



## 4.666 based expressions

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### Abstract

Maclaurin series and series of Euler number are basic series and are taught all over the world to children. The series based on Maclaurin and Euler number are fundamental in nature. But an important development to the Maclaurin series and Euler number is the use of consecutive numbers. Consecutive numbers are not just a mindset of all of us. They help in the development of functions similar to Maclaurin series and Euler number. 4.666 is the number that we get when consecutive numbers are used following the Maclaurin series and the Euler Number e series. In this paper the basic Maclaurin series and e series are outlined and also rigorous explanation and the similarity to the consecutive number based 4.666 series are explained.

### Keywords

Maclaurin series, Euler number e, e series, 4.666 series, sin(x) series, cos(x) series, one(x) series, zero(x) series

### Introduction

Maclaurin made series named after him after considering the Taylor series around zero. Some of the Maclaurin series are sin(x), cos(x) and log(x). The following are the expressions of  $e^x$ , ln(x), cos(x) and sin(x)

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$\ln(x) = \sum_{n=1}^{\infty} (-1)^{(n+1)} \frac{(x-1)^n}{n}$$

$$\cos(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$$

$$\sin(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{(2n+1)}}{(2n+1)!}$$

While working on the relationship between consecutive numbers and factorials, the following was observed. This is an approximate expression and works in selected range only.

$$(2n)! \approx \left( \frac{(n+1)(n+2)}{2} \right)^n 3.3^{(-0.00000002375*n^4 + 0.00000596458*n^3 - 0.000595531*n^2 + 0.1006565*n - 0.588125)}$$

Then the factor  $\left( \frac{\left( \frac{(n+1)(n+2)}{2} \right)^n}{(2n)!} \right)$  was used in an infinite series with  $\left( \frac{1}{x} \right)^n$ .

This gave rise to new possibilities and results of great importance were obtained.

**4.666 to the power  $\frac{1}{x}$**

$$4.666^{\frac{1}{x}} \approx \sum_0^{\infty} \frac{\left( \frac{(n+1)(n+2)}{2} \right)^n}{(2n)!} \left( \frac{1}{x} \right)^n$$

**Log equivalent to 4.666 to the power  $\frac{1}{x}$**

$$4.666 \log = (x+1) \ln \left( \frac{x+1}{x} \right) - 1 = \sum_1^{\infty} (-1)^{(n+1)} \frac{\left( \frac{1}{x} \right)^n}{n(n+1)}$$

**One(x) function**

$$one(x) = \sum_0^{\infty} (-1)^n \frac{\left( \frac{(n+2)(n+3)}{2} \right)^{(2n)}}{(4n)!} \left( \frac{1}{x} \right)^{(2n)}$$

**Zero(x) function**

$$zero(x) = \sum_0^{\infty} (-1)^n \frac{\left( \frac{(n+3)(n+4)}{2} \right)^{(2n+1)}}{(4n+2)!} \left( \frac{1}{x} \right)^{(2n+1)}$$

**Integral for 2n factorial**

$$(2n)! \approx \left( \frac{(n+1)(n+2)}{2} \right)^n \int \frac{1}{4.666^x} \frac{1}{x^n} dx \left( 0.915961 - \frac{1654391}{\left( 1 + \left( \frac{n}{0.000001011} \right)^{0.8329756} \right)} \right)$$

**Ln(x) function**

$$\ln(x) \approx \sum_{n=1}^x \frac{0.5+n}{(n(n+1))} - 0.077$$

4.666 to the power  $\frac{1}{x}$

$$4.666^{\frac{1}{x}} \approx (x+1) \left( \frac{0.544 + (x-1)}{(x-1)x} \right)$$

## Wolfram Alpha Screenshots

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**WolframAlpha**

sum of  $\frac{\left(\left(\frac{(n+1)(n+2)}{2}\right)^n\right)}{(2n)!} \left(\frac{1}{115}\right)^n$ , for n=0 to infinity

NATURAL LANGUAGE MATH INPUT

★  $\sqrt{\quad}$   $\partial f$   $(::)$   $\sqrt[n]{\quad}$   $a_\omega$  ...

$\frac{\square}{\square}$   $\square^\square$   $\sqrt{\square}$   $\sqrt[n]{\square}$   $\sqrt[n]{\square}$   $\frac{d}{d\square}$   $\frac{d^2}{d\square^2}$   $\int \square$   $\int \square$   $\sum \square$   $\lim_{\square \rightarrow \square}$   $[a,b,c]$   $\left(\begin{smallmatrix} a & b & c \\ d & e & f \end{smallmatrix}\right)$

Input interpretation

$$\sum_{n=0}^{\infty} \frac{\left(\frac{1}{2}((n+1)(n+2))\right)^n}{(2n)!} \left(\frac{1}{115}\right)^n$$

$n!$  is the factorial function

$$\sum_{n=0}^{\infty} \frac{\left(\frac{1}{2}(n+1)(n+2)\right)^n \left(\frac{1}{115}\right)^n}{(2n)!} = 1.01316$$

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**WolframAlpha**

$(4.666)^{\frac{1}{115}}$

NATURAL LANGUAGE MATH INPUT

★  $\sqrt{\quad}$   $\partial f$   $(::)$   $\sqrt[n]{\quad}$   $a_\omega$  ...

