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Performance Analysis of M^X/ (G_1, G_2)/1 Queue with Repairable Server, Bernoulli Vacation Schedule under N-Policy

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Abstract

Proposed work aims to analyze M^x/G/1 queuing system for unreliable server queue with two phases of heterogeneous services one after the other to the arriving batch under Bernoulli schedule vacation. The server operates under N-policy according to which he remains idle till queue size becomes N ($\geq$1); i.e. when N customers are accumulated in the system, the server is immediately turn on and provides service. While servicing the customers the server may breakdown and is sent for repair immediately. As soon as the server is repaired, it starts to serve the customers. After completion of both phases of service, the server either goes for a vacation or may continue to serve the next customer, if any. The server's vacations are based on Bernoulli schedule under a single vacation policy. We obtain explicit queue size distribution at random epoch as well as at the departure epoch under the steady state conditions. The numerical results for various performance measures are summarized in tables and displayed via graphs.

Keywords: $M^X/(G_1,G_2)/1$ queue, Bernoulli schedule, Vacation, N-Policy, Unreliable Server, Phase Services, Supplementary variable, Queue Size.

1. Introduction

The single server queues in different frameworks have been studied by numerous authors including Burke (1975), Choudhury (1979), Madan (2000) and Medhi (2002), many others. N-Policy is highly applicable to the queueing systems where the server's closedown, startup and setup costs are high and it is economically infeasible to start service until a certain minimum number of units are accumulated. It is worthwhile to have a look on some important works done by researchers on queues under N-policy. Some characteristics of N-policy have been studied by Lee et al. (1994). Lee et al. (1995) considered an M^X/G/1 queueing system with N-Policy and single vacation. The effect of different arrival rates on the N-policy M/G/1 queue was examined by Hur and Paik (1999). Nobel and Tijms (1999) considered a practically important model with controllable service rate, where switch over times are involved while changing the service mode. Two phase queueing system with N-policy was considered by Kim and Park (2003). An M^X/G/1 queueing system with two phases of heterogeneous service under N-policy was studied by Choudhury and Paul (2004). The customers are assumed to receive the batch service in the first phase followed by individual service in the second phase. M/G/1 queueing system was considered by Lee and Kim (2006) where the speed of the server depends on the amount of work. Sikdar and Gupta (2008) considered an M^x/G^x/1/N queue to obtain various performance measures for single and multiple vacation models. Banik (2009) obtained queue length distributions at various epochs for a finite buffer single server queueing model under N-policy.

Various authors studied the queues with server vacation under various vacation policies including Bernoulli schedules. Various aspects of Bernoulli vacation model for single server queueing system have been studied by Keilson and Servi (1986). Bernoulli vacation model for two stage heterogeneous service queueing system has been studied by Madan (2001). Single server queues with Bernoulli vacation schedule and a general retrial time was considered by Kumar and Arivudainambi (2002). Madan et al. (2003) considered two models for a two server queue and single vacation with Bernoulli schedules. A batch arrival queueing system was studied by Choudhury and Madan (2004). The server was assumed to provide two phases of heterogeneous service in succession. Atencia and Moreno (2005) studied an M/G/1 retrial queue with general retrial times and Bernoulli schedule. A single server Poisson arrival queue along with Bernoulli schedule vacation has been examined by Choudhury and Paul (2006). Choudhury et al. (2007) studied the steady state behavior of batch arrival queue with two phases of heterogeneous service under multiple vacation policy. Choudhury (2008) obtained the queue size distribution of an $M^x/G/1$ queue with random set up time and Bernoulli vacation schedule under a restricted admissibility policy. Ke and Chang (2009) constructed a mathematical model for a batch arrival queue where the server provides two phases of heterogeneous service to all customers under Bernoulli vacation schedule.

