



Rough Set Theory: Past, Present, and Future

A Comprehensive Review

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Abstract : Rough Set Theory (RST) has emerged as a powerful mathematical tool for dealing with vagueness and uncertainty in data analysis. Since its inception, RST has evolved through numerous extensions and hybrid models that aim to enhance its capacity in complex decision-making environments. This review presents a comprehensive overview of the evolution of RST—from its classical foundations to modern developments such as fuzzy rough sets, probabilistic rough sets, covering-based models, and information-theoretic variants. The study critically examines the inherent limitations of traditional RST, explores its integration with contemporary computational frameworks, and identifies open challenges and promising research directions for future advancement.

Index Terms - Rough Set Theory; Attribute Reduction; Uncertainty Modeling; Hybrid Rough Models; Extensions of RST; Knowledge Discovery

1. INTRODUCTION

Rough Set Theory (RST), introduced by Prof. Zdzisław Pawlak in 1982, provides a mathematical framework for reasoning with uncertain, vague, and incomplete data. It defines sets using **lower approximations** (definite members), **upper approximations** (possible members), and a **boundary region** (uncertain classification), without requiring prior knowledge of probability distributions or membership functions [1].

RST has been widely applied in machine learning, data mining, pattern recognition, and decision support systems, especially in domains where transparency and interpretability are essential. It supports explainable artificial intelligence (XAI) and knowledge discovery in complex domains.

Popular implementations include:

- **Python:** *PyRoughSets*, *scikit-roughsets*
- **R:** *RoughSets* package
- **Standalone Tools:** *ROSETTA*, *RSES*

Indian research groups at ISI Kolkata, IITs, IISc, and Jadavpur University have contributed significantly to the field, focusing on hybrid models integrating fuzzy logic and neural networks. Globally, active research continues in Poland, China, USA, UK, and Japan, with applications in biomedicine, cybersecurity, and interpretable AI systems.

2. FUNDAMENTALS OF ROUGH SET THEORY

Definition 2.1 (Information System [1]). A rough set information system is a 4-tuple $IS = \langle \mathcal{U}, \mathcal{A}, V, f \rangle$ where:

- $\mathcal{U}$ is a non-empty finite set of objects called the **universe**.
- $\mathcal{A} = \mathcal{C} \cup \mathcal{D}$ is a finite set of attributes partitioned into condition attributes $\mathcal{C}$ and decision attributes $\mathcal{D}$.
- $V = \bigcup_{a \in \mathcal{A}} V_a$, where V_a is the value domain of attribute a .
- $f: \mathcal{U} \times \mathcal{A} \rightarrow V$ is the **information function** such that $f(x, a) \in V_a$ for all $x \in \mathcal{U}$ and $a \in \mathcal{A}$.

Definition 2.2 (Indiscernibility Relation [1]). Given an information system $IS = \langle \mathcal{U}, \mathcal{A}, V, f \rangle$ and any subset $B \subseteq \mathcal{A}$, the **indiscernibility relation** induced by B is defined as: $\text{ind}(B) = \{(x, y) \in \mathcal{U} \times \mathcal{U} \mid \forall a \in B, f(x, a) = f(y, a)\}$. The relation $\text{ind}(B)$ is an equivalence relation on $\mathcal{U}$. The equivalence class of an object $x \in \mathcal{U}$ under $\text{ind}(B)$ is denoted by $[x]_B$.

Definition 2.3 (Approximation Space [1]). Given an information system $IS = \langle \mathcal{U}, \mathcal{A}, V, f \rangle$ and $B \subseteq \mathcal{A}$, the pair $(\mathcal{U}, \text{ind}(B))$ is called an **approximation space**, where $\text{ind}(B)$ is the indiscernibility relation induced by B .

Definition 2.4 (Lower Approximation [1]). Given an approximation space $(\mathcal{U}, \text{ind}(B))$ and a target set $X \subseteq \mathcal{U}$, the **lower approximation** of X with respect to B is defined as: $\underline{\text{apr}}_B(X) = \{x \in \mathcal{U} \mid [x]_B \subseteq X\}$. The lower approximation consists of all objects that can be certainly classified as members of X using the knowledge in B .

Definition 2.5 (Upper Approximation [1]). Given an approximation space $(\mathcal{U}, \text{ind}(B))$ and a target set $X \subseteq \mathcal{U}$, the **upper approximation** of X with respect to B is defined as: $\overline{\text{apr}}_B(X) = \{x \in \mathcal{U} \mid [x]_B \cap X \neq \emptyset\}$. The upper approximation consists of all objects that can be possibly classified as members of X using the knowledge in B .

Definition 2.6 (Positive Region [1]). Given an approximation space $(\mathcal{U}, \text{ind}(B))$ and a target set $X \subseteq \mathcal{U}$, the **positive region** of X with respect to B is: $\text{POS}_B(X) = \underline{\text{apr}}_B(X)$. If $x \in \text{POS}_B(X)$, then x certainly belongs to X based on the information in B .