$\frac{\square}{\square}$   $\square^\square$   $\sqrt{\square}$   $\sqrt[n]{\square}$   $\sqrt[n]{\square}$   $\frac{d}{d\square}$   $\frac{d^2}{d\square^2}$   $\int \square$   $\int \square$   $\sum \square$   $\lim_{\square \rightarrow \square}$   $[a,b,c]$   $\left(\begin{smallmatrix} a & b & c \\ d & e & f \end{smallmatrix}\right)$

Input

$$\sqrt[115]{4.666}$$

Result

1.0134840...

More digits

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sum of  $\frac{\left(\left(\frac{(n+1)(n+2)}{2}\right)^n\right)}{(2n)!} \left(-\frac{1}{215}\right)^n$ , for n = 0 to infinity

NATURAL LANGUAGE MATH INPUT

$\frac{\square}{\square}$ 
 $\square^\square$ 
 $\sqrt{\square}$ 
 $\sqrt[n]{\square}$ 
 $\sqrt[n]{\square}$ 
 $\frac{d}{d\square}$ 
 $\frac{d^2}{d\square^2}$ 
 $\int \square$ 
 $\int \square$ 
 $\sum \square$ 
 $\lim_{\square \rightarrow \square}$ 
 $[\square, \square, \square]$ 
 $\left(\begin{smallmatrix} \square & \square & \square \\ \square & \square & \square \end{smallmatrix}\right)$

Input interpretation

$$\sum_{n=0}^{\infty} \frac{\left(\frac{1}{2}(n+1)(n+2)\right)^n}{(2n)!} \left(-\frac{1}{215}\right)^n$$

$n!$  is the factorial function

$$\sum_{n=0}^{\infty} \frac{\left(\frac{1}{2}(n+1)(n+2)\right)^n \left(-\frac{1}{215}\right)^n}{(2n)!} = 0.993056$$

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$(4.666)^{-\frac{1}{215}}$

NATURAL LANGUAGE MATH INPUT

$\frac{\square}{\square}$ 
 $\square^\square$ 
 $\sqrt{\square}$ 
 $\sqrt[n]{\square}$ 
 $\sqrt[n]{\square}$ 
 $\frac{d}{d\square}$ 
 $\frac{d^2}{d\square^2}$ 
 $\int \square$ 
 $\int \square$ 
 $\sum \square$ 
 $\lim_{\square \rightarrow \square}$ 
 $[\square, \square, \square]$ 
 $\left(\begin{smallmatrix} \square & \square & \square \\ \square & \square & \square \end{smallmatrix}\right)$

Input

$4.666^{-1/215}$

Result

0.99286141...

More digits

sum of  $(-1)^{(n+1)} \frac{\left(\frac{1}{x}\right)^n}{n(n+1)}$ , for n = 1 to infinity

⚙️ NATURAL LANGUAGE
🔢 MATH INPUT

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$\frac{\square}{\square}$ 
 $\square^\square$ 
 $\sqrt{\square}$ 
 $\sqrt[n]{\square}$ 
 $\frac{d}{d\square}$ 
 $\frac{d^2}{d\square^2}$ 
 $\int \square$ 
 $\int \square^\square$ 
 $\sum \square$ 
 $\lim_{\square \rightarrow \square}$ 
 $[\square, \square, \square]$ 
 $\left(\begin{smallmatrix} \square & \square & \square \\ \square & \square & \square \end{smallmatrix}\right)$

Input interpretation

$$\sum_{n=1}^{\infty} (-1)^{n+1} \times \frac{\left(\frac{1}{x}\right)^n}{n(n+1)}$$

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1} \left(\frac{1}{x}\right)^n}{n(n+1)} = x \log\left(\frac{x+1}{x}\right) + \log\left(\frac{x+1}{x}\right) - 1 \quad \text{when } \frac{1}{|x|} < 1$$

log(x) is the natural logarithm  
|z| is the absolute value of z

Result

$$(x+1) \log\left(\frac{1}{x} + 1\right) - 1 \quad \text{for } |x| > 1$$

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$$\frac{0.1}{\left( \left( (4.666 \times 888 + 1) \ln\left(\frac{4.666 \times 888 + 1}{4.666 \times 888}\right) - 1 \right) \left(\frac{4.666}{5}\right) \right)}$$