In many real cases, the server may experience breakdowns, so that a more realistic queueing model is that which incorporates the assumption of unreliable server. The minimum expected cost and the optimal operating policy was investigated by Wang et al. (1999) for a single removable and non reliable server. Wang et al. (2005) used the maximum entropy principle to develop the approximate formulae for the probability distribution of the number of customers in a single removable server M/G/1 queueing system operating under N-policy. An M/G/1 G-queue with preemptive resume and feedback under N-policy where the server is subject to breakdown and repair was investigated by Liu et al. (2009). $M^x/G/1$ queue with two types of services and modified Bernoulli schedule server vacations is studied by Jain and Chauhan (2016). Maximum entropy analysis of unreliable queue with bernoulli vacation schedule is studied by Chauhan (2018). Ayyappan and Karpagam (2019) proposed an analysis of a bulk queue with unreliable server, immediate feedback, n-policy, bernoulli schedule multiple vacation and stand-by server. An unreliable retrial system with bernoulli schedule is proposed by Choudhury and Tadj (2020). Matrix-geometric solution of multi-server queueing systems with bernoulli scheduled modified vacation and retention of reneged customers is given by Shekhar et al. (2021). Analysis of a bulk arrival n-policy queue is considered by Begum and Chaudhury (2022) with two service genre, breakdown, delayed repair under bernoulli vacation and repeated service policy.

In the present paper, we consider $M^x/G/1$ queue with batch arrivals, two types of general heterogeneous service. The concept of Bernoulli vacation schedule and unreliable server are incorporated. The paper is organized as follows. In section 2, we define the underlying assumptions and notations of the system under study and also construct the steady state equations. The analysis based on supplementary variables and generating function approach, is given in section 3. The queue size distributions at random epoch and departure epoch are obtained in sections 4 and 5, respectively. Section 6 is meant for sensitivity analysis. In the last section 7, the conclusions are drawn.

2. Model Description

We consider an $M^X/G/1$ queueing system with the following assumptions:

- The customers arrive at the system according to a compound Poisson process with random batch size denoted by variable 'X'. Let λ be the mean arrival rate of the customers.
- When N customers are accumulated in the system, the server starts service to the customers.
- There is a single unreliable server who provides two kinds of general heterogeneous services in the sequence to the customers on a first come first served (FCFS) basis i.e. first stage service (FSS) followed by second stage service (SSS).
- As soon as the service of a customer is completed, the server may take a vacation with probability r or else with probability (1-r), he may continue servicing the next customer, if any. Otherwise the system is turned off.
- We assume that the service time random variable S_i (i=1,2) of the i^{th} type of service follows a general probability law with $S_i(x)$ as the distribution function. Let Laplace Stieltjes transform (LST) of $S_i(x)$ is $S_i^*(\theta)$ with finite k^{th} moment $E(S_i^k)$, $k\geq1$, i=1,2.

- The vacation time V of the server follows a general probability law with distribution function V(x), LST V*(θ) and finite moment $E(V^k)$, k=1,2.
- The server may breakdown while servicing the customer and it is sent for repair immediately. The repair time is generally distributed with probability distribution function $G_i(x)$, i=1,2 while server fails during i^{th} phase service. Immediately after the server is fixed, it starts to serve the customers.

Let $N_Q(t)$ be the queue size at time 't'. To make it Markov process we introduce supplementary variables $S_1^0(t)$, $S_2^0(t)$, $G_1^0(t)$, $G_2^0(t)$ and $V^0(t)$, where $S_1^0(t)$ and $S_2^0(t)$ be the elapsed FSS time and elapsed SSS time respectively at time 't', $G_1^0(t)$ and $G_2^0(t)$ be the elapsed repair times while the server failed during FSS and SSS, respectively and $V^0(t)$ be the elapsed vacation time at time 't'. Let the status of the server at time t is denoted by Y(t) as

$$Y(t)=\begin{cases}0, & server\ is\ idle\\ 1, & server\ is\ busy\ with\ FSS\\ 2, & server\ is\ busy\ with\ SSS\\ 3, & server\ breaks\ down\ while\ rendering\ FSS\\ 4, & server\ breaks\ down\ while\ rendering\ SSS\\ 5, & server\ is\ on\ vacation\end{cases}$$