Definition 2.7 (Negative Region [1]). Given an approximation space $(\mathcal{U}, \text{ind}(B))$ and a target set $X \subseteq \mathcal{U}$, the **negative region** of X with respect to B is: $\text{NEG}_B(X) = \mathcal{U} \setminus \overline{\text{apr}}_B(X)$. If $x \in \text{NEG}_B(X)$, then x certainly does not belong to X based on the information in B .

Definition 2.8 (Boundary Region [1]). Given an approximation space $(\mathcal{U}, \text{ind}(B))$ and a target set $X \subseteq \mathcal{U}$, the **boundary region** of X with respect to B is: $\text{BND}_B(X) = \overline{\text{apr}}_B(X) \setminus \underline{\text{apr}}_B(X)$. If $x \in \text{BND}_B(X)$, then the membership of x in X is uncertain based on the information in B .

Remark 1. The three regions partition the universe:

$$\mathcal{U} = \text{POS}_B(X) \cup \text{BND}_B(X) \cup \text{NEG}_B(X),$$

and they are pairwise disjoint.

Definition 2.9 (Definable Set [1]). Given an approximation space $(\mathcal{U}, \text{ind}(B))$ and a target set $X \subseteq \mathcal{U}$, the set X is called **definable** (or **crisp**) with respect to B if and only if: $\underline{\text{apr}}_B(X) = \overline{\text{apr}}_B(X)$. Equivalently, X is definable if $\text{BND}_B(X) = \emptyset$.

Definition 2.10 (Rough Set [1]). Given an approximation space $(\mathcal{U}, \text{ind}(B))$ and a target set $X \subseteq \mathcal{U}$, the set X is called a **rough set** with respect to B if and only if: $\underline{\text{apr}}_B(X) \neq \overline{\text{apr}}_B(X)$. Equivalently, X is rough if $\text{BND}_B(X) \neq \emptyset$.

Definition 2.11 (Dependency Degree [2]). Given an information system $IS = \langle \mathcal{U}, \mathcal{A}, V, f \rangle$ with $B \subseteq \mathcal{C}$ and decision attribute set $\mathcal{D}$, the **degree of dependency** of $\mathcal{D}$ on B is defined as: $\gamma_B(\mathcal{D}) = \frac{|\text{POS}_B(\mathcal{D})|}{|\mathcal{U}|}$, where $\text{POS}_B(\mathcal{D}) = \bigcup_{X \in \mathcal{U}/\text{ind}(\mathcal{D})} \underline{\text{apr}}_B(X)$ is the positive region of the partition induced by $\mathcal{D}$.

Remark 2. The dependency degree $\gamma_B(\mathcal{D})$ satisfies $0 \leq \gamma_B(\mathcal{D}) \leq 1$, and can be interpreted as follows:

- $\gamma_B(\mathcal{D}) = 1$ implies that $\mathcal{D}$ is **fully dependent** on B (all decisions can be uniquely determined by B);
- $\gamma_B(\mathcal{D}) = 0$ implies that $\mathcal{D}$ is **independent** of B (knowledge of B provides no information about $\mathcal{D}$);
- $0 < \gamma_B(\mathcal{D}) < 1$ implies that $\mathcal{D}$ is **partially dependent** on B .

Definition 2.12 (Significance of an Attribute [2]). Given an information system $IS = \langle \mathcal{U}, \mathcal{A}, V, f \rangle$, the **significance** of an attribute $a \in B \subseteq \mathcal{C}$ with respect to the decision $\mathcal{D}$ is defined as: $\text{SIG}_a(B, \mathcal{D}) = \gamma_B(\mathcal{D}) - \gamma_{B \setminus \{a\}}(\mathcal{D})$.

Remark 3. The significance measure indicates the change in classification power when attribute a is removed:

- $\text{SIG}_a(B, \mathcal{D}) = 0$ implies that removing a does not reduce dependency, so a is **redundant** in B ;
- $\text{SIG}_a(B, \mathcal{D}) > 0$ implies that removing a reduces dependency, so a is **significant** (indispensable) in B .

Definition 2.13 (Classification Uncertainty [2]). Given an information system $IS = \langle \mathcal{U}, \mathcal{A}, V, f \rangle$ with condition attributes $\mathcal{C}$ and decision attributes $\mathcal{D}$, the **classification uncertainty** is defined as: $U_{\mathcal{C}}(\mathcal{D}) = 1 - \gamma_{\mathcal{C}}(\mathcal{D}) = \frac{|\mathcal{U}| - |\text{POS}_{\mathcal{C}}(\mathcal{D})|}{|\mathcal{U}|}$.

