



# Magnetohydrodynamic Similarity Analysis of Nanofluid Flow over a Moving flat plate

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**Abstract :** This study investigates the steady, two-dimensional forced convection laminar boundary layer flow of a nanofluid over a dynamically moving surface in the presence of a magnetic field. The governing partial differential equations are transformed into ordinary differential equations using a scaling transformation technique, allowing for a similarity solution. A one-parameter transformation method is employed to simplify the system, considering cases where the surface moves either in the direction of or against the free stream. The effects of key parameters on velocity, temperature, and nanoparticle concentration profiles are analyzed. Additionally, three-dimensional representations of temperature and velocity distributions provide further insight into the thermal and flow characteristics of the nanofluid. The graphical results highlight the influence of magnetic fields and flow dynamics on heat and mass transfer, offering valuable implications for magnetohydrodynamics and nanofluid applications.

**Keywords:** Nanofluid, Scaling transformation technique, Magnetic field, Flat plate, Laminar flow, Moving surface.

## I. INTRODUCTION

Magnetohydrodynamics is the study of the behaviour of electrically conducting fluids in the presence of a magnetic field. This field of study is significant in many industrial and natural applications, including plasma physics, astrophysics, and engineering. A particular area of interest within MHD is the study of nanofluid flow over moving flat plates, which has applications in cooling systems, biomedical engineering, and advanced manufacturing processes.

Nanofluids, which are fluids containing nanometer-sized particles (nanoparticles), have attracted significant attention due to their enhanced thermal properties compared to conventional fluids. By dispersing nanoparticles such as alumina, copper, titanium dioxide, and magnetite into base fluids like water, ethylene glycol, or oils, one can achieve higher thermal conductivity and improved heat transfer performance. This makes nanofluids ideal for use in energy systems, cooling technologies, and microfluidic devices.

When such nanofluids interact with a moving flat plate in the presence of a magnetic field, the flow characteristics and heat transfer behavior become significantly altered due to the interplay between the Lorentz force and the fluid's momentum. The study of these interactions through similarity analysis is crucial for understanding how fluid velocity, temperature distribution, and boundary layer thickness behave under different conditions.

Magnetohydrodynamics flow of nanofluids has applications in many industrial and scientific fields. For example, in nuclear reactors, cooling devices, biomedical applications, and aerospace technologies, the ability to control and optimize fluid flow under a magnetic field is essential for performance enhancement and energy efficiency. The introduction of nanoparticles into the fluid not only improves heat transfer but also influences the drag force, stability, and boundary layer characteristics.

One of the key aspects of MHD nanofluid flow over a moving flat plate is the ability to control the velocity and thermal boundary layers. The presence of a magnetic field can either suppress or enhance fluid movement depending on the Hartmann number ( $Ha$ ), which measures the strength of the magnetic force relative to viscous forces. The interaction between the magnetic field and nanofluid creates an additional resistive force known as the Lorentz force, which can modify the flow characteristics significantly.

Moran and Gaggioli [1], presents a systematic approach to reduce the number of independent variables in systems of partial differential equations, incorporating auxiliary conditions. The methodology leverages group theory techniques to simplify complex systems, making it particularly useful for practical applications like analyzing boundary layer flows. Ioan Pop, Nield D.A. [2], explores the magnetohydrodynamic mixed convection boundary layer flow near a vertical surface embedded in a porous medium saturated with a nanofluid. Using Buongiorno's model, the authors analyze the effects of thermophoresis and Brownian motion on heat and mass transfer. The study provides valuable insights into the behaviour of nanofluids in porous media, with applications in engineering and industrial processes.

Roslinda Nazar, Ioan Pop [3], focuses on the magnetohydrodynamic (MHD) stagnation-point flow of an electrically conducting, viscous fluid over a stretching sheet, influenced by the presence of a transverse magnetic field. The authors employ mathematical techniques to analyze the flow behavior and heat transfer characteristics. Their findings contribute to the understanding of MHD flow phenomena, which are relevant to various industrial and engineering applications. Sheikholeslami M, Gorji-Bandpy, Ganji D [4], examines magnetohydrodynamic (MHD) natural convection in a nanofluid-filled inclined enclosure with sinusoidal walls. Using the Control Volume Finite Element Method (CVFEM), the authors analyze the impact of various parameters, such as Hartmann number, Rayleigh number, and nanoparticle volume fraction, on heat transfer and fluid flow. The study provides valuable insights into enhancing thermal performance in engineering applications.