Define the limiting probabilities as:

$$I_n = Lim_{t\to\infty}\Pr[N_Q(t)=n, Y(t)=0]$$

$$P_{1,n}(x)\,dx = Lim_{t\to\infty}\Pr[N_Q(t)=n,\ Y(t)=1, x<S_1^0(t)\le x+dx]$$

$$P_{2,n}(x)\,dx = Lim_{t\to\infty}\Pr[N_Q(t)=n, Y(t)=2, x<S_2^0(t)\le x+dx]$$

$$R_{1,n}(x,y)\,dxdy = Lim_{t\to\infty}\Pr[N_Q(t)=n, Y(t)=3, S_1^0(t)=x, y<R_1^0(t)\le y+dy]$$

$$R_2(x,y)\,dxdy = Lim_{t\to\infty}\Pr[N_Q(t)=n, Y(t)=4, S_2^0(t)=x, y<R_2^0(t)\le y+dy]$$

$$Q_n(x)\,dx = Lim_{t\to\infty}\Pr[N_Q(t)=n, Y(t)=5, x<V^0(t)\le x+dx]$$

Hazard rates are given by;

$$\upsilon(x)=\frac{dV(x)}{1-V(x)}\ ;\quad \mu_i(x)=\frac{dB_i(x)}{1-B_i(x)},\quad i=1,2\ ;\quad \beta_i(x)=\frac{dG_i(x)}{1-G_i(x)},\quad i=1,2$$

Steady State Equations

Now Chapman Kolmogorov equations governing the models are constructed as follows:

$$\lambda I_0 = \int_0^\infty \upsilon(x)Q_0(x)\,dx + (1-r)\int_0^\infty \mu_2(x)P_{2,1}(x)\,dx, \tag{1}$$

$$\lambda I_n = \lambda\sum_{i=1}^{n} a_i I_{n-i},\ n=1,2,\ldots,N-1 \tag{2}$$

$$\frac{d}{dx}P_{1,n}(x)+(\lambda+\mu_1(x)+\alpha_1)P_{1,n}(x)=\lambda\sum_{i=1}^{n}a_iP_{1,n-i}(x)+\int_0^\infty R_{1,n}(x,y)\,\beta_1(y)\,dy,\ \ n\ge 1 \tag{3}$$

$$\frac{d}{dx}P_{2,n}(x)+(\lambda+\mu_2(x)+\alpha_{21})P_{2,n}(x)=\lambda\sum_{i=1}^{n}a_iP_{2,n-i}(x)+\int_0^\infty R_{2,n}(x,y)\,\beta_{21}(y)\,dy,\ n\ge 1 \tag{4}$$

$$\frac{d}{dx}R_{1,n}(x,y)+(\lambda+\beta_1(y))R_{1,n}(x,y)=\lambda\sum_{i=1}^{n}a_iR_{1,n-i}(y),\ n\ge 1 \tag{5}$$

$$\frac{d}{dx}R_{2,n}(x,y)+(\lambda+\beta_2(y))R_{2,n}(x,y)=\lambda\sum_{i=1}^{n}a_iR_{2,,n-i}(y),\ n\ge 1 \tag{6}$$

$$\frac{d}{dx}Q_n(x)+(\lambda+\nu(x))Q_n(x)=\lambda\sum_{i=1}^{n}a_iQ_{n-i}(x), \qquad n\geq 1 \tag{7}$$

$$\frac{d}{dx}Q_0(x)+(\lambda+\nu(x))Q_0(x)=0 \tag{8}$$

These equations are to be solved subject to the following boundary conditions:

$$P_{1,n}(0)=(1-r)\int_0^\infty \mu_2(x)P_{2,n+1}(x)dx+\int_0^\infty \upsilon(x)Q_n(x)\,dx, \qquad n=1,2,\ldots,N-1 \tag{9}$$

$$P_{1,n}(0)=(1-r)\int_0^\infty \mu_2(x)P_{2,n+1}(x)dx+\int_0^\infty \upsilon(x)Q_n(x)\,dx+\lambda\sum_{i=0}^{n}a_iI_{n-i}, \quad n\geq N \tag{10}$$

