\sigma = \sqrt{6}\left[\frac{\alpha-1}{\alpha+2}\frac{H_0}{(\eta+1)}\left[\eta(1+z)^{\alpha+1} + 1\right]\right. \quad (46)$$

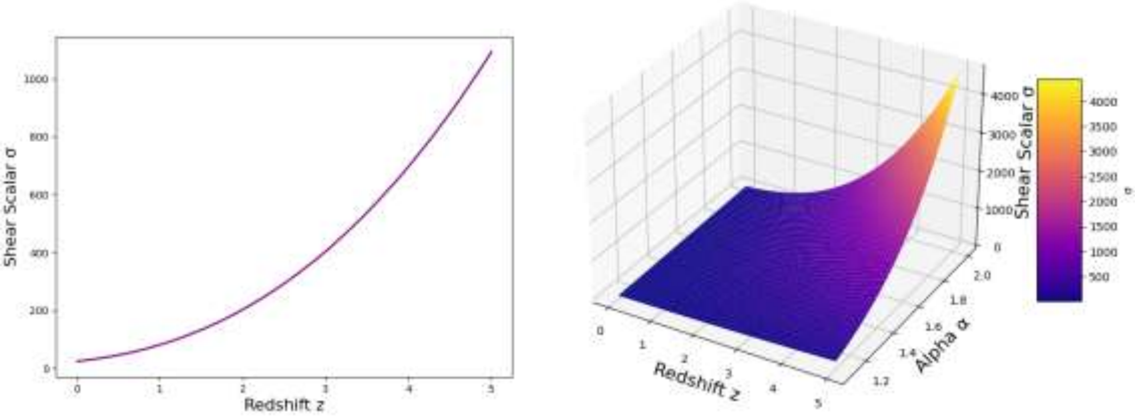

Figure 4. Shear Scalar (σ) vs. Redshift (z)

$$A_m = 4\left(\frac{\alpha-1}{\alpha+2}\right)^2 \tag{47}$$

which is constant.

The value of the deceleration parameter is found to be

$$q(z) = \frac{b(1+z)^{(b+1)}-\eta}{(1+z)^{(b+1)}+\eta} \tag{48}$$

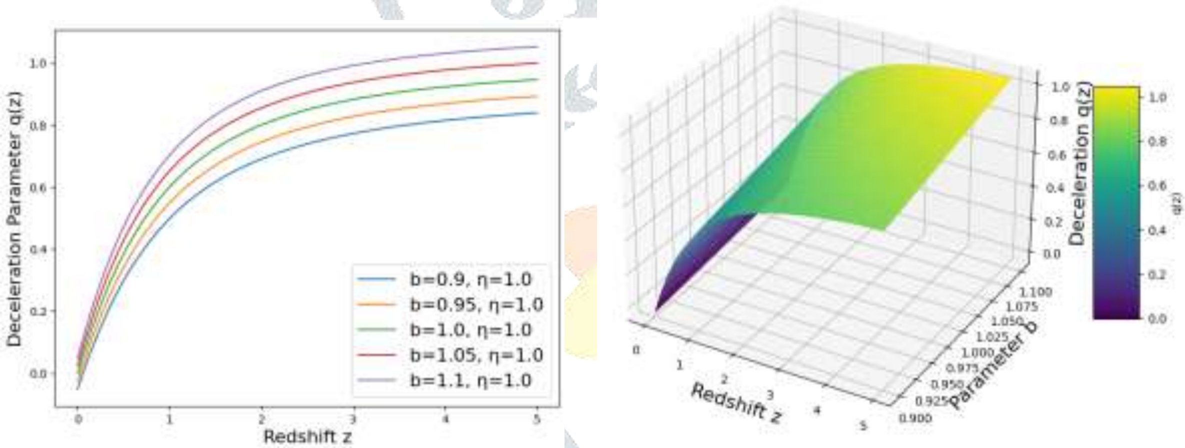

Figure 5. Deceleration Parameter (q) vs. Redshift (z)

The figure 5 shows that b controls the strength of the deceleration phase in the cosmological model. For smaller value of b indicates lower deceleration. It tends toward acceleration earlier. Larger value of b gives us stronger deceleration in the past, which indicates delays transition to acceleration.

