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Scaling Group Invariant Solutions for Pseudoplastic Prandtl Fluids over a Stretching Sheet

[1] **Sonawane Pankaj M**
[1]Associate Professor
[1]Department of Mathematics
[1]Dhanaji Nana Mahavidyalaya, Faizpur, India

Abstract : This study presents a similarity analysis of the two-dimensional steady flow and heat transfer of a pseudoplastic visco-inelastic non-Newtonian Prandtl fluid over a stretching permeable surface. By employing a one-parameter scaling group transformation, the governing system of partial differential equations is reduced to a set of nonlinear ordinary differential equations. The scaling symmetry is systematically derived using group-theoretic methods to identify invariant solutions and corresponding similarity variables. The resulting boundary value problem is solved MATLAB ode solver. Effects of flow parameters on the velocity and temperature profiles are examined. Also, this study investigates the influence of suction/injection and the Prandtl number on velocity and temperature profiles in a boundary layer flow.

Keywords: Scaling Group Transformation, Prandtl Non-Newtonian Fluid, Stretching Permeable Surface, Skin Friction Coefficient, Nusselt Number.

I. Introduction

The study of boundary layer flows over stretching surfaces has garnered significant attention due to its wide-ranging applications in industrial processes such as polymer extrusion, glass fiber production, and metal forming. The classical work by Sakiadis laid the foundation for analyzing such flows, particularly emphasizing the significance of stretching surfaces in fluid dynamics. Subsequent studies have extended this framework to encompass non-Newtonian fluids, which exhibit complex rheological behaviors not captured by Newtonian models.

Among non-Newtonian fluids, the Prandtl fluid model stands out for its ability to describe pseudoplastic behavior, where the apparent viscosity decreases with increasing shear rate. This characteristic is particularly relevant in processes involving shear-thinning fluids. Understanding the flow and heat transfer characteristics of Prandtl fluids over stretching sheets is crucial for optimizing industrial applications where such fluids are prevalent.

The mathematical modeling of these flows involves solving nonlinear partial differential equations (PDEs) that describe momentum and energy conservation. Due to the complexity of these equations, analytical solutions are often unattainable, necessitating the use of similarity transformations to reduce the PDEs to ordinary differential equations (ODEs). The scaling group transformation technique, rooted in Lie group analysis, provides a systematic approach to identify such similarity variables, facilitating the reduction process.

This study focuses on employing the scaling group transformation technique to derive similarity solutions for the boundary layer flow and heat transfer of pseudoplastic Prandtl fluids over a stretching sheet. By transforming the governing PDEs into a set of nonlinear ODEs, we aim to analyze the effects of various physical parameters on the flow and thermal fields. The transformed equations are solved numerically using appropriate methods, and the results are discussed in detail to provide insights into the behavior of Prandtl fluids in such configurations.

Bariya H. G., & Patel M. P. [1], study explores the boundary layer flow of Prandtl-Eyring fluid over a stretching sheet, a crucial aspect in fluid dynamics. By transforming governing equations into nonlinear ordinary differential equations, the research provides numerical solutions using MATLAB. The findings highlight the influence of fluid parameters on velocity profiles, offering insights applicable to industrial processes. This work contributes to advancements in materials science, engineering, and polymer manufacturing.

Kandasamy R., Muhaimin I., Kamachi G. [2], investigated the impact of temperature-dependent viscosity in magnetohydrodynamic nanofluid flow over a porous stretching surface. Using scaling group transformation, the study formulates and solves nonlinear differential equations governing the boundary layer behavior. The findings reveal how viscosity variations influence velocity, temperature, and concentration profiles, providing insights into optimizing industrial applications involving nanofluids. This research contributes to advancements in heat transfer, fluid mechanics, and engineering processes.

Suali M., Nik Long, N. M. A., Ariffin, N. M. [3] examined, the unsteady stagnation point flow and heat transfer over a stretching or shrinking sheet with suction or injection. The governing partial differential equations are transformed into nonlinear ordinary differential equations using a similarity transformation and solved numerically. The research considers both stretching and shrinking cases, revealing that dual solutions exist for the shrinking scenario, while the stretching case yields a unique solution. Results are presented for skin friction coefficient, local Nusselt number, velocity, and temperature profiles under varying governing parameters.