⚙️ NATURAL LANGUAGE
🔢 MATH INPUT

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$\frac{\square}{\square}$ 
 $\square^\square$ 
 $\sqrt{\square}$ 
 $\sqrt[n]{\square}$ 
 $\frac{d}{d\square}$ 
 $\frac{d^2}{d\square^2}$ 
 $\int \square$ 
 $\int \square^\square$ 
 $\sum \square$ 
 $\lim_{\square \rightarrow \square}$ 
 $[\square, \square, \square]$ 
 $\left(\begin{smallmatrix} \square & \square & \square \\ \square & \square & \square \end{smallmatrix}\right)$

Input

$$\frac{0.1}{\left( (4.666 \times 888 + 1) \log\left(\frac{4.666 \times 888 + 1}{4.666 \times 888}\right) - 1 \right) \times \frac{4.666}{5}}$$

log(x) is the natural logarithm

Result

888.071435903700043253975290116478642153340813493429129951725826  
923666364661...

More digits

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$$(4.666)^{\left(\frac{1}{\ln(4.666)} \cdot \ln\left(\frac{0.5}{778 \cdot \ln\left(\frac{778}{777}\right) - 1}\right)\right)}$$

NATURAL LANGUAGE  $\int_0^\pi$  MATH INPUT

$\frac{\square}{\square}$   $\square^\square$   $\sqrt{\square}$   $\sqrt[3]{\square}$   $\sqrt[n]{\square}$   $\frac{d}{d\square}$   $\frac{d^2}{d\square^2}$   $\int \square$   $\int \square^\square$   $\sum \square$   $\lim_{\square \rightarrow \square}$   $[\square, \square, \square]$   $\left(\begin{smallmatrix} \square & \square & \square \\ \square & \square & \square \end{smallmatrix}\right)$

Input

$$4.666^{1/\log(4.666) \log(0.5 / (778 \log(\frac{778}{777}) - 1))}$$

log(x) is the natural logarithm

Result More digits

777.333...



$$\text{sum of } (-1)^n \frac{\left(\left(\frac{(2n+1)(2n+2)}{2}\right)^{(2n)}\right)}{(4n)!} \left(\frac{1}{376}\right)^{2n}, \text{ for } n = 0 \text{ to infinity}$$

NATURAL LANGUAGE  $\int_0^\pi$  MATH INPUT

$\frac{\square}{\square}$   $\square^\square$   $\sqrt{\square}$   $\sqrt[3]{\square}$   $\sqrt[n]{\square}$   $\frac{d}{d\square}$   $\frac{d^2}{d\square^2}$   $\int \square$   $\int \square^\square$   $\sum \square$   $\lim_{\square \rightarrow \square}$   $[\square, \square, \square]$   $\left(\begin{smallmatrix} \square & \square & \square \\ \square & \square & \square \end{smallmatrix}\right)$

Input interpretation

$$\sum_{n=0}^{\infty} (-1)^n \times \frac{\left(\frac{1}{2} ((2n+1)(2n+2))\right)^{2n}}{(4n)!} \left(\frac{1}{376}\right)^{2n}$$

n! is the factorial function

$$\sum_{n=0}^{\infty} \frac{(-1)^n \left(\frac{1}{2} (2n+1)(2n+2)\right)^{2n} \left(\frac{1}{376}\right)^{2n}}{(4n)!} = 0.999989$$



$$\text{sum of } (-1)^n \frac{\left( \left( \frac{(2n+2)(2n+3)}{2} \right)^{(2n+1)} \right)}{(4n+2)!} \left( \frac{1}{376} \right)^{2n+1}, \text{ for } n = 0 \text{ to infinity}$$

 NATURAL LANGUAGE

 MATH INPUT

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Input interpretation

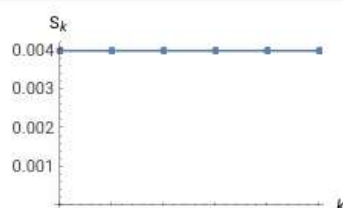
$$\sum_{n=0}^{\infty} (-1)^n \times \frac{\left( \frac{1}{2} ((2n+2)(2n+3)) \right)^{2n+1}}{(4n+2)!} \left( \frac{1}{376} \right)^{2n+1}$$

$n!$  is the factorial function

Partial sums

More terms

Show points





$$\left( \left( \frac{(71+1)(71+2)}{2} \right)^{71} \right) \int_{0.845217}^{\text{infinity}} \frac{1}{4.666^x} \left( \frac{1}{x^{71}} \right) dx$$

$$\left( 0.9159651 - \frac{1654391}{1 + \left( \frac{71}{0.0000001011} \right)^{0.8329756}} \right)$$


 NATURAL LANGUAGE

 MATH INPUT

★      











Assuming multiplication | Use [a list](#) instead

Input interpretation

$$\left( \frac{1}{2} ((71+1)(71+2)) \right)^{71} \int_{0.845217}^{\infty} \frac{1}{4.666^x} \times \frac{1}{x^{71}} dx$$

Computation result

$$\left( \frac{1}{2} ((71+1)(71+2)) \right)^{71} \int_{0.845217}^{\infty} \frac{1}{4.666^x x^{71}} dx = 3.06902 \times 10^{245}$$

142!