Hatami M., Ganji D.D [5], investigated the heat transfer and nanofluid flow in a suction and blowing process between parallel disks under the influence of a variable magnetic field. The authors employ analytical and numerical methods to study the effects of key parameters, such as magnetic field strength and nanofluid properties, on flow and thermal behaviour. The findings have practical implications for optimizing industrial processes involving nanofluids and magnetic fields. Ioan Pop [6], examines the magnetohydrodynamic mixed convection flow near the stagnation point of a vertical permeable surface. The authors analyze the interplay between mixed convection and the influence of permeability on the flow and heat transfer characteristics. The study offers valuable insights into boundary layer behavior, with applications in engineering processes involving MHD flows and permeable surfaces.

Fazle Mabood, Anuar Ishak, Ioan Pop [7], investigates the stagnation-point flow of a nanofluid over a moving plate under the influence of a convective boundary condition and a magnetic field. The authors analyze the effects of key parameters, such as nanoparticle concentration and magnetic field strength, on the flow and heat transfer behavior. The study contributes to the understanding of nanofluid dynamics in engineering and industrial applications involving magnetohydrodynamics. Fazle Mabood, Ashwinkumar G.P., Sandeep N. [8], analyzed the unsteady flow dynamics of magnetohydrodynamic (MHD) hybrid nano liquids over flat and slendering surfaces. The authors investigate the influence of physical parameters, such as nanoparticle interactions and magnetic fields, on heat transfer characteristics. The findings contribute to advancements in thermal analysis and enhance understanding of fluid behavior in engineering applications.

Meena O.P., Janapatla P., Meena M.K. [9], presented new similarity solutions for magnetohydrodynamic flow over a horizontal plate, incorporating nonlinear hydrodynamic slip and linear thermal and mass slips. Using Lie group analysis, the authors transform the governing equations into a system of nonlinear ordinary differential equations. The study provides insights into the effects of slip parameters and magnetic fields on flow, heat, and mass transfer, with applications in advanced fluid dynamics and engineering. Umar Khan, Naveed Ahmed, Syed Tauseef Mohyud-Din [10], investigates the magnetohydrodynamic flow and heat transfer of a Cu–water nanofluid confined between parallel plates, considering the influence of different nanoparticle shapes. The authors analyze the effects of various factors, such as nanoparticle geometry and magnetic fields, on thermal and flow behavior. The study provides valuable insights into optimizing nanofluids for enhanced heat transfer performance in engineering applications.

Sheikholeslami M., Ganji D.D. [11], delves into the heat transfer analysis of unsteady nanofluid flow confined between moving parallel plates under the influence of a magnetic field. Employing an analytical approach, the authors investigate the effects of key parameters, such as magnetic field strength and nanofluid properties, on thermal performance. The study offers valuable insights for optimizing heat transfer in engineering systems involving nanofluids and magnetic fields.

## II Formation of problem

Consider the phenomenon of two-dimensional forced convection involving a nanofluid as it flows past a flat plate in motion and introducing the stream function the governed equations of flow by Timol M. G. [12]

$$\psi_y \psi_{yx} - \psi_x \psi_{yy} - \psi_{yyy} - U_e (U_e)_x = M(x) [\psi_y - U_e] \quad (1)$$

$$\psi_y \theta_x - \psi_x \theta_y - (Pr)^{-1} \theta_{yy} = 0 \quad (2)$$

$$S_c (\psi_y \phi_x - \psi_x \phi_y) = \phi_{yy} \quad (3)$$

With boundary conditions

$$\text{For } y = 0, \quad \psi_y = u_\omega \lambda, \quad \psi_x = 0, \quad \theta = 1, \quad \phi = 1 \quad (4)$$

$$\text{For } y = \infty, \quad \psi_y = U_e, \quad \theta = \phi = 0 \quad (5)$$

### III Generalized group theoretic method

By using linear group transformation,

$$\begin{aligned} \bar{x}^* &= K^{A_1} x^*, & \bar{y}^* &= K^{A_2} y^*, & \bar{\psi}^* &= K^{A_3} \psi^*, & \bar{U}^* &= K^{A_4} U^* \\ \bar{M}^* &= K^{A_5} M^*, & \bar{\theta}^* &= K^{A_6} \theta^*, & \bar{\phi}^* &= K^{A_7} \phi^* \end{aligned} \quad (6)$$

