JETIR.ORG ISSN: 2349-5162 | ESTD Year : 2014 | Monthly Issue
JOURNAL OF EMERGING TECHNOLOGIES AND INNOVATIVE RESEARCH (JETIR)
An International Scholarly Open Access, Peer-reviewed, Refereed Journal

Linear Algebra, its Origin and Applications

CHINU GOYAL
Assistant Professor in Maths
Khalsa College For Women, Civil Lines Ludhiana

Abstract:

This is compilation of historical information from various sources, about what linear algebra is? , Its origin and applications. The information has been put together for students who are curious about the origin and applications of the subject. Linear algebra is study of certain algebraic structure called a vector space, study of linear sets of equations and its transformation properties. It is branch of mathematics that deals with the finite dimensional vector spaces and linear mappings. Around 4000 years ago, the people of Babylon knew how to solve a simple 2x2 system of linear equations with two unknowns. Around 200 BC, the Chinese published that "Nine chapters of Mathematical Art "they displayed ability to solve 3x3 systemof equations.Linear Algebra is widely used in the fields of Math, Science, and Engineering.Linear algebra plays an important role to determine unknown quantities. Some real life applications of linear algebra are calculation of speed, time and distance, used for projecting a three dimensional view into a two dimensional plane, handled by linear maps. Cryptography and Game theory are its other applications.

Introduction of Linear Algebra:

Linear algebra is a study of linear combinations. It is the study of vector spaces,lines and planes, and some mappings that are required to perform the linear transformations. It includes vectors, matrices and linear functions. It is study of linear setsof equations and its transformation properties.

Linear Algebra Equations

The general linear equations is represented as $a_1 x_1 + a_2 x_2 + a_3 x_3 + \ldots\ldots\ldots\ldots\ldots + a_n x_n = b$

Here a's – represents the coefficients

X's – represents the unknowns

B – represents the constants

There exists a system of linear algebraic equations, which is set of equations, The system of equations can be solved using the matrices.

It obeys the linear functions such as

$$(x_1\, x_2\, x_3 \ldots\ldots\ldots\ldots x_n) \rightarrow a_1 x_1 + a_2 x_2 + a_3 x_3 + \ldots\ldots\ldots\ldots\ldots + a_n x_n$$

Various topics of linear algebra Euclidian Vector Spaces, Eigen Values and Eigen Vectors, Orthogonal matrices, Linear transformations, Projections, Solving system of equations with matrices, Mathematical operations with matrices (i.e. addition , multiplication), Matrix inverse and determinants, Positive – definite matrices, Singular value decomposition, Linear dependence and independence.

Three main concepts which are the prerequisite to linear algebra are explained in detail. They are:

- Vector spaces
- Linear Functions
- Matrix

All these three concepts are interrelated such that a system of linear equations can be represented using these concepts mathematically. In general terms, vectors are elements that we can add, and linear functions are the functions of vectors that include the addition of vectors.

Vector Spaces: Let < F, +, .> be a field and <V,+> be an abelian group. Then V is equipped with a map F x V → V such that (a, x) → ax, satisfying following axioms for all a,b ∈ F and x,y ∈ V:

1. a (x+y) = ax+ ay
2. (a+b) = ax + bx
3. (ab)x = a(bx)
4. 1x=x, where 1 is unity of F.

The elements of V are called vectors and elements of F are called scalars

Linear Function:

A function L: $R^n \rightarrow R^m$ is linear if

(i) $L(x + y) = L(x) + L(y)$

(ii) $L(\alpha x) = \alpha L(x)$

For all x, y ∈ R^n, α ∈ R

Matrix:

Matrices are linear functions of a certain kind. Matrix is the result of organizing information related to certain linear functions. Matrix almost appears in linear algebra because it is the central information of linear algebra.

Mathematically, this relation can be defined as follows.

A is an m × n matrix, then we get a linear function L: $R^n \rightarrow R^m$ by defining

$L(x) = Ax$

Or

$Ax = B$

Origin:

- Around 4000 years ago, the people of Babylon knew how to solve a simple 2X2 system of linear equations with two unknowns. Around 200 BC, the Chinese published that “Nine Chapters of the Mathematical Art,” they displayed the ability to solve a 3X3 system of equations.