Remark 4. The classification uncertainty measures the proportion of objects that cannot be classified with certainty:

- If $\gamma_{\mathcal{C}}(\mathcal{D}) = 1$, then $U_{\mathcal{C}}(\mathcal{D}) = 0$, indicating **no uncertainty** (perfect classification).
- If $0 < \gamma_{\mathcal{C}}(\mathcal{D}) < 1$, then $0 < U_{\mathcal{C}}(\mathcal{D}) < 1$, indicating **partial uncertainty**.
- If $\gamma_{\mathcal{C}}(\mathcal{D}) = 0$, then $U_{\mathcal{C}}(\mathcal{D}) = 1$, indicating **maximum uncertainty**.

Definition 2.14 (Reduct [2]). Given an information system $IS = \langle \mathcal{U}, \mathcal{A}, V, f \rangle$ with condition attributes $\mathcal{C}$ and decision attributes $\mathcal{D}$, a subset $\mathcal{R} \subseteq \mathcal{C}$ is called a **reduct** of $\mathcal{C}$ if:

1. $\gamma_{\mathcal{R}}(\mathcal{D}) = \gamma_{\mathcal{C}}(\mathcal{D})$ (preservation of classification power), and
2. $\forall a \in \mathcal{R}, \gamma_{\mathcal{R} \setminus \{a\}}(\mathcal{D}) < \gamma_{\mathcal{R}}(\mathcal{D})$ (minimality).

In other words, a reduct is a minimal subset of attributes that preserves the dependency degree of the full attribute set.

Definition 2.15 (Core [2]). Given an information system $IS = \langle \mathcal{U}, \mathcal{A}, V, f \rangle$ with condition attributes $\mathcal{C}$ and decision attributes $\mathcal{D}$, the **core** of $\mathcal{C}$ is defined as: $\text{CORE}(\mathcal{C}) = \bigcap \{ \mathcal{R} \subseteq \mathcal{C} \mid \mathcal{R} \text{ is a reduct of } \mathcal{C} \}$. The core consists of all attributes that appear in every reduct, i.e., the absolutely indispensable attributes.

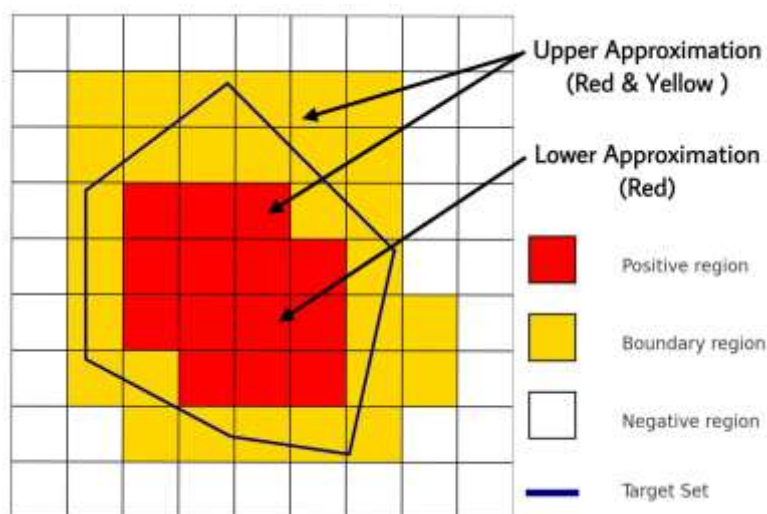


Figure 1: Illustration of lower and upper approximations in Rough Set Theory

The inner region represents the lower approximation (certain membership), the outer boundary represents the upper approximation (possible membership), and the gap between them is the boundary region (uncertain membership).

Definition 2.16 (Definable and Rough Sets [1])

A set $X \subseteq \mathcal{U}$ is definable with respect to attribute set B if $\underline{\text{apr}}_B(X) = \overline{\text{apr}}_B(X)$, meaning there is no uncertainty in classifying objects into X . Otherwise, X is a rough set due to the presence of boundary elements that cannot be definitively classified.

Definition 2.17 (Roughness Measure [1])

The roughness of a set quantifies the degree of uncertainty in its classification. For a non-empty set $X \subseteq \mathcal{U}$ and attribute set $B \subseteq \mathcal{A}$, the **roughness measure** is defined as:

$$\rho_B(X) = 1 - \frac{|\underline{\text{apr}}_B(X)|}{|\overline{\text{apr}}_B(X)|}.$$

Alternatively, the **accuracy measure** is given by:

$$\alpha_B(X) = \frac{|\underline{\text{apr}}_B(X)|}{|\overline{\text{apr}}_B(X)|}.$$

The accuracy measure $\alpha_B(X) \in (0,1]$, where $\alpha_B(X) = 1$ indicates perfect definability (no roughness), and smaller values indicate greater uncertainty.

Illustrative Example: Patient Information System

The following table presents a simple patient dataset used to demonstrate fundamental rough set operations such as indiscernibility relations, lower and upper approximations, positive and boundary regions, and roughness computation.