The Ricci scalar for this solution turn out to be

$$R = -2\left[(\alpha+2)\frac{(-3n^2)(1+z)^{(b+1)}}{\eta^2(\alpha+2)(b+1)}\left[(1+z)^{(b+1)}+\eta\right] + (\alpha^2+2\alpha+3)\frac{9H_0^2}{(\alpha+2)^2(1+\eta)^2}\left[\eta(1+z)^{(b+1)}+1\right]^2\right] \tag{49}$$

The Gauss–Bonnet term is a curvature invariant that plays a crucial role in modified gravity theories. It encodes contributions from higher-order curvature corrections to Einstein's general relativity. We have obtained GB invariant in terms of redshift.

$$G = 24\alpha \times \frac{9H_0^2}{(\alpha+2)^2(1+\eta)^2}\left[\eta(1+z)^{(b+1)}+1\right]^2 - \left[\frac{-3n^2(1+z)^{(b+1)}}{\eta^2(\alpha+2)(b+1)}\left[(1+z)^{(b+1)}+\eta\right] + \frac{9H_0^2}{(\alpha+2)^2(1+\eta)^2}\left[\eta(1+z)^{(b+1)}+1\right]^2\right] \tag{50}$$

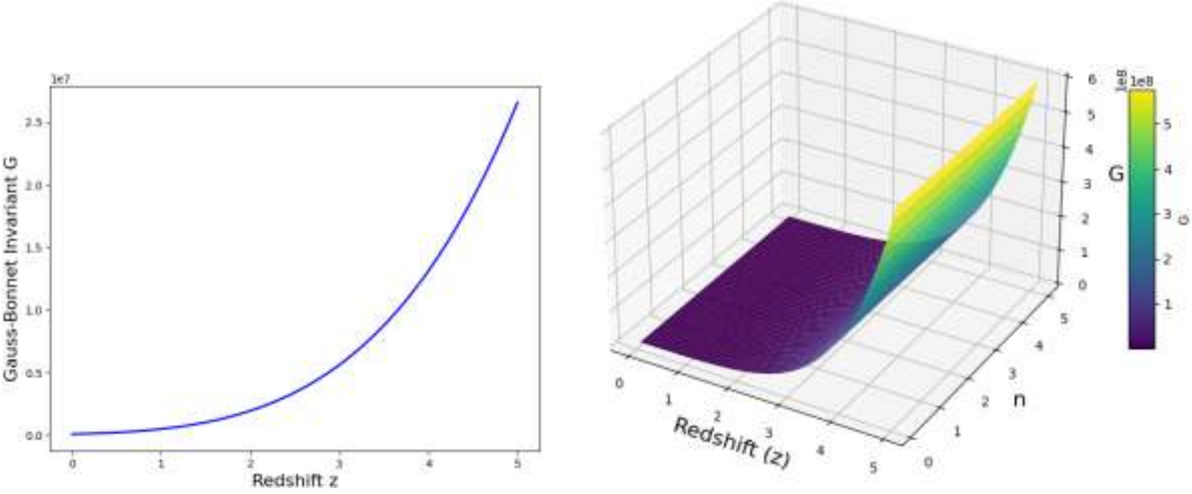

Figure 6. GB Invariant (G) vs. Redshift (z) for, 0 < n < 5

At low redshift $(z \approx 0)$, G is small, indicating weak curvature effects in the present, late-time universe. It means higher-order curvature effects are negligible in the present epoch. Thus, the universe is dominated by standard dark energy contributions rather than strong curvature corrections. As redshift increases, G rises steeply, showing that the Gauss–Bonnet term becomes very large in the early universe where curvature and energy densities are high. It shows that higher-order curvature corrections were dominant in the early universe. This is consistent with inflationary or high-energy epochs where spacetime curvature is very large.

4. Jerk parameter:

In the field of cosmology, the term "jerk parameter" denotes the third time derivative of the scale factor of the Universe concerning cosmic time. This parameter, expressed as a dimensionless quantity, serves as a crucial metric for quantifying the pace of alteration in the acceleration of the Universe's expansion. Researchers employ the jerk parameter to delve into the intricate dynamics of the cosmos and to differentiate among various cosmological models. Within the realm of cosmology, the precise value of the jerk parameter takes on significant importance as it plays a pivotal role in unraveling the enigma of dark energy.