Mohamed M. K. A., Salleh M. Z., Ishak A., Pop I. [4], study investigates the stagnation point flow and heat transfer over a stretching or shrinking sheet in a viscoelastic fluid, specifically Walter's liquid-B model, with a convective boundary condition and partial slip velocity. The governing nonlinear partial differential equations are transformed into ordinary differential equations using a similarity transformation and solved numerically using the Runge-Kutta-Fehlberg method.

Abel M. S., Begum G. [5], examined the heat transfer characteristics in magnetohydrodynamic viscoelastic fluid flow over a stretching sheet, incorporating the effects of heat source/sink, viscous dissipation, stress work, and radiation, particularly for large Prandtl numbers. The governing equations are transformed into nonlinear ordinary differential equations using similarity transformations and solved numerically. The investigation highlights the influence of various physical parameters on velocity and temperature profiles, demonstrating the significance of thermal radiation and viscous dissipation in heat transfer enhancement.

Govindaraj N., Singh A. K., Shukla P. [6], investigates fluid flow and heat transfer over a stretching sheet, considering temperature-dependent variations in Prandtl number and viscosity. Using similarity transformations, the governing partial differential equations are converted into non-dimensional form and solved numerically. The research highlights how viscosity and Prandtl number influence velocity and temperature profiles, demonstrating that viscosity decreases with increasing temperature while Prandtl number follows an inverse linear relationship.

Swain S., Sarkar G. M., Sahoo B. [7], explores the flow and heat transfer characteristics of a special third-grade fluid over a stretchable surface in a parallel free stream. Using Lie group analysis, the governing equations are transformed into nonlinear ordinary differential equations and solved numerically. The investigation highlights the existence of dual solutions under certain conditions, emphasizing the impact of stretching and suction parameters on fluid behavior.

Jain, Nita and Timol, M. G. [8], study focuses on the similarity solutions of quasi three-dimensional power-law fluids using the Method of Satisfaction of Asymptotic Boundary Conditions (MSABC). By applying deductive group-theoretic transformations, the governing nonlinear partial differential equations are reduced to a self-similar form, enabling numerical solutions. The research examines steady, laminar, incompressible boundary layer flow and highlights the influence of power-law fluid properties on skin friction and velocity profiles. The findings contribute to theoretical advancements in fluid mechanics, particularly in predicting boundary layer behavior for complex aerodynamic configurations.

Darji R. M. and Timol M.G. [9], studied deductive group-theoretic analysis to investigate magnetohydrodynamic (MHD) flow of a Sisko fluid through a porous medium. By employing similarity transformations, the governing nonlinear differential equations are reduced to a more manageable form and solved numerically. The research highlights the influence of fluid parameters on velocity and temperature distributions, contributing to advancements in non-Newtonian fluid mechanics and industrial applications involving porous media.

Sonawane P. [10], investigates the behavior of non-Newtonian Reiner–Philippoff fluids through a detailed analysis of boundary layer flow using a similarity transformation technique. The impact of a dimensionless fluid parameter γ is examined on three key flow characteristics.

Timol, M. G. [11], explores similarity methods for analyzing laminar forced convection over a horizontal plate. By employing group-theoretic transformations, the governing nonlinear partial differential equations are reduced to a more manageable form and solved numerically. The research compares different similarity transformation techniques, highlighting their effectiveness in simplifying complex fluid flow problems. The findings contribute to advancements in heat transfer analysis and engineering applications.

Mukhopadhyay, S., Layek, J. C. and Samad, S. A. [12], examined magnetohydrodynamic boundary-layer flow over a heated stretching sheet with variable viscosity. The fluid viscosity is model as a function of temperature, influencing velocity and thermal profiles. Using similarity transformations, the governing equations are reduced to ordinary differential equations and solved numerically. The findings provide insights into optimizing heat transfer in industrial applications involving electrically conducting fluids.

Patel M., Timol M. G. [13], study investigates the steady two-dimensional magneto-hydrodynamic (MHD) stagnation point flow of a power-law fluid over a stretching surface. The surface is stretched in its own plane with a velocity proportional to the distance from the stagnation point, while the fluid impinges orthogonally. Numerical and analytical solutions are obtained for different cases, highlighting the influence of fluid parameters on velocity and temperature distributions. The findings contribute to advancements in non-Newtonian fluid mechanics and industrial applications involving electrically conducting fluids.