Table 1: Patient symptom data demonstrating rough set approximations

Patient	Fever	Cough	Headache	Diagnosis
P1	High	Yes	Yes	Flu
P2	High	Yes	No	Flu
P3	Normal	No	Yes	Cold
P4	High	No	Yes	Flu
P5	Normal	No	No	Cold

Let $\mathcal{U} = \{P1, P2, P3, P4, P5\}$ be the universe of patients, $\mathcal{C} = \{\text{Fever}, \text{Cough}, \text{Headache}\}$ be the condition attributes, and $\mathcal{D} = \{\text{Diagnosis}\}$ be the decision attribute.

Consider $B = \{\text{Fever}, \text{Cough}\} \subseteq \mathcal{C}$. The indiscernibility classes induced by B are:

$$\begin{aligned}[P1]_B &= [P2]_B = \{P1, P2\}, \\ [P3]_B &= [P5]_B = \{P3, P5\}, \\ [P4]_B &= \{P4\}.\end{aligned}$$

For the decision class $X = \{\text{Patients with Flu}\} = \{P1, P2, P4\}$, we can compute:

$$\begin{aligned}\underline{\text{apr}}_B(X) &= \{P1, P2, P4\}, \\ \overline{\text{apr}}_B(X) &= \{P1, P2, P4\}, \\ \text{BND}_B(X) &= \emptyset.\end{aligned}$$

Since $\underline{\text{apr}}_B(X) = \overline{\text{apr}}_B(X)$, the decision class “Flu” is definable with respect to attributes $\{\text{Fever}, \text{Cough}\}$, and the accuracy is $\alpha_B(X) = 1$.

3. CHRONOLOGICAL EVOLUTION OF ROUGH SET THEORY EXTENSIONS

Since the inception of classical rough set theory (RST) by Zdzisław Pawlak in 1982, various extensions have been proposed to overcome its limitations in modeling uncertainty, handling imprecise or noisy data, and adapting to different types of information granules. This section presents a chronological overview of these extensions, highlighting their theoretical motivations, core principles, and practical significance.

3.1. Classical Rough Set Theory (1982) [1]

Proposed by: Z. Pawlak

Foundation: Based on *equivalence relations* and *lower and upper approximations*.

Limitation: Assumes complete and discrete data; cannot handle vague or noisy information.

3.2. Variable Precision Rough Sets (1993) [3]

Proposed by: W. Ziarko

Motivation: Introduced a precision parameter (β) to tolerate classification errors.

Core Idea: Approximation regions are defined based on the allowed misclassification rate.

Application: Effective in knowledge discovery from imperfect data.

3.3. Probabilistic Rough Sets (Mid-1990s) [4] [5]

Contributors: S. Slowinski, J. Grzymala-Busse

Extension: Introduced (α, β) probabilistic approximations, allowing probabilistic interpretation of membership.

Relevance: Bridges RST with probability theory; widely used in risk analysis and medical domains.

3.4. Fuzzy Rough Sets (Late 1990s) [6] [7]

Proposed by: D. Dubois, H. Prade

Concept: Combines fuzzy logic with rough set approximations.

Advantage: Supports continuous-valued or vague attributes.

Application: Fuzzy pattern recognition, image classification.

3.5. Tolerance-Based and Similarity Rough Sets (Late 1990s – Early 2000s) [8] [9]

Idea: Replace equivalence relations with tolerance or similarity relations.

Motivation: Enable soft classification for objects that are nearly, but not exactly, indiscernible.

Use Case: Real-valued data, pattern clustering.

3.6. Covering-Based Rough Sets (Early 2000s) [10] [11]

Proposed by: A. Skowron, M. Zakowski

Concept: Based on coverings instead of partitions, allowing overlapping granules.

Strength: Better suited to incomplete, missing, or overlapping information.

Application: Web mining, sensor networks.

3.7. Dominance-Based Rough Set Approach (DRSA) (Early 2000s) [12]

Proposed by: F. Greco, B. Matarazzo, R. Slowinski

Extension: Uses dominance relations to model preference-based decision making.

Importance: Key in multi-criteria decision analysis (MCDA).

Application: Business, social sciences, environmental management.

3.8. Hybrid Rough Set Models (2000s Onward) [13] [14]

Nature: Integration with neural networks, evolutionary algorithms, swarm intelligence, etc.

Goal: Enhance computational performance and scalability.

Application: Feature selection, classification, bioinformatics.

3.9. Granular and Multi-Granulation Rough Sets (2010s) [15] [16]

Concept: Employ multiple granules (multi-view or multi-source approximations).

Contributors: Y. Y. Yao and others

Advantage: Flexible modeling of hierarchical or multi-source information.

Trend: Aligned with granular computing and knowledge fusion.