$$P_{2,n}(0)=\int_0^\infty \mu_1(x)P_{1,n}(x)dx, \quad n\geq 1 \tag{11}$$

$$Q_n(0)=r\int_0^\infty \mu_2(x)P_{2,n+1}(x)dx, \quad n\geq 0 \tag{12}$$

$$R_{1,n}(x,0)=\alpha_1\,P_{1,n}(x), \quad n\geq 1 \tag{13}$$

$$R_{2,n}(x,0)=\alpha_2\,P_{2,n}(x), \ n\geq 1 \tag{14}$$

The normalizing condition yields

$$\sum_{n=0}^{N-1}I_n+\sum_{i=1}^{2}\sum_{n=1}^{\infty}\int_0^\infty P_{i,n}(x)dx+\sum_{i=1}^{2}\sum_{n=1}^{\infty}\int_0^\infty\int_0^\infty R_{i,n}(x,y)dx\,dy+\sum_{n=0}^{\infty}\int_0^\infty Q_n(x)dx=1 \tag{15}$$

Define the following generating functions:

$$P_i(x;z)=\sum_{n=1}^{\infty}z^nP_{i,n}(x), i=1,2;\ P_i(0;z)=\sum_{n=1}^{\infty}z^nP_{i,n}(0), i=1,2$$

$$R_i(x;y;z)=\sum_{n=1}^{\infty}z^nR_{i,n}(x,y), i=1,2;\ R_i(0;y;z)=\sum_{n=1}^{\infty}z^nR_{i,n}(0,y), i=1,2$$

$$Q(x;z)=\sum_{n=0}^{\infty}z^nQ_n(x);\quad Q(0;z)=\sum_{n=0}^{\infty}z^nQ_n(0);\quad R(z)=\sum_{n=0}^{\infty}z^nR_n$$

3. The Analysis

In this section, we obtain joint and marginal generating functions of queue size as follows:

Theorem 1: The joint probability generating functions when the server is busy with FSS and SSS, under breakdown while rendering service during FSS and SSS and on vacations respectively, are given by

$$P_1(x,z)=P_1(0,z)\exp[\{-\xi_1(z)\}x]\{1-B_1(x)\} \tag{16}$$

$$P_2(x,z)=P_2(0,z)\exp[\{-\xi_2(z)\}x]\{1-B_2(x)\} \tag{17}$$

$$R_1(x,y,z)=R_1(x,0,z)\exp[\{-\lambda(1-X(z)\}x]\{1-G_1(y)\} \tag{18}$$

$$R_2(x,y,z)=R_2(x,0,z)\exp[\{-\lambda(1-X(z)\}x]\{1-G_2(y)\} \tag{19}$$

$$Q(x,z)=Q(0,z)\exp[\{-\lambda(1-X(z)\}x]\{1-V(x)\} \tag{20}$$

where $\xi_i(z)=\lambda+\alpha_i-\lambda X(z)-\alpha_i g_i*(\lambda-\lambda X(z))$.

Theorem 2: The marginal generating functions are given by

$$P_1(z) = P_1(0,z)\left\{\frac{1-b^*(\xi_1(z))}{\xi_1(z)}\right\} \tag{21}$$

$$P_2(z) = P_1(0,z)b^*(\xi_1(z))\left\{\frac{1-b^*(\xi_2(z))}{\xi_2(z)}\right\} \tag{22}$$

$$R_1(z) = \alpha_1 P_1(0,z)\left\{\frac{1-b^*(\xi_1(z))}{\xi_1(z)}\right\}\left\{\frac{1-g_1^*(\lambda-\lambda X(z))}{(\lambda-\lambda X(z))}\right\} \tag{23}$$

$$R_2(z) = \alpha_2 P_1(0,z)b^*(\xi_1(z))\left\{\frac{1-b^*(\xi_2(z))}{\xi_2(z)}\right\}\left\{\frac{1-g_2^*(\lambda-\lambda X(z))}{(\lambda-\lambda X(z))}\right\} \tag{24}$$

$$Q(z) = rP_1(0,z)b^*(\xi_1(z)b^*(\xi_2(z))z^{-1}\left\{\frac{1-\upsilon^*(\lambda-\lambda X(z))}{(\lambda-\lambda X(z))}\right\} \tag{25}$$

where

$$P_1(0,z) = \frac{z\lambda I(z)\{X(z)-1\}}{z-\{(1-r)+r\upsilon^*(\lambda-\lambda X(z))\}b^*(\xi_1(z))b^*(\xi_2(z))} \tag{26}$$

and $I(z) = \dfrac{(1-\phi)\sum_{n=0}^{N-1} z^n \psi_n}{\sum_{n=0}^{N-1}\psi_n}$

$\phi = \lambda E[X]\{rE[V] - E[B_1](\alpha_1 E[G_1]+1) - E[B_2](\alpha_2 E[G_2]+1)\}$