Where  $A_1, A_2, A_3, A_4, A_5, A_6, A_7$ , and  $K$  are constants

For the dependent and independent variables. From equation (6) one obtains

$$\left(\frac{\bar{x}^*}{x^*}\right)^{\frac{1}{A_1}} = \left(\frac{\bar{y}^*}{y^*}\right)^{\frac{1}{A_2}} = \left(\frac{\bar{\psi}^*}{\psi^*}\right)^{\frac{1}{A_3}} = \left(\frac{\bar{U}^*}{U^*}\right)^{\frac{1}{A_4}} = \left(\frac{\bar{M}^*}{M^*}\right)^{\frac{1}{A_5}} = \left(\frac{\bar{\theta}^*}{\theta^*}\right)^{\frac{1}{A_6}} = \left(\frac{\bar{\phi}^*}{\phi^*}\right)^{\frac{1}{A_7}} = K \quad (7)$$

By utilizing the linear transformation defined in equation (6) on equations (1)-(3), we observe that the differential equations retain their complete invariance under the given linear transformations. Given that the exponent  $A_1 \neq 0$ , we subsequently establish the following relationships as,

$$A_1 = 3A_2 = \frac{3}{2}A_3 = 3A_4 = -\frac{3}{2}A_5 \text{ and } A_6 = A_7 = 0 \quad (8)$$

Substituting equation (8) into equation (7) results in

$$\begin{aligned} \eta &= \frac{y}{x^{\frac{1}{3}}}, & \psi &= \frac{1}{x^{\frac{1}{3}}} f_1(\eta), & U_e x^{-\frac{1}{3}} &= f_2(\eta) = C \\ M(x) x^{\frac{2}{3}} &= f_3(\eta) = M_0, & \theta(\eta) &= f_4(\eta), & \phi(\eta) &= f_5(\eta) \end{aligned} \quad (9)$$

### IV Establishing Ordinary Differential Equations

Rewriting the similarity variables (9) in terms of equations (1)-(5) results in the following ordinary differential equations

$$3f_1'''(\eta) - f_1'^2(\eta) - M_0[f_1'(\eta) - C] + C^2 = 0 \quad (10)$$

$$\frac{1}{Pr} \theta''(\eta) + 2f_1(\eta) \theta'(\eta) = 0 \quad (11)$$

$$\frac{1}{Sc} \phi''(\eta) + \frac{2}{3} f_1(\eta) \phi'(\eta) = 0 \quad (12)$$

With boundary conditions

$$\text{For } \eta = 0, \quad f_1'(\eta) = \lambda, \quad f_1(\eta) = 0, \quad \theta(\eta) = 1, \quad \phi(\eta) = 1 \quad (13)$$

$$\text{For } \eta = \infty, \quad f_1'(\eta) = C, \quad \theta(\eta) = 0, \quad \phi(\eta) = 0 \quad (14)$$