3.10. Summary of Timeline

Table 2: Chronological summary of major rough set extensions

Year	Extension	Core Concept	Advantage	Contributors
1982	Classical RST	Equivalence-based approximations	Exact classification	Pawlak
1993	VPRS	Tolerates error (β)	Noise resistance	Ziarko
~1995	PRS	Probabilistic thresholds	Uncertainty modeling	Slowinski
~1998	FRS	Fuzzy sets + RST	Vagueness handling	Dubois, Prade
~2000	TRS/SRS	Tolerance/similarity relations	Partial indiscernibility	Multiple
2000s	Covering RST	Overlapping covers	Incomplete data	Zakowski
2000s	DRSA	Dominance preference	MCDA capability	Greco
2000s+	Hybrid RST	RST + AI/ML	Improved learning	Lingras
2010s	Multi-Granulation	Multi-level granules	Hierarchical modeling	Yao

4. RECENT EXTENSIONS OF ROUGH SET THEORY

As data has become more complex, researchers have introduced new versions of Rough Set Theory (RST) to improve its ability to handle real-world problems. Many of these extensions have been developed in the last decade to work better with uncertain, incomplete, or fast-changing data. Below are some of the most important recent developments:

- **Causal Rough Sets (2023):** This is a new type of rough set that not only finds patterns but also looks for possible cause-effect relationships in data. It is helpful in areas like healthcare and policymaking, where understanding “why” something happens is as important as knowing “what” is happening [37].
- **Multi-Granulation Probabilistic Rough Sets (2019):** This model combines two ideas—multi-granulation (looking at data from different perspectives) and probability—to make better decisions when data comes from different sources or levels. It is useful in sensor networks and ensemble learning systems [21].
- **Interval-Valued Fuzzy Rough Sets (2020):** In this version, values are allowed to vary within a range (interval) instead of being exact. This is helpful when data is vague or subjective, such as human opinions in surveys or expert systems [40].
- **Neighborhood Covering-Based Rough Sets (2021):** Instead of using fixed groups (partitions), this model looks at the “neighborhood” of each data point to form flexible and overlapping groups. This makes it better at handling noisy or continuous data, especially in image and pattern recognition tasks [41].
- **Dynamic Rough Sets (2019):** These are designed for situations where data keeps changing over time, like in live sensor feeds or real-time systems. Dynamic rough sets can update their results on the fly, which is useful for applications like cybersecurity or IoT [42].
- **Granular Rough Set Neural Networks (2020):** This hybrid model connects rough sets with deep learning. It keeps the interpretability of rough sets while using the powerful learning abilities of neural networks. It has been used in finance and medical applications where both accuracy and transparency are needed [49].

5. INTERNATIONAL RESEARCH ACTIVITIES IN ROUGH SET THEORY (2015–2025)

Over the past decade, Rough Set Theory (RST) has grown through global collaboration, academic conferences, and technological integration. The timeline below highlights key international developments from 2015 to 2025:

- **2015 – Growth of Multi-Granulation Models:** China, Poland, and Canada advanced multi-granulation rough sets for hierarchical decision-making in medical and sensor systems [21].
- **2016 – High-Dimensional Feature Selection:** Researchers from China, India, and the UK introduced scalable rough set-based feature selection for biomedical and text data [22].

- **2017 – Probabilistic and Tolerance-Based Models:** European teams enhanced probabilistic rough set theory and tolerance-based relations for applications in image processing and finance [4] .
- **2018 – Explainable AI Integration:** Canada, Poland, and the UK integrated RST with machine learning models for explainability in artificial intelligence (XAI) [31].
- **2019 – Dynamic Rough Sets and Streaming Data:** Germany and China proposed dynamic RST for evolving data in real-time systems like IoT and cybersecurity [42] .
- **2020 – Virtual Conferences and Medical Focus:** Global participation in IJCRS and RSCTC centered around rough-fuzzy neural models and healthcare decision systems [32].
- **2021 – Hybrid AI Models:** Collaborations in Poland and Germany focused on combining RST with neural networks, swarm intelligence, and evolutionary computation [33] .
- **2022 – Python-Based Tool Development:** Key tools such as *roughset*, *scikit-roughsets*, and *RoughPy* were released and adopted internationally [66] .
- **2023 – Causal Rough Sets:** Causal inference models based on RST were introduced by research teams in the USA and China [37] .
- **2024–2025 – Edge Computing and Policy Applications:** Ongoing research in Canada, the UK, and the EU explores RST in edge-AI, environmental modeling, and public policy analytics [38] [39].

6. INDIAN RESEARCH ACTIVITIES IN ROUGH SET THEORY (2015–2025)

India has actively contributed to the advancement of Rough Set Theory, with impactful research in soft computing, agriculture, healthcare, education, and policy analytics. The following highlights key developments in India during the last decade:

- **2015 – Feature Selection in Medical Data:** Indian institutions like ISI Kolkata and IITs worked on rough set-based feature selection for high-dimensional datasets in medical and genomic studies [22] .
- **2016 – Fuzzy-Rough Decision Systems:** Universities such as Jadavpur and Anna University applied fuzzy-rough models to build interpretable systems in decision support and pattern recognition [23] [24].
- **2017 – Agricultural Informatics:** Collaborations with ICAR and state agricultural universities used RST for crop disease detection, weather-based alerts, and satellite image interpretation [25] [26].
- **2018 – Academic Training Initiatives:** National workshops and training programs at IITs and central universities promoted rough computing, fuzzy logic, and granular reasoning [27] [28].
- **2019 – E-Governance and Social Data Analytics:** RST was used in sentiment mining of regional languages, welfare targeting, and government classification models based on incomplete data [29] [30].
- **2020 – COVID-19 Healthcare Decision Tools:** Indian researchers developed lightweight, rule-based RST tools for the diagnosis and triage of COVID-19 using limited symptoms [34].
- **2021 – Hybrid RST with AI and Optimization:** RST was integrated with genetic algorithms and neural models for use in smart grid forecasting, microfinance and anomaly detection [35] [36].
- **2022 – Python Tool Customization:** Indian teams adopted and customized tools like *scikit-roughsets* and *RoughPy* for educational and industrial AI applications [66].
- **2023 – Research Cluster Expansion at Tripura University:** Ph.D. research clusters at Tripura University, BHU, and University of Hyderabad focused on soft computing, causal rough sets, and interpretable decision systems [61].
- **2024–2025 – National Policy Analytics and DST Projects:** RST is now used in Indian educational assessment, public policy modeling, and interdisciplinary projects funded by DST and UGC for social-scale decision making [43].

7. APPLICATIONS OF ROUGH SET THEORY IN AGRICULTURE, ENVIRONMENT, URBAN SYSTEMS & MODERN TECHNOLOGIES

Rough Set Theory (RST) has become a powerful mathematical tool for analyzing imprecise, vague, or incomplete data. Its ability to handle uncertainty without requiring prior probability makes it especially valuable in domains where traditional models fall short. RST supports rule extraction, pattern discovery, and decision-making in fields ranging from agriculture and climate science to AI, medicine, and smart infrastructure.

7.1. Groundwater Monitoring and Mineral Exploration [26]

RST has proven particularly effective in groundwater studies, where geological and hydrological datasets are often incomplete or noisy. It helps in identifying aquifer zones, mapping potential groundwater recharge areas, and prioritizing regions for mineral exploration. By integrating terrain data, field samples, and satellite images (e.g., Bhuvan, USGS, FAO AQUASTAT), RST simplifies complex spatial relationships and aids planners in reducing uncertainty in environmental assessments. Geospatial platforms such as QGIS and tools such as RoughPy enable visual representation and processing of these patterns.

7.2. Weather Forecasting [46]

In meteorology, where data loss and sensor variability are common, RST is used to develop simplified models for forecasting rainfall, temperature, and cloud behavior. It handles missing or inconsistent weather data efficiently, producing interpretable rules that support early warning systems and climate risk assessments. Datasets from IMD, NASA POWER, and NOAA are commonly processed using tools like *xarray*, *cdsapi*, and QGIS.



Figure 2: Visual Summary of Application in Ground Water Monitoring, Weather Forecasting & Crop Disease Management

7.3. Crop Disease Management [25] [48]

RST is highly valuable in detecting crop diseases from field imagery and spectral data. It supports pattern recognition even when disease symptoms overlap or gradually evolve, helping to ensure early intervention. Agricultural institutes (ICAR, KVK) and open datasets such as PlantVillage allow RST to be trained for use in disease classification using tools like TensorFlow, OpenCV, and RoughPy.

7.4. Urban Planning and Traffic Management [60]

In smart city planning, RST is used to evaluate incomplete demographic and land use data to guide zoning and infrastructure development. It also supports traffic control systems by predicting congestion patterns from fluctuating input data in real time.

7.5. Medical and Industrial Applications [55] [56]

In healthcare, RST enables disease diagnosis and clinical decision support by deriving rules from incomplete patient data. It's widely used in cardiology, oncology, and metabolic disorders to select the most relevant features and avoid data overload. In industrial systems, RST helps to detect faults, optimize processes, and ensure quality even when sensors fail or misreport.

7.6. Artificial Intelligence and Explainability [57]

RST contributes to explainable AI by generating transparent rule-based outputs from large datasets. Its integration with fuzzy logic and neural networks improves the interpretability of the model without sacrificing performance. It is especially helpful in domains where black-box AI models are not acceptable, such as finance or healthcare.



Figure 3: Visual Summary of Applications in Healthcare Analysis, Road Traffic Managements & Image Classifications

7.7. Everyday Decision Support Systems [58]

In everyday tech like e-commerce, RST supports personalized recommendations despite vague or incomplete user input. It is also used in financial systems for fraud detection and credit scoring, adapting well to fast-changing or incomplete data streams. Voice assistants and e-learning platforms apply RST to adapt to learner behavior or preferences dynamically.

7.8. Image Processing and Classification [9]

RST is used in satellite image classification, medical imaging, and object recognition. It simplifies high-dimensional image data by reducing redundancy and identifying the most important features. When paired with deep learning or fuzzy models, it improves image segmentation and pattern detection.