Theorem 3: The probability generating function of the number of the customers in the queue is given by

$$P(z) = \frac{(1-z)I(z)\{1-r+r\upsilon^*(\lambda-\lambda X(z))\}b^*(\xi_1(z))b^*(\xi_2(z))}{\{(1-r)+r\upsilon^*(\lambda-\lambda X(z))\}b^*(\xi_1(z))b^*(\xi_2(z))-z} \tag{27}$$

4. Mean Queue Size at Random Epoch

Let ψ_n (n=0,1,2,…,N-1) be the probability that a batch of customers find at least n customers in the system during an idle period where ψ_n is given by the following recursive equation,

$$\psi_n = \sum_{i=1}^{n} a_i \psi_{n-i} \qquad (n = 0,1,2,\ldots,(N-1)) \text{ and } \qquad \psi_0=1 \tag{28}$$

$I_n=I_0\ \psi_n$, where I_0 is normalizing constant, therefore

$$I(Z) = I_0 \sum_{n=0}^{N-1} Z^n \psi_n \tag{29}$$

To determine I_0, we use normalizing condition P(1)=1 and get

I(z)=(1-Φ), thus

$$I(z) = \frac{(1-\phi)\sum_{n=0}^{N-1} Z^n \psi_n}{\sum_{n=0}^{N-1}\psi_n} \tag{30}$$

where $\phi = \lambda E[X]\{rE[V] - E[B_1](\alpha_1 E[G_1]+1) - E[B_2](\alpha_2 E[G_2]+1)\}$.

and $\sum_{n=0}^{N-1}\psi_n$ is the mean number of batches arriving during the idle period.

Let g_n be the probability that there are n customers in the system during the idle period, then

$$g_n = \frac{I_n}{\sum_{n=0}^{N-1} I_n} = \frac{\psi_n}{\sum_{n=0}^{N-1} \psi_n} \tag{31}$$

PGF of g_n is given by

$$G(z) = \sum_{n=0}^{N-1} z^n g_n = \frac{(1-\phi)\sum_{j=0}^{N-1} z^n \psi_n}{\sum_{n=0}^{N-1} \psi_n} \tag{32}$$

Therefore

$$I(z)=(1-\Phi)G(z) \tag{33}$$

Combining (27), (31) and (33), we have the following stochastic decomposition property:

$$P(z) = \frac{(1-z)I(z)\{(1-r)+rv^*(\lambda-\lambda X(z))\}b^*\xi_1(z)b^*\xi_2(z)}{\{(1-r)+rv^*(\lambda-\lambda X(z))\}b^*(\xi_1(z))b^*(\xi_2(z))-z}$$

$$= \frac{(1-z)(1-\phi)\{(1-r)+rv^*(\lambda-\lambda X(z))\}b^*\xi_1(z)b^*\xi_2(z)}{\{(1-r)+rv^*(\lambda-\lambda X(z))\}b^*(\xi_1(z))b^*(\xi_2(z))-z} \times \frac{\sum_{n=0}^{N-1} z^n \psi_n}{\sum_{n=0}^{N-1} \psi_n}$$

$$\equiv G(z)P_0(z) \tag{34}$$

where $G(z) = \frac{\sum_{n=0}^{N-1} z^n \psi_n}{\sum_{n=0}^{N-1} \psi_n}$.

and $P_0(z) = \frac{(1-z)(1-\phi)\{(1-r)+rv^*(\lambda-\lambda X(z))\}b^*\xi_1(z)b^*\xi_2(z)}{\{(1-r)+rv^*(\lambda-\lambda X(z))\}b^*(\xi_1(z))b^*(\xi_2(z))-z}$

The queue size distribution at random epoch given in equation (34) decomposes into the distributions of two independent random variables as

(i) First term $P_0(z)$ represents the queue size distribution of $M^x/G/1$ queue with vacation time under Bernoulli schedule.