The jerk parameter (j), can be defined as follows:

$$j = \frac{\dddot{a}}{aH^3}$$

$$j = \frac{\eta(b+1)^2(1+z)^{(b+1)} + [b(1+z)^{(b+1)} - \eta][(2b+1)(1+z)^{(b+1)} - \eta}{[(1+z)^{(b+1)} + \eta]^2} \quad (51)$$

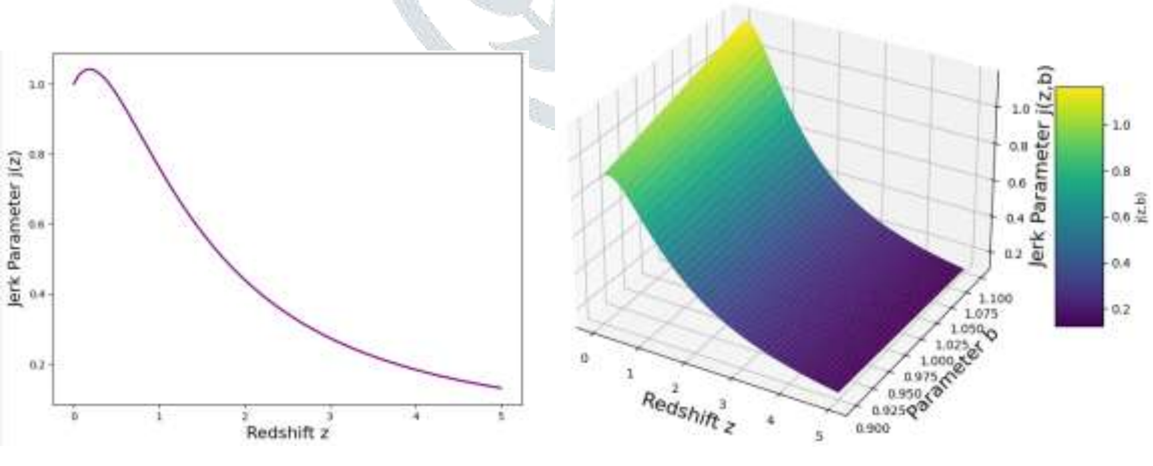

Figure 7. Jerk Parameter (j) vs. Redshift (z)

5. Statefinder parameters:

In cosmology, the concept of a "statefinder pair" refers to a pair of dimensionless parameters that can help distinguish between different cosmological models based on the evolution of the cosmic scale factor $a(t)$ and its time derivatives. These parameters were introduced by Sahni et al. [49]. The statefinder pair $\{r, s\}$ can be calculated from observational data to probe the nature of dark energy and the expansion history of the Universe. Different cosmological models, including those with different forms of dark energy or modified gravity, can produce distinct trajectories in the $\{r, s\}$ plane, allowing researchers to constrain and compare these models with observations. The statefinder parameters are defined as follows:

$r = 1 + 3\frac{\dot{H}}{H^2} + \frac{\ddot{H}}{H^3}, \qquad s = \frac{r-1}{3(q-0.5)}$

$$r = 2\left[\frac{b(1+z)^{(b+1)}-\eta}{(1+z)^{(b+1)}+\eta}\right]^2 + \frac{b(1+z)^{(b+1)}-\eta}{(1+z)^{(b+1)}+\eta} + \frac{\eta(b+1)^2(1+z)^{(b+1)}}{\left[(1+z)^{(b+1)}+\eta\right]^2} \tag{52}$$

$$s = \frac{2(2b^2+b-1)(1+z)^{2(b+1)}+\eta(b^2-b-1)(1+z)^{(b+1)}-\eta^2}{3[(2b-1)(1+z)^{(b+1)}-3\eta]} \tag{53}$$