7.9. High-Stakes Decision-Making Systems [58]

In critical environments such as airports, radar systems, and banking infrastructure, RST handles real-time decisions with partial input. It is used in scheduling, surveillance, and risk management where conventional systems may fail due to incomplete or fluctuating data.

8. MODERN ROUGH SET-BASED SOFTWARE AND LIBRARIES

Several Python-based tools now make RST accessible to both researchers and practitioners:

- **roughset** – Basic tool for reducts and decision rule extraction; ideal for learning and experimentation [62].
- **scikit-roughsets** – Integrates with scikit-learn for rough set-based feature selection [63].
- **Rough-Set-Clustering-in-Python** – For tolerance-based clustering in ambiguous or overlapping datasets [64].
- **fuzzy-rough-learn** – Implements fuzzy-rough logic for classification and explainable ML [65].

9. CURRENT CHALLENGES IN ROUGH SET THEORY

Rough Set Theory (RST) has made significant strides in modeling uncertainty and generating interpretable rules. However, the rapidly evolving landscape of data-driven applications presents several limitations for traditional RST models. In this section, we explore the most pressing challenges and research directions.

9.1. Scalability in High-Dimensional Spaces

Rough Set Theory (RST) relies on the indiscernibility relation based on an attribute subset $B \subseteq \mathcal{A}$:

$$\text{ind}(B) = \{(x, y) \in \mathcal{U} \times \mathcal{U} \mid \forall a \in B, f(x, a) = f(y, a)\}$$

As the number of attributes increases, the calculation of $\text{ind}(B)$ and the construction of the discernibility matrix becomes computationally intensive:

$$\text{Size} \sim O(n^2 \cdot m)$$

where $n = |\mathcal{U}|$ is the number of objects and $m = |\mathcal{A}|$ is the number of attributes. As m increases, computing $\text{ind}(B)$ becomes increasingly expensive [2] [12].

This reflects practical scalability issues in rough set theory applications. Researchers have proposed distributed computing frameworks and rough set-based feature selection techniques to address this challenge [20] [21].

9.2. Real-Time and Dynamic Decision Systems

Traditional Rough Set Theory (RST) was originally designed for static datasets, where all data is available upfront. However, in today's world, data is constantly changing as new records are added, old ones are updated, and patterns evolve over time. To keep up, modern systems need models that can evolve dynamically. That's where Dynamic Rough Sets come in. They make it possible to

update approximations incrementally instead of recalculating everything from scratch, making them especially useful in real-time environments like fraud detection and industrial monitoring.

For a target set $X \subseteq \mathcal{U}$:

$$\underline{\text{apr}}_B(X) = \{x \in \mathcal{U} \mid [x]_B \subseteq X\}, \quad \overline{\text{apr}}_B(X) = \{x \in \mathcal{U} \mid [x]_B \cap X \neq \emptyset\}$$

This capability is crucial for applications such as fraud detection and industrial monitoring [19] [42].

9.3. Noise Tolerance and Missing Data

RST assumes complete and noise-free data. In practice, this is rarely the case. Variable Precision Rough Sets (VPRS) introduce a relaxation threshold $\beta \in [0,1]$:

$$\underline{\text{apr}}_B(X)^\beta = \left\{x \in \mathcal{U} \mid \frac{|[x]_B \cap X|}{|[x]_B|} \geq \beta\right\}$$

This model, along with fuzzy-rough sets, helps manage ambiguity and imprecise observations [3] [6].

9.4. Integration with Explainable AI (XAI)

Although RST naturally generates interpretable rules, integration with deep learning and other black-box AI models is still developing. Research focuses on extracting symbolic rules from trained networks and combining them with rough approximations [49] [31].

9.5. Hybrid and Multi-Granular Models

Combining RST with soft computing methods, such as fuzzy logic, neural networks, or genetic algorithms, improves both prediction power and interpretability. Multi-granular models allow reasoning across abstraction layers:

$$\mathcal{G} = \{Gr_1, Gr_2, \dots, Gr_k\}$$

where each Gr_i represents a granularity level with its own indiscernibility relation. These approaches are effective in hierarchical and uncertain environments [7] [5].

9.6. Lack of Modern Tools and Benchmarks

Most existing rough set tools (e.g., ROSETTA, RSES) lack support for real-time analytics, cloud integration, and scalable processing. Open-source libraries like RoughPy are in active development but need community support and standard benchmarks.

10. FUTURE DIRECTIONS IN ROUGH SET THEORY

As intelligent systems grow more advanced, Rough Set Theory (RST) also needs to evolve beyond its traditional form. This section presents some promising future directions that can make RST models more powerful, easier to understand, and better suited for handling large or fast-changing data.

10.1 Explainable Hybrid Models

One major trend is to combine RST with strong models such as neural networks and ensemble learning methods. These models are good at making accurate predictions, but they are often difficult to interpret. RST can help make the decision-making process more understandable.