(ii) Second term G(z) represents the additional queue size distribution due to N-policy.

The number of customers in the system can be obtained by using $L = \lim_{z\to 1} P'(z)$. Now the mean queue size at random epoch is:

$$L = \left.\frac{dP(z)}{dz}\right|_{z=1}$$

$$= \phi + \frac{\phi\lambda E(X(X-1))}{2(1-\phi)} + \frac{\lambda r E(X)E^2(V)}{2(1-\phi)} - \frac{\{E^2(B_1)(\alpha_1 E(G_1)+1)+E^2(B_2)(\alpha_2 E(G_2)+1)\}}{2(1-\phi)}$$

$$-\frac{\{\alpha_1 E(B_1)E^2(G_1)+\alpha_2 E(B_2)E^2(G_2)\}}{2(1-\phi)} - \frac{r\lambda^2 E^2(X)E(V)\{E(B_1)(\alpha_1 E(G_1)+1)+E(B_2)(\alpha_2 E(G_2)+1)\}}{(1-\phi)} + \frac{\sum_{j=0}^{N-1} n\psi_n}{\sum_{n=0}^{N-1}\psi_n} \tag{35}$$

Corollary 1: If the system is in steady state, then

Pr [The server is in idle state]=$\text{Lim}_{z\to 1} I(z) = (1-\phi)$ (36)

Pr [The server is busy with FSS]=$\text{Lim}_{z\to 1} P_1(z) = \lambda E[X]E[S_1]$ (37)

Pr [The server is busy with SSS]=$\text{Lim}_{z\to1}P_2(z)=\lambda E[X]E[S_2]$ (38)

Pr [The server is broken-down while FSS]=$\text{Lim}_{z\to1}R_1(z)=\alpha_1\lambda E[X]E[G]$ (39)

Pr [The server is broken-down while SSS]=$\text{Lim}_{z\to1}R_2(z)=\alpha_2\lambda E[X]E[G_2]$ (40)

Pr [The server is on vacation]=$\text{Lim}_{z\to1}Q(z)=r\lambda E[X]E[V]$ (41)

Proof: The corresponding steady state results can be obtained by applying L'Hospital rule in eqs (21)-(25), respectively, and letting z=1.

5. Queue Size Distribution at Departure Epoch

According the argument of PASTA (Poisson arrival see time average), we state that a departing customer will see 'n' customers in the queue just after a departure if and only if there were (n+1) customers in the queue just before the departure. Let π_n be the probability that there are 'n' customers in the queue at a departure epoch, and then we have

$$\pi_n = k_0(1-r)\int_0^\infty \mu_2(x)P_{2,n+1}(x)dx + k_0\int_0^\infty \upsilon(x)Q_n(x)dx, \quad n \ge 0 \tag{42}$$

where k_0 is normalizing constant. Next we define the PGF of π_n as

$$\pi(z) = \sum_0^\infty z^n \pi_n$$

$$= \frac{\lambda k_0(1-\phi)(1-X(z))\{(1-r)+rV^*(\lambda-\lambda X(z))\}S^*\theta_1(z)S^*\theta_2(z)\left\{\sum_{n=0}^{N-1} z^n\psi_n\right\}}{\{(1-r)+rV^*(\lambda-\lambda X(z))\}S^*(\theta_1(z))S^*(\theta_2(z))-z\}\left\{\sum_{n=0}^{N-1}\psi_n\right\}} \tag{43}$$

By using the normalizing condition π(1)=1, we get

$$k_0 = [\lambda E(X)]^{-1} \tag{44}$$

Using equation (43) into (42), we get

$$\pi(z) = \frac{\lambda(1-\phi)(1-X(z))\{(1-r)+rV^*(\lambda-\lambda X(z))\}S^*\theta_1(z)S^*\theta_2(z)\left\{\sum_{n=0}^{N-1} z^n\psi_n\right\}}{E(X)\{(1-r)+rV^*(\lambda-\lambda X(z))\}S^*(\theta_1(z))S^*(\theta_2(z))-z\ \}\left\{\sum_{n=0}^{N-1}\psi_n\right\}} \tag{45}$$

Hence the relationship between P(z) and π(z) is given by

$$\pi(z) = \frac{\{1-X(z)\}}{E(X)(1-z)}P(z) = A(z)P_0(z)G(z) \tag{46}$$

where

$A(z) = \frac{\{1-X(z)\}}{E(X)(1-z)}$, is the PGF of the number of customers placed before an arbitrary customer (tagged customer) in a batch in which the tagged customer arrives.