Jain N.R., Timol M. G. [14] analyzed the velocity and temperature distribution in the flow of a viscous incompressible Prandtl fluid over a stretching permeable surface. The governing partial differential equations are transformed into ordinary differential equations using deductive group transformation, and numerical solutions are obtained through the shooting method. The results, presented graphically, highlight the influence of fluid parameters on velocity and temperature profiles, as well as the corresponding skin friction coefficient and Nusselt number

So motivated by Jain N.R., Timol M.G. [14], investigate by scaling group transformation technique, the scaling symmetry is systematically determined, leading to the identification of invariant solutions and corresponding similarity variables. The resulting boundary value problem is solved using MATLAB's ODE solver. The study examines the effects of various flow parameters on velocity and temperature distributions, while also evaluating the variations in skin friction coefficient and Nusselt number. These analyses highlight the role of suction/injection and thermophysical parameters in shaping the boundary layer characteristics.

II Mathematical Formulation

Consider the steady, two-dimensional boundary layer flow of an incompressible, viscous-inelastic Prandtl non-Newtonian fluid over a stretching, permeable surface and applying stream function $u = \frac{\partial \psi}{\partial y}$ and $v = -\frac{\partial \psi}{\partial x}$. Introducing dimensionless quantities and equation of continuity is identical as by Jain and Timol M.G. [14] are given by

$$\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial x\partial y}-\frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y^2}=\frac{\partial}{\partial y}(\tau_{yx}) \tag{1}$$

$$\frac{\partial\psi}{\partial y}\frac{\partial\theta}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial\theta}{\partial y}=\frac{1}{Pr}\frac{\partial^2\theta}{\partial y^2} \tag{2}$$

Therefore, shearing stress is,

$$\xi\left(\tau_{yx},\frac{\partial^2\psi}{\partial y^2}\right)=0 \tag{3}$$

With boundary conditions

$$For\ y=0 \qquad \frac{\partial\psi}{\partial y}=u_w, \qquad \frac{\partial\psi}{\partial x}=C, \qquad \theta=1 \tag{4}$$

$$For\ y=\infty \qquad \frac{\partial\psi}{\partial y}=0\,, \qquad \theta=0 \tag{5}$$

III Method of Solution for mathematical formulation

Reformulate the system of partial differential equations as a system of ordinary differential equations by applying a one-parameter transformation group.

$$x=\mathfrak{L}^{\beta_1}x^*, \qquad y=\mathfrak{L}^{\beta_2}y^*, \qquad \psi=\mathfrak{L}^{\beta_3}\psi^*$$
$$\tau_{yx}=\mathfrak{L}^{\beta_4}\tau_{yx}^*\,, \qquad \theta=\mathfrak{L}^{\zeta_2}\theta^* \tag{6}$$

Where $\beta_1,\ \beta_2,\beta_3,\beta_4,\ \beta_5\ and\ \mathfrak{L}$ are constants.

By deriving from equation (6), we obtain

$$\left(\frac{x}{x^*}\right)^{\frac{1}{\beta_1}}=\left(\frac{y}{y^*}\right)^{\frac{1}{\beta_2}}=\left(\frac{\psi}{\psi^*}\right)^{\frac{1}{\beta_3}}=\left(\frac{\tau_{yx}}{\tau_{yx}^*}\right)^{\frac{1}{\beta_4}}=\left(\frac{\theta}{\theta^*}\right)^{\frac{1}{\beta_5}}=\mathfrak{L} \tag{7}$$

By applying the linear transformation defined in equation (6) to equations (1) and (2), we find that the differential equations remain completely invariant under this transformation. Assuming the exponent α_1 is nonzero, we then derive the following relationships among the exponents as

$$\beta=\beta_1=-3\beta_2=\frac{3}{2}\beta_3=\beta_4=\beta_5 \tag{8}$$

Substituting equation (8) into equation (7) yields the following result. This substitution gives rise to the derived expression.

$$\eta=yx^{-\frac{1}{3}}, \qquad \psi=x^{\frac{2}{3}}F_1(\eta), \qquad \theta=F_2(\eta), \qquad \tau_{yx}=F_3(\eta) \tag{9}$$