NATURAL LANGUAGE

MATH INPUT

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$\frac{\square}{\square}$

$\square^\square$

$\sqrt{\square}$

$\sqrt[3]{\square}$

$\sqrt[n]{\square}$

$\frac{d}{d\square}$

$\frac{d^2}{d\square^2}$

$\int \square$

$\int \square^\square$

$\sum \square$

$\lim_{\square \rightarrow \square} \square$

$[ \square, \square, \square ]$

$\begin{pmatrix} \square & \square & \square \\ \square & \square & \square \end{pmatrix}$

Input

142!

n! is the factorial function.

Result

269 536 413 788 816 277 658 850 750 803 729 026 709 400 851 689 139 611 079`  
208 959 973 446 471 469 886 705 721 287 973 193 419 489 734 024 361 060 974`  
102 771 686 730 776 482 217 285 444 779 897 586 125 484 868 294 790 044 340`  
222 989 800 738 079 091 821 598 557 128 192 667 156 480 000 000 000 000 000`  
000 000 000 000 000 000

Decimal form

More digits

2.69536413788816277658850750803729026709400851689139611079208959`  
97344... × 10<sup>245</sup>

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(sum of  $\frac{(0.5+n)}{n(n+1)}$ , for n = 1 to 1596)-0.077

×

=

NATURAL LANGUAGE

MATH INPUT

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(::)

√<sub>v</sub>

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...

$\frac{\square}{\square}$

$\square^\square$

$\sqrt{\square}$

$\sqrt[3]{\square}$

$\sqrt[n]{\square}$

$\frac{d}{d\square}$

$\frac{d^2}{d\square^2}$

$\int \square$

$\int \square^\square$

$\sum \square$

$\lim_{\square \rightarrow \square} \square$

$[ \square, \square, \square ]$

$\begin{pmatrix} \square & \square & \square \\ \square & \square & \square \end{pmatrix}$

Input interpretation

$$\sum_{n=1}^{1596} \frac{0.5+n}{n(n+1)} - 0.077$$

Result

7.3761

JETIR2509424

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e231



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Input:  $\ln(1596)$

NATURAL LANGUAGE MATH INPUT

Buttons:  $\frac{\square}{\square}$ ,  $\square^\square$ ,  $\sqrt{\square}$ ,  $\sqrt[\square]{\square}$ ,  $\sqrt[\square]{\square}$ ,  $\frac{d}{d\square}$ ,  $\frac{d^2}{d\square^2}$ ,  $\int \square$ ,  $\int \square^\square$ ,  $\sum \square$ ,  $\lim_{\square \rightarrow \square}$ ,  $[\square, \square, \square]$ ,  $\left( \begin{smallmatrix} \square & \square & \square \\ \square & \square & \square \end{smallmatrix} \right)$

Input

$\log(1596)$

$\log(x)$  is the natural logarithm.

Decimal approximation More digits

7.37525577800975407534408965516991210767295481742424996697340473...  
897115410197...

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Input:  $(2298) \frac{(0.5+2296)}{(2296*2297)}$

NATURAL LANGUAGE MATH INPUT

Buttons:  $\frac{\square}{\square}$ ,  $\square^\square$ ,  $\sqrt{\square}$ ,  $\sqrt[\square]{\square}$ ,  $\sqrt[\square]{\square}$ ,  $\frac{d}{d\square}$ ,  $\frac{d^2}{d\square^2}$ ,  $\int \square$ ,  $\int \square^\square$ ,  $\sum \square$ ,  $\lim_{\square \rightarrow \square}$ ,  $[\square, \square, \square]$ ,  $\left( \begin{smallmatrix} \square & \square & \square \\ \square & \square & \square \end{smallmatrix} \right)$

Input

$2298 \times \frac{0.5 + 2296}{2296 \times 2297}$

Result More digits

1.00065321529824540113676527025858603632370050922351377876612275...  
669370289075...

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## Conclusion

In total five expressions have been submitted in this paper which are similar to Maclaurin series and e series. The expressions have been tested in Wolframalpha. The results have been found to be valid in most ranges of natural numbers. The one(x) and zero(x) numbers are very interesting and their values goes towards 1 and 0 respectively as the value of x increases. The 4.666 to the power  $\frac{1}{x}$  and its log equivalent show a surprising similarity to e function and log function relationship.

## References

1. [www.wolframalpha.com](http://www.wolframalpha.com)
2. [www.wikipedia.org](http://www.wikipedia.org)