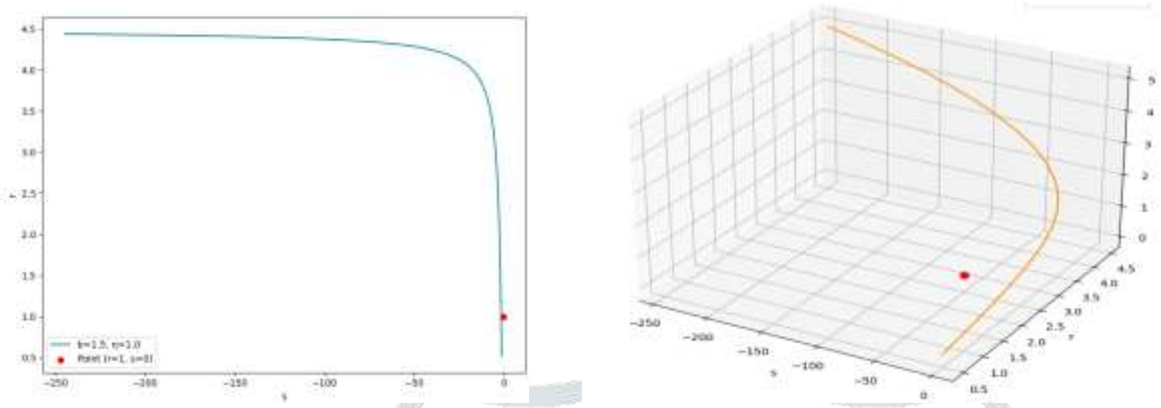

Figure 7. r vs. s

It can be observed from figure 7 that the values of r and s ultimately correspond to the derived model to ΛCDM.

6. Discussion and conclusion:

In cosmology, redshift describes how light from faraway galaxies or celestial bodies becomes stretched, shifting toward longer wavelengths closer to the red part of the electromagnetic spectrum. This effect occurs because the universe is expanding. As space itself grows, the light traveling through it is elongated, leading to this wavelength shift. The farther away a galaxy is, the more its light is redshifted. This phenomenon is a vital tool for astronomers, helping them determine both the distance to cosmic objects and their motion through space.

In this research, we explore the characteristics of the Bianchi type-I space-time within the framework of $f(G)$ gravity theory. The model is constructed based on specific assumptions. The first assumption posits a power low relation between f and G. i.e. $f(G) = \lambda G^{\mu}$ is employed. where λ & μ are arbitrary constants. The second assumption sets the relation between Hubble parameter and deceleration parameter i.e. $= b - \frac{n}{H}$, where b and n are constants. Additionally, we have employed the linear relationship between the directional Hubble parameters H_1 and H_2.
Figure 1 demonstrates the evolution of the Hubble parameter with redshift and shows that the proposed cosmological model, once parameters are tuned, provides an excellent fit to observational Hubble data. This validates the model as a suitable description of the universe's expansion history and constrains key cosmological parameters such as H_0, η and b.

The figure 2 shows that the cosmic density grows steeply with redshift, reaching very high values in the early universe and small values at the present epoch. The parameter mmm has negligible effect on the density evolution, implying that density is mainly governed by the expansion factor, while mmm influences pressure instead.

The figure 3 shows how cosmological pressure evolves with redshift for different values of parameter m. Pressure grows rapidly in the early universe, while differences in m shift the magnitude of this growth. This illustrates how the parameter m influences the dynamical behavior of the universe, with smaller m corresponding to higher pressure and larger m leading to lower pressure at all redshifts.

The behavior of deceleration parameter (q) confirms that the model can describe the transition from a decelerating universe in the past to a nearly accelerating universe at present, consistent with observational cosmology. The deceleration parameter evolves with redshift for different values of b. The universe was strongly decelerating at high redshift, but at low redshift, it approaches acceleration. The parameter b tunes the strength of past deceleration, with higher b giving stronger deceleration and lower b leading to an earlier approach to acceleration.

The steep growth of G suggests that modifications of gravity through the Gauss–Bonnet term primarily affect the early-time universe, while at late times, they naturally weaken, making the model consistent with current observations. The figure 6 shows that the Gauss–Bonnet invariant G grows rapidly with redshift, being negligible at present but very large in the early universe. This means higher-order curvature effects were significant in the past and faded away with cosmic expansion, consistent with the role of f(G) gravity in describing early inflation and allowing smooth transition to late-time acceleration.

The figure 7 shows that the jerk parameter $j(z)$ is close to 1 at the present epoch, consistent with ΛCDM-like behavior. but deviates significantly at higher redshift, decreasing with z. This means the model mimics standard cosmology today while incorporating dynamical effects in the early universe, consistent with a transition from decelerated to accelerated expansion.

The deceleration parameter and jerk parameter confirm that the universe was decelerating in the past, entered a transition phase at intermediate redshifts, and is accelerating today.

The Hubble parameter H plotted vs redshift z. The graph shows the values of Hubble parameter in the range of standard dataset which supports the current observational data. The Hubble parameter in terms of redshift is a crucial observational quantity that informs us about the current state and past history of the universe's expansion. Studying its behavior with redshift provides valuable information about the underlying cosmological model and the influence of various components like matter, radiation, and dark energy on the evolution of the cosmos.

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