IV Formulation of Ordinary Differential Equations

By applying the similarity variables defined in equation (9) to equations (1) through (5), the original system is transformed into a set of ordinary differential equations (ODEs). This transformation preserves the essential properties of the system, ensuring that the resulting ODEs faithfully represent the behavior of the original equations under the new variables are

$$(F_1'(\eta))^2-2F_1(\eta)F_1''(\eta)-3F_3'(\eta)=0 \tag{10}$$

$$(F_2''(\eta))^2-\frac{2}{3}F_1(\eta)F_2'(\eta)=0 \tag{11}$$

$$\xi\big(F_3(\eta),F_1''(\eta)\big)=0 \tag{12}$$

With boundary conditions

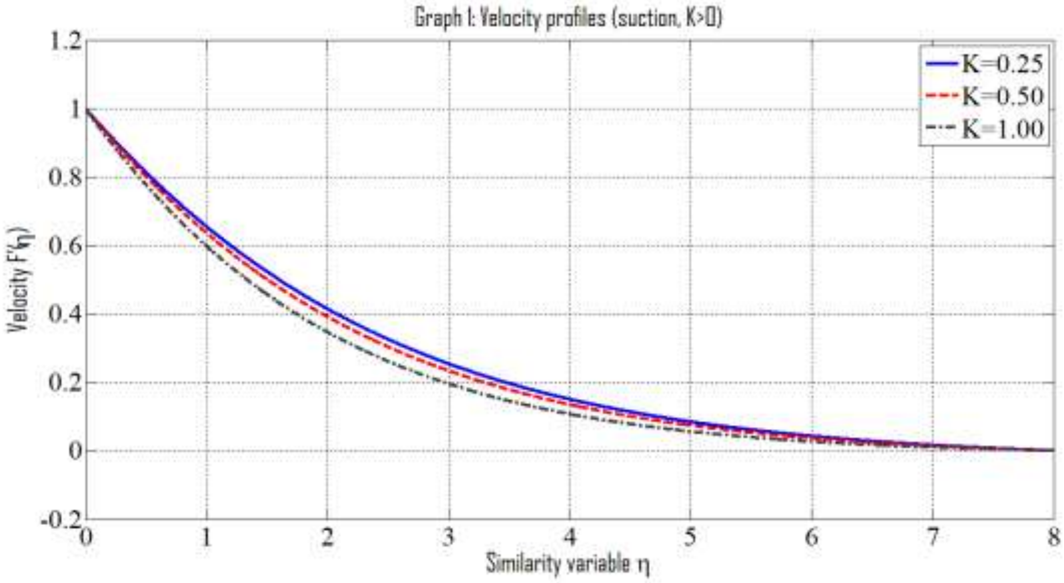
$$\eta=0, \qquad F_1(\eta)=C, \qquad F_1'(\eta)=1, \qquad F_2(\eta)=1 \tag{13}$$

$$\eta=\infty, \qquad F_1'(\eta)=0, \quad F_2'(\eta)=0 \tag{14}$$

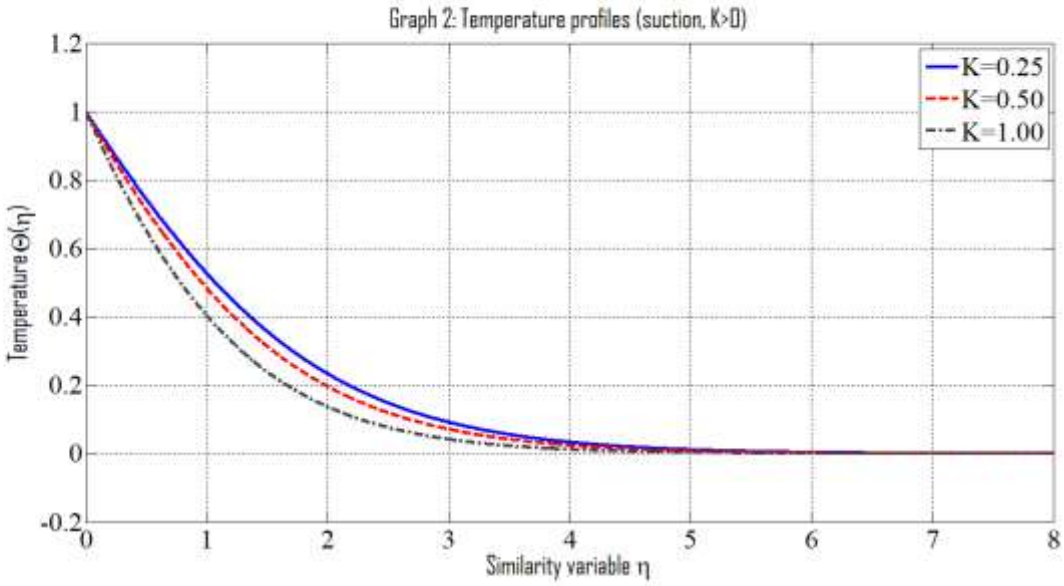
V Results and Discussion

The transformed system of nonlinear ordinary differential equations (10) – (12), subject to the boundary conditions (13)-(14), is solved using MATLAB's ODE solver. The resulting graphical plots depict the behavior and variation of the flow parameters, providing insights into the dynamics of the system.