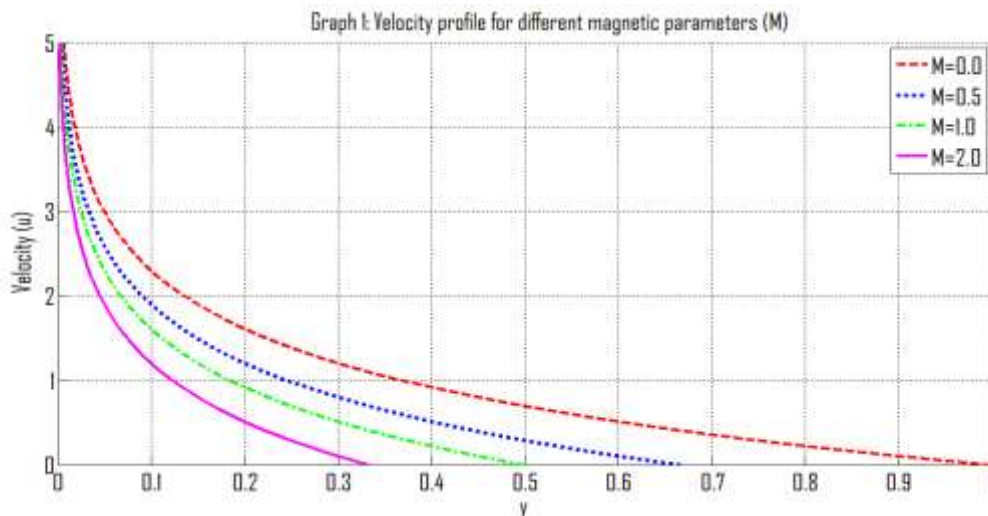
### V Results and Discussion

The set of non-linear ordinary differential equations, identified as equations (10), (11), and (12) in the problem statement, along with the associated boundary conditions specified as equations (13) and (14), has been successfully addressed and solved. This solution has been achieved through the application of MATLAB's built-in ordinary differential equation solver. By employing this computational tool, the numerical integration of the equations and the satisfaction of the boundary conditions have been ensured, enabling an effective resolution of the given mathematical system.

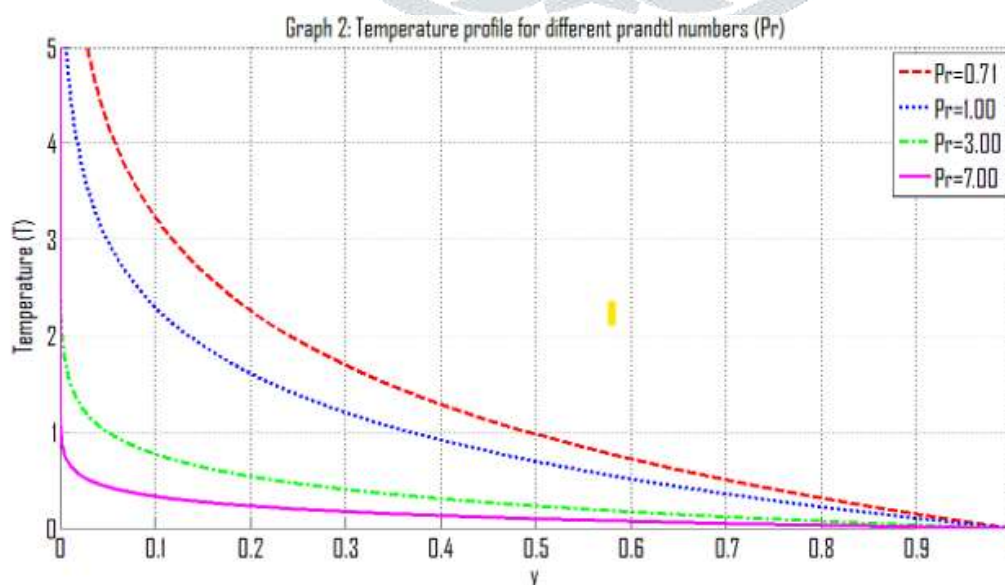
The graph (1) illustrates the **velocity profile** for different **magnetic parameters** ( $M$ ) in a boundary layer flow. The **y-axis** represents the distance from the moving surface, while the **x-axis** represents the velocity ( $u$ ). The plot shows how the velocity decreases as the distance from the surface increases, with different curves corresponding to different values of  $M$ . As the magnetic parameter  $M$  increases, the velocity decreases more rapidly, indicating that the presence of a stronger magnetic field creates a **retarding (resistive) effect on the fluid flow** due to the **Lorentz force**, which acts as a drag force. The red dashed line



( $M = 0.0$ ) represents the case without a magnetic field, showing the highest velocity near the surface. In contrast, the magenta solid line ( $M = 2.0$ ) depicts the case with the highest magnetic field, showing the most significant reduction in velocity. This trend is consistent with **Magnetohydrodynamics** theory, where the application of a transverse magnetic field **suppresses fluid motion**, leading to a thicker boundary layer and reduced flow velocity.