Eq. (45) illustrates that the departure point queue size distribution of given model is the convolution of three independent random variables as

(i) First term A(z) is the number of customers places before a tagged unit in the batch.

(ii) Second term $P_0(z)$ represents the queue size distribution of $M^x/G/1$ queue with vacation time under Bernoulli schedule.

(iii) Third term G(z) represents additional queue size distribution due to N-Policy.

Applying L'Hospital rule repetitively in eq. (45), we have L_D, the mean queue size at departure epoch as follows;

$$L_D = \left.\frac{d\pi(z)}{dz}\right|_{z=1}$$
$$= \phi + \frac{\phi\lambda E(X(X-1))}{2(1-\phi)} + \frac{\lambda r E(X)E^2(V)}{2(1-\phi)} - \frac{\left\{E^2(S_1)(\alpha_1 E(G_1)+1) + E^2(S_2)(\alpha_2 E(G_2)+1)\right\}}{2(1-\phi)} - \frac{\left\{\alpha_1 E(S_1)E^2(G_1) + \alpha_2 E(S_2)E^2(G_2)\right\}}{2(1-\phi)}$$
$$- \frac{r\lambda^2 E^2(X)E(V)\left\{E(S_1)(\alpha_1 E(G_1)+1) + E(S_2)(\alpha_2 E(G_2)+1)\right\}}{(1-\phi)} + \frac{\sum_{j=0}^{N-1} n\psi_n}{\sum_{n=0}^{N-1}\psi_n} + \frac{E(X(X-1))}{2E(X)} \tag{47}$$

Thus

$$L_D = L_Q + \frac{E(X(X-1))}{2E(X)} \tag{48}$$

6. Sensitivity Analysis

In this section, we validate our analytical results by taking numerical examples. The sensitivity analysis is performed to visualize the effect of different parameters on the average queue length and long run state probabilities of the server.

The default parameters are fixed as N=4, α_1=0.04, α_2=0.03, β_1=3, β_2=4, μ_1=8, μ_2=6, v=0.6, r=0.2 for computational results demonstrated in figures 1-6 which plot the variation in the mean queue length for various system input parameters. In all figures 1-6, we observed that as λ increases, there is remarkable increase in the queue length; the impact is more prominent for higher values of λ in comparison to lower values of λ. From fig. 1 we notice that higher service rates have a significant impact on the queue length as it leads to considerably decrement in the queue length. Fig. 2 illustrates the fact that that increased failure rates result in increased queue length. Fig. 3 indicates that sufficient repair facility can be helpful in reducing the queue length. For higher values of vacation rate, the lower values of average queue length can be seen in fig. 4. The variation in queue length with respect to probability of opting second phase service (r) is shown in fig. 5. It is found that by increasing the probability there is remarkable decrease in L. Finally in fig. 6, we exhibit the variation in L with respect to threshold parameter; we see that for higher values of N, the average queue length is higher.

7. Conclusion

In the present paper, we have considered $M^x/G/1$ queue with batch arrival, two types of general heterogeneous service and modified Bernoulli schedule server vacations. The concepts of server breakdown and server repair were incorporated into the model to make the study more realistic. In many congestion situations just before a service starts, the customer has the option to choose one of two kinds of services. Such a model may find applications in many day to day real life queueing situations. Further our model assumes that the server vacations are based on Bernoulli schedule which means that just after completing a service selected by the customer, the server may take vacation of random length or may continue staying in the system. The concepts of Bernoulli schedule vacation, batch arrival and unreliable server have been incorporated together in our queueing model which has potential applicability in manufacturing, computer and communication systems, etc..