Example: A hybrid model might use deep learning methods to detect patterns and then use RST to explain those patterns with clear symbolic rules.

In the future, researchers may explore the following.

- Using RST to extract rules from trained neural networks [49].
- Designing systems where RST lower and upper approximations guide the attention of deep models.
- Creating frameworks that connect RST with methods such as SHAP, LIME, or Causal Inference Tools [50] [51] [52] .

10.2. Causal Rough Set Theory [37]

Although traditional rough set theory (RST) is effective in identifying patterns and classifying uncertain data, it does not inherently capture *causal relationships* that is, understanding what factors bring about a particular outcome.

Causal Rough Set Theory (CRST) extends classical RST by introducing mechanisms to model and reason about causality. Rather than just grouping objects with similar characteristics, CRST aims to determine how changes in certain attributes (conditions) can lead to changes in decision outcomes .

Let $\mathcal{C}$ be a set of condition attributes that may causally influence a decision class X . Then:

$$\text{Causal Lower Approximation: } \underline{P}_{\text{cause}}(X) = \{x \in \mathcal{U} \mid \text{cause}(x) \Rightarrow X\}$$

This approximation identifies objects for which the observed attribute configuration can be reliably considered a cause of X . Unlike classical approximations, causal approximations are intended to reflect not just correlation, but influence.

Future research directions include:

- Defining new forms of causal granules and dependency models.
- Applying causal RST in domains like healthcare (e.g., identifying causes of disease), economics (e.g., impact of policies) and public decision making.
- Integrating CRST with probabilistic causal inference or structural equation modeling for better interpretability.

10.3. Real-Time and Edge-Aware Rough Sets

As more systems move to mobile devices and IoT (Internet of Things), RST must adapt to run efficiently on these limited platforms.

Some key questions are:

- How can we update RST results on-the-fly (in real-time) on small devices?
- Can we develop new methods that work well with fast incoming data streams?

Recent research looks at using smart techniques like sketching, quantization, and sampling to make quick and approximate decisions [42].

10.4. Standardization and Tool Development

Even though RST is a well-developed theory, it still lacks easy-to-use and standardized software tools. In the future, the focus should be on the following:

- Creating complete and user-friendly Python libraries (like RoughPy) [66].
- Developing benchmarks to measure how well RST models explain results.
- Integrating RST into popular machine learning tools like TensorFlow, PyTorch, or Scikit-learn.

10.5. Quantum-Inspired and Topological Extensions

Some future directions will explore more advanced math ideas to improve RST:

- **Quantum Rough Sets:** Use ideas from quantum computing, like superposition, to handle uncertainty in new ways.
- **Topological Rough Sets:** Use neighborhood systems and topology (a branch of mathematics) to create flexible boundary regions [19].

These new ideas could connect RST with cutting-edge areas like quantum computing and formal geometry.

10.6. Ethical and Fairness-Aware RST

As AI is increasingly used in sensitive areas, it's important to ensure fairness and avoid bias. RST, with its rule-based transparency, can help create ethical decision systems.

Goals for future research include:

- Developing fairness-aware lower approximations that are both explainable and unbiased.
- Understanding how bias can spread through decision rules in RST, especially for sensitive features like gender, caste, or income.

This is a growing area with a lot of potential for impactful work.

11. CONCLUSION

Rough Set Theory has evolved significantly since its origins as a symbolic framework for handling uncertainty. Today, it stands as a flexible and evolving tool, capable of addressing a wide range of problems where data are incomplete, noisy, and hard to interpret. Its ability to offer transparent and rule-based insights gives it a valuable place in fields that demand both precision and explainability.

In this survey, we have explored not only the foundations and extensions of RST but also its real-world impact, from artificial intelligence to healthcare, where it works alongside actual datasets to extract meaningful patterns. Our in-depth study of current challenges identified key roadblocks, including scaling to high-dimensional data, integrating with deep learning systems, and facilitating real-time decision-making in dynamic environments.

However, the progress is promising. Innovations in dynamic rough sets, causal inference, and explainable AI are steadily pushing boundaries. With stronger interdisciplinary collaboration and more user-friendly tools, RST is not only staying relevant but also

finding new ground. As data continues to grow more complex, the core strengths of rough set theory—interpretability, adaptability, and handling uncertainty — will only become more important in shaping the future of intelligent systems.

AUTHOR STATEMENTS AND DECLARATIONS

Author Contributions:

Arunava Bhattacharjee conceptualized the research, conducted the literature review, and drafted the initial manuscript. Prof. Sharmistha Bhattacharya Halder provided critical revisions, theoretical insights, and guidance throughout the study. Dr. Tandra Sarkar contributed to the interdisciplinary aspects, enhanced the discussion on computational applications, and assisted in manuscript refinement. All authors read and approved the final manuscript.

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The authors confirm that this work did not use any third-party data, materials, or objects. All data and materials discussed or analyzed in this study are original and fully attributed to the authors' own research and findings.

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