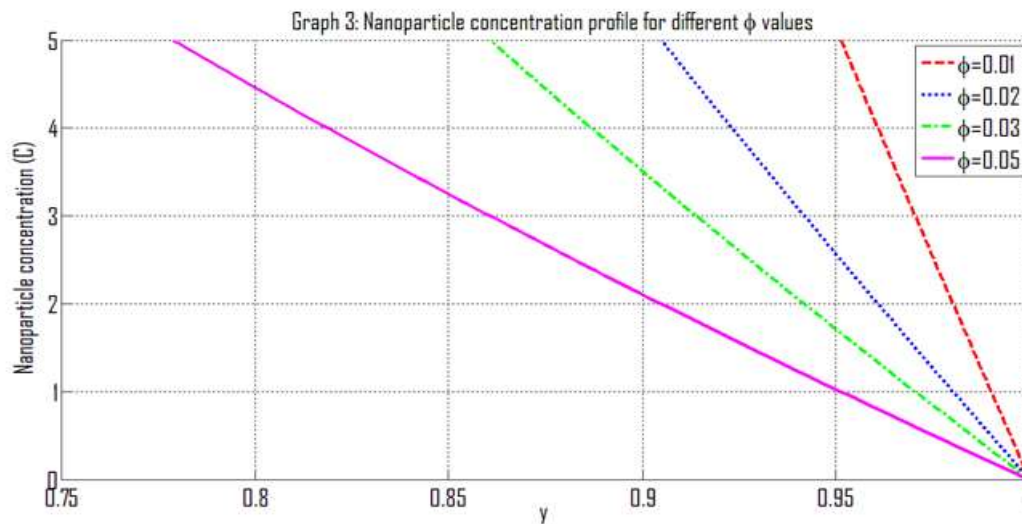


The graph (2) illustrates the temperature profile for different Prandtl numbers ( $Pr$ ) in a boundary layer flow. The  $y$ -axis represents the distance from the heated surface, while the  $x$ -axis represents the temperature ( $T$ ). The curves show how temperature decreases with increasing distance from the surface, with different lines corresponding to different Prandtl numbers. A higher Prandtl number ( $Pr$ ) results in a steeper temperature gradient, meaning that the fluid temperature drops more rapidly as  $y$  increases. This is because higher  $Pr$  values correspond to fluids with lower thermal diffusivity (e.g., oils), causing heat to diffuse slower and confining the thermal boundary layer near the surface. The red dashed line ( $Pr = 0.71$ ) represents a fluid with high thermal diffusivity (like air), leading to a thicker thermal boundary layer and a more gradual temperature decrease. In contrast, the magenta solid line ( $Pr = 7.00$ ) represents a fluid with low thermal diffusivity (like water), where heat remains close to the surface and the temperature decreases sharply. This behavior is consistent with heat transfer theory, where fluids with higher  $Pr$  numbers have higher temperature gradients and thinner thermal boundary layers, making them more effective for localized heat transfer applications.

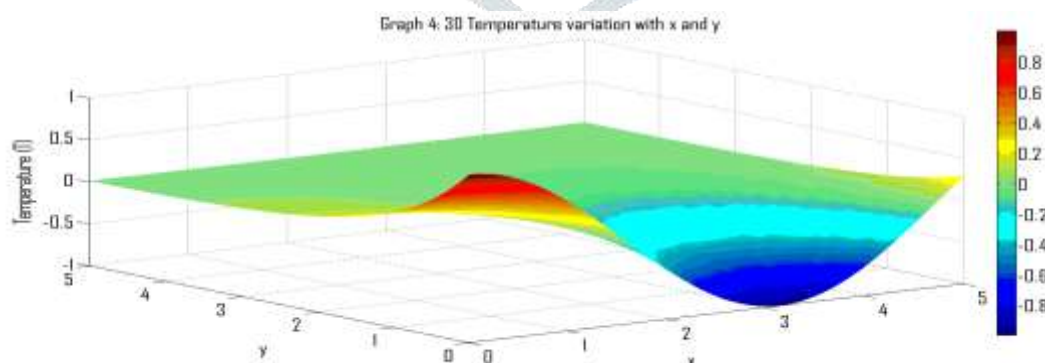


The graph (3) illustrates the nanoparticle concentration profile for different nanoparticle volume fractions ( $\phi$ ) in a boundary layer flow. The  $y$ -axis represents the distance from the surface, while the  $x$ -axis represents the nanoparticle concentration ( $C$ ). The curves show how the concentration decreases as the distance from the surface increases, with different

lines corresponding to different values of  $\phi$ . A higher nanoparticle volume fraction ( $\phi$ ) results in a steeper concentration gradient, indicating that more nanoparticles are concentrated near the surface and the concentration decreases more rapidly with distance. The red dashed line ( $\phi = 0.01$ ) represents a lower nanoparticle fraction, leading to a slower concentration decay. In contrast, the magenta solid line ( $\phi = 0.05$ ) shows a faster decrease in concentration, meaning that higher nanoparticle concentrations enhance mass transport effects. This behavior aligns with nanofluid dynamics, where increasing the nanoparticle volume fraction leads to enhanced thermal and mass transport properties, but also results in a thinner concentration boundary layer, affecting diffusion characteristics.

