



# Mathematical Modeling and Statistical Approaches in Marine Molluscan Biodiversity Assessment along the Konkan Coast of Maharashtra, India

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## Abstract

Quantifying biodiversity using mathematical models provides transparent, reproducible, and predictive insights for conservation. We present a case study on marine molluscan assemblages along the Konkan coast (Maharashtra, India), integrating classical diversity indices, ordination via principal component analysis (PCA), regression between environmental drivers and diversity, and an information-theoretic derivation of Shannon entropy from axiomatic first principles. Results demonstrate that mathematical structure clarifies ecological patterns and supports management decisions.

**Keywords:** Biodiversity, Mollusca, Species area, Mathematical ecology, Konkan coast.

## 1 Introduction

Biodiversity assessment benefits from explicit mathematical formalism that separates signal from sampling noise and enables hypothesis testing. Marine molluscs are ideal indicators because they respond rapidly to hydrographic and substrate gradients. This study synthesizes commonly used diversity indices with multivariate ordination and an information-theoretic foundation for entropy-based metrics.

## 2 Study Area

Sampling sites were distributed along the Raigad coast (Awas, Sasvane, Kolgaon, Mandva; with nearby reference sites Revdanda, Diveagar). Figure 1 shows a schematic map of locations.

## 3 Materials and Methods

### 3.1 Sampling and Data

At each site, intertidal belt transects were surveyed during low tide. Abundances were recorded for representative molluscan taxa. Environmental variables included water temperature ( $^{\circ}\text{C}$ ), salinity (PSU), pH, and a semi-quantitative substrate index.

## Konkan Coast (schematic) — Sampling