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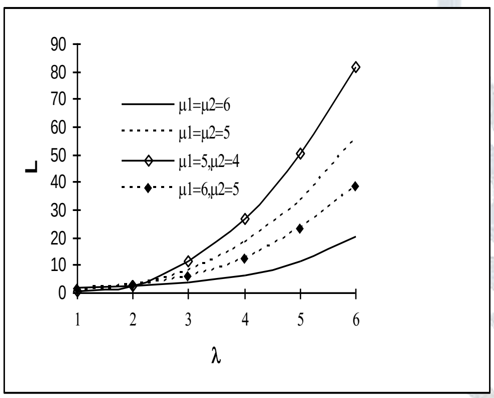


Fig. 1: Queue length vs λ

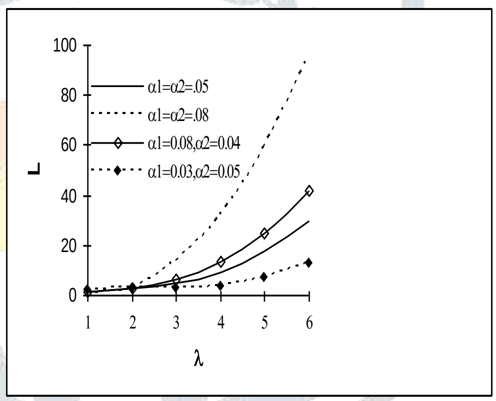
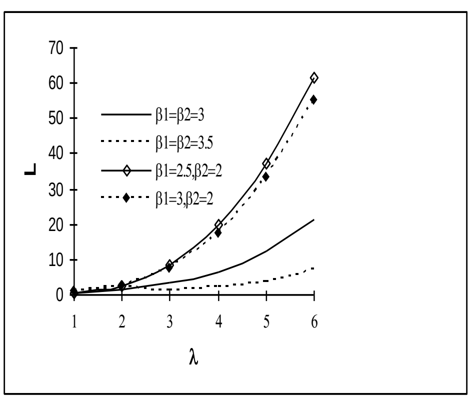


Fig. 2: Queue length vs λ



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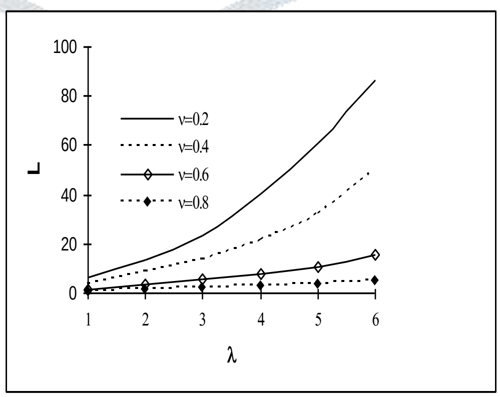

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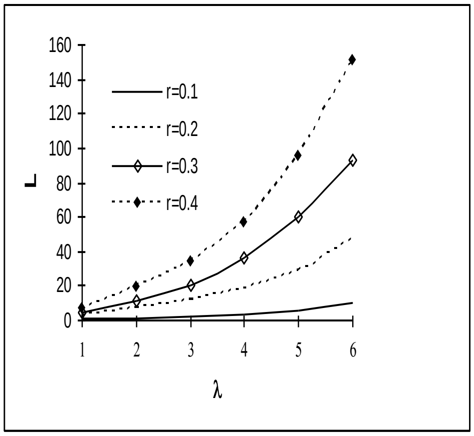

Fig. 5: Queue length vs λ

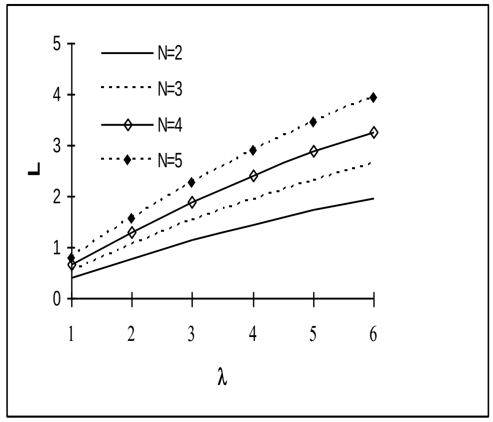

Fig. 6: Queue length vs λ