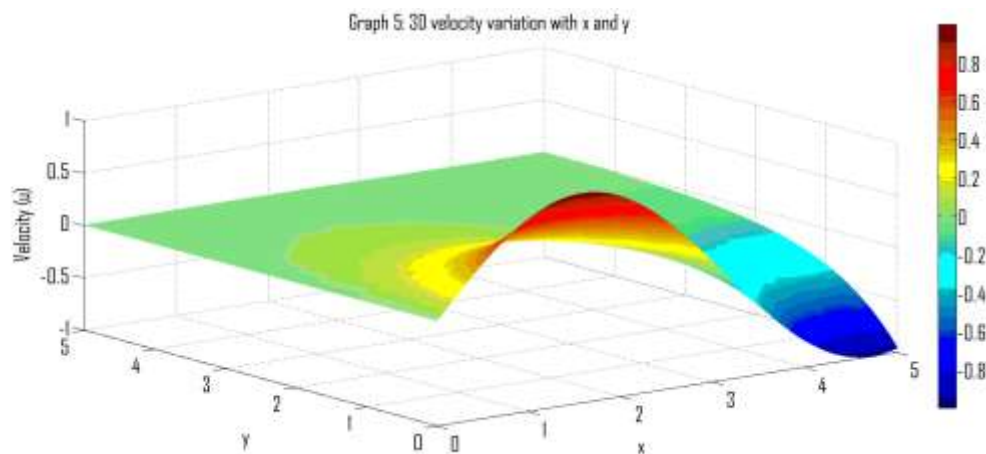


The graph (4) presents a **3D temperature variation** with respect to  $x$  and  $y$  in a boundary layer flow. The  $x$ -axis represents the horizontal distance along the surface, the  $y$ -axis represents the vertical distance from the surface, and the  $z$ -axis (color-coded) represents the temperature ( $T$ ). The color bar on the right provides a scale for temperature variations, with **red regions indicating higher temperatures** and **blue regions indicating lower temperatures**. The temperature distribution shows a **wave-like behavior**, where certain regions experience a rise in temperature (red areas) while others show a cooling effect (blue areas). The **steep temperature gradient near the surface** suggests **heat transfer effects**, likely influenced by parameters such as the **Prandtl number ( $Pr$ )**, **thermal diffusivity**, and **convective boundary conditions**. The presence of contour levels enhances the visualization of **temperature gradients and heat diffusion trends**, providing insights into the **thermal boundary layer behaviour**. This type of graph is useful in **nanofluid and MHD studies**, where temperature control is critical for optimizing heat transfer performance.



The graph (5) presents a 3D velocity variation with respect to  $x$  and  $y$  in a boundary layer flow. The  $x$ -axis represents the horizontal distance along the surface, the  $y$ -axis represents the vertical distance from the surface, and the  $z$ -axis (color-coded) represents the velocity ( $u$ ). The color bar on the right provides a scale for velocity variations, where red regions indicate higher velocity magnitudes and blue regions indicate lower velocity or flow reversal. The contour patterns highlight areas of strong velocity gradients, indicating regions of rapid deceleration or acceleration in the flow. The wave-like behavior suggests flow instability or oscillatory motion, possibly due to factors such as magnetic field effects (MHD), shear forces, or external

disturbances. The green region represents areas with near-zero velocity, indicating the transition between positive and negative flow regions. This type of visualization is useful for understanding fluid dynamics, shear layer interactions, and velocity boundary layer development, particularly in the study of nanofluid flow, heat transfer, and magnetohydrodynamic effects.



## Conclusion

This study successfully analyzed the magnetohydrodynamic (MHD) flow of nanofluids over a moving flat plate using similarity transformation techniques. The results highlight the significant influence of the magnetic field, Prandtl number, and nanoparticle concentration on velocity, temperature, and mass transfer characteristics. The presence of a magnetic field creates a resistive Lorentz force, reducing fluid motion, while higher Prandtl numbers and nanoparticle fractions enhance thermal and mass transfer properties. These findings provide valuable insights for optimizing MHD nanofluid applications in energy systems, cooling technologies, and biomedical engineering.

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