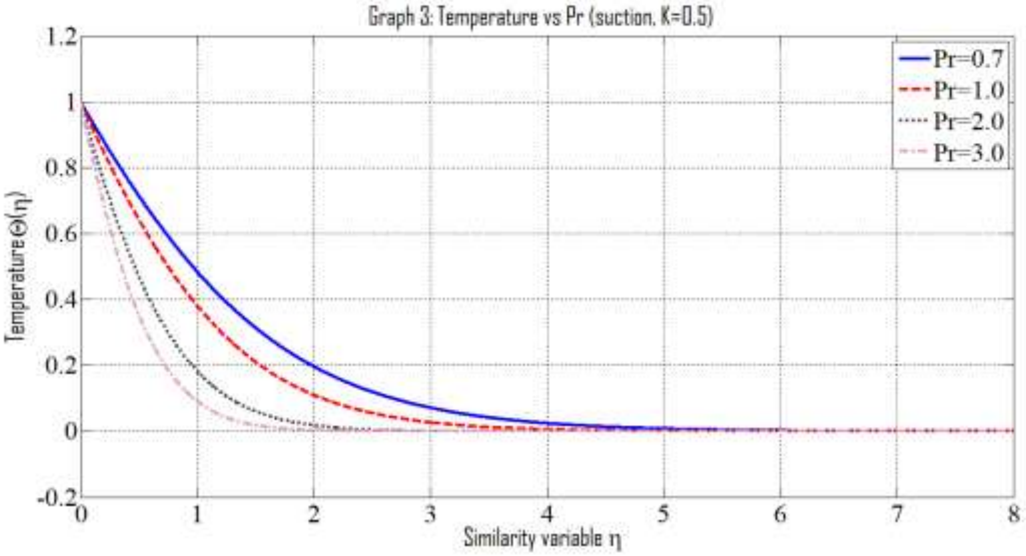
The graph (1) illustrates the **velocity profiles F′(η)** of a fluid subject to **suction** ($\boldsymbol{K>0}$) as a function of the similarity variable η. Three different values of the suction parameter are considered: $K=0.25, K=0.50$, and $K=1.00$. In all cases, the velocity decreases monotonically with increasing η, approaching zero asymptotically, which indicates that the fluid velocity diminishes far away from the surface due to viscous effects. A higher suction parameter (K) enhances the rate of velocity decay, meaning that fluid suction at the boundary layer suppresses velocity more rapidly and reduces the boundary layer thickness. This behavior is consistent with the physical expectation that stronger suction stabilizes the boundary layer by drawing fluid particles closer to the surface and weakening the velocity field farther away. The use of different line styles and colors clearly distinguishes the impact of each suction level, with higher K values producing sharper velocity reduction.