- German mathematician Gottfried Leibniz established the use of determinants to solve linear systems in 1693. In 1750, Swiss mathematician Gabriel Cramer used this concept to solve linear systems and develop what is now known as Cramer's rule.
- Lagrange came out with his work regarding Lagrange multipliers, a way to "characterize the maxima and minima multivariate functions."
- With the turn into the 19th century Gauss introduced a procedure to be used for solving a system of linear equations. His work dealt mainly with the linear equations and had yet to bring in the idea of matrices or their notations. Gauss' work is summed up in the term Gaussian elimination. This method uses the concepts of combining, swapping, or multiplying rows with each other in order to eliminate variables from certain equations. After variables are determined, the student is then to use back substitution to help find the remaining unknown variables
- In 1848 in England, J.J. Sylvester first introduced the term "matrix," which was the Latin word for womb, as a name for an array of numbers. Matrix algebra was nurtured by the work of Arthur Cayley in 1855. The famous Cayley-Hamilton theorem which asserts that a square matrix is a root of its characteristic polynomial was given by Cayley in his 1858 Memoir on the Theory of Matrices.
- Matrices continued to be closely associated with linear transformations. By 1900 they were just a finite-dimensional subcase of the emerging theory of linear transformations. The modern definition of a vector space was introduced by Peano in 1888.

Applications of Linear Algebra:

1. Game theory

It is an application of linear algebra, which is a mathematical study that describes the number of possible options. The players make these options during the game playing. As per psychologists, the social interaction theory is used to consider the player's options against other players in the competition

1. Application of Linear Algebra in Real Life:

- Linear Algebra is used to check the distribution of microwave energy in a microwave oven.
- It is used to create ranking algorithms in search engines such as Google, Yahoo, etc.
- Used to recover the codes that have been tampered with during processing or transmission.
- Used for space studies.
- It is used for projecting a three-dimensional view into a two-dimensional plane, handled by linear maps.
- Used to examine the digital signals and encode or decode them. These can be the signals of audio or video.
- It is used to optimize in the field of linear programming.
- Used to check the energy levels of atoms.

2. Applications of Linear Algebra in Computer Science

Linear algebra is essential for things like:

- Pattern Recognition.
- Graph theory (social graphs, for example).
- Data Classification and Clustering.
- Singular Value Decomposition for recommendation systems.
- Graphics Programming.

- Various forms of Artificial Intelligence (AI)

3. Cryptography

- It is the study of decoding and encoding of the secret messages. Using electronic transactions and communications, solid encryption methods can be applied. Those methods involve modular arithmetic to decode/encode the messages. And the simpler encoding methods apply using the concept of matrix transformation.

- I mentioned above that the linear algebra concept is used to decode the secret message (a cryptography method).

Conclusion:

Linear algebra basically is the study of the planes and lines, mapping, and vector spaces, which are needed for linear transformations. Therefore, it is necessary to know what is history behind linear algebra and what are the applications of linear algebra in real life.

References:

1.Darkwing. (n.d.). A brief history of linear algebra and matrix theory. Retrieved from http://darkwing.uoregon.edu/~vitulli/441.sp04/LinAlgHistory.html Perotti. (n.d.).

2.History of linear algebra. Retrieved from http://www.science.unitn.it/~perotti/History of Linear Algebra.pdf Strang, G. (1993). The fundamental theorem of linear algebra.

3.S. Athloen and R. McLaughlin, Gauss-Jordan reduction: A brief history, American Mathematical Monthly 94 (1987) 130-142.

4.A. Tucker, The growing importance of linear algebra in undergraduate mathematics, The College Mathematics Journal, 24 (1993) 3-9.

5.Jeff Christensen April 2012 Final Project Math 2270 Grant Gustafson University of Utah.

6.Bhattacharya P.B., Jain S.K. and Nagpaul S.R.

7.First Course in Linear Algebra, Wiley Eastern Ltd., 1991.