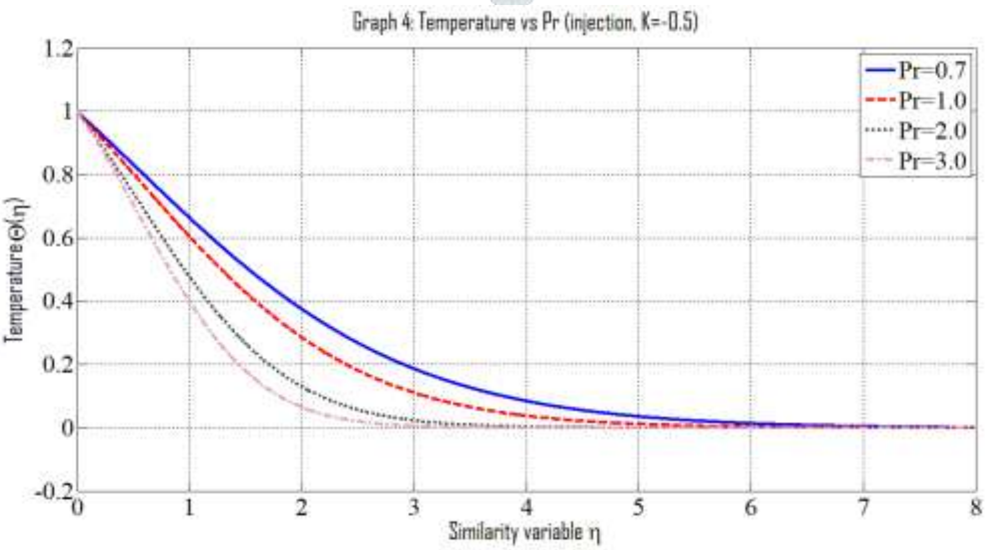
The graph (2) presents the **temperature profiles $\boldsymbol{\theta}$(η)** of a fluid under *suction* $(\boldsymbol{K > 0})$ plotted against the similarity variable η. It shows results for three suction parameter values: $K = 0.25, K = 0.50$ and $K = 1.00$. In all cases, the temperature decreases rapidly from its maximum at the surface $(\eta = 0)$ toward zero as η increases, signifying that thermal energy dissipates away from the wall into the fluid. As the suction parameter K increases, the temperature decays faster, resulting in thinner thermal boundary layers. This is because stronger suction removes heated fluid near the wall more effectively, thereby enhancing cooling and suppressing temperature rise in the boundary layer region. Consequently, higher suction levels improve thermal stability by reducing heat penetration into the fluid.



The graph (3) illustrates the **temperature profiles $\theta(\eta)$** of a fluid for different **Prandtl numbers (Pr)** under **suction conditions ($K = 0.5$)**, plotted against the similarity variable η. Four values of Pr are shown $Pr = 0.7, 1.0, 2.0$ and 3.0. In all cases, the temperature decreases from unity at the wall ($\eta = 0$) to zero as η increases, indicating heat dissipation away from the surface. As the Prandtl number increases, the temperature falls more steeply and the thermal boundary layer becomes thinner. This happens because higher Prandtl numbers correspond to fluids with lower thermal diffusivity, meaning heat is conducted less effectively and is confined closer to the wall. Conversely, at lower Pr ($e.g., Pr = 0.7$), the thermal boundary layer is thicker, reflecting greater thermal diffusion into the fluid. Overall, the graph highlights the significant role of the Prandtl number in controlling heat transfer characteristics in the boundary layer.



The graph (4) shows the **temperature profiles θ(η)** for different **Prandtl numbers** (Pr) under **injection conditions (K=−0.5)**, plotted against the similarity variable η. Four cases are presented: Pr=0.7,1.0,2.0 and 3.0. In all profiles, temperature starts at unity at the wall (η=0) and decays toward zero with increasing η, reflecting the diffusion of heat into the fluid. As the Prandtl number increases, the thermal boundary layer becomes thinner, and the temperature falls more rapidly. This is because higher Pr corresponds to lower thermal diffusivity, restricting heat conduction to a narrow region near the wall. In contrast, lower Pr fluids (e.g., Pr=0.7) exhibit thicker thermal boundary layers due to stronger heat diffusion. The effect of **injection** (negative K) is to push more heated fluid away from the wall, which slightly broadens the temperature distribution compared to suction cases, but the overall influence of Pr remains dominant in controlling the rate of thermal decay.



Conclusion

In this study, the boundary layer flow and heat transfer of pseudoplastic Prandtl fluids over a stretching permeable sheet were analyzed using scaling group transformation techniques. The governing nonlinear partial differential equations were reduced to similarity forms and solved numerically to examine the effects of suction/injection and thermophysical parameters on velocity and temperature fields.

The results demonstrate that suction enhances boundary layer stability by accelerating velocity and temperature decay, thereby reducing both momentum and thermal boundary layer thicknesses. Conversely, injection broadens the thermal boundary layer by driving heated fluid away from the surface, though the influence of the Prandtl number remains the dominant factor in heat transfer behavior. Higher Prandtl numbers yield sharper temperature gradients and thinner thermal layers due to reduced thermal diffusivity, while lower values promote stronger thermal diffusion.

Overall, the findings highlight the critical roles of suction/injection and the Prandtl number in shaping boundary layer characteristics of pseudoplastic Prandtl fluids. These insights not only strengthen the theoretical understanding of non-Newtonian fluid dynamics but also provide practical implications for industrial processes such as polymer extrusion, cooling technologies, and coating flows, where control of boundary layer stability and heat transfer efficiency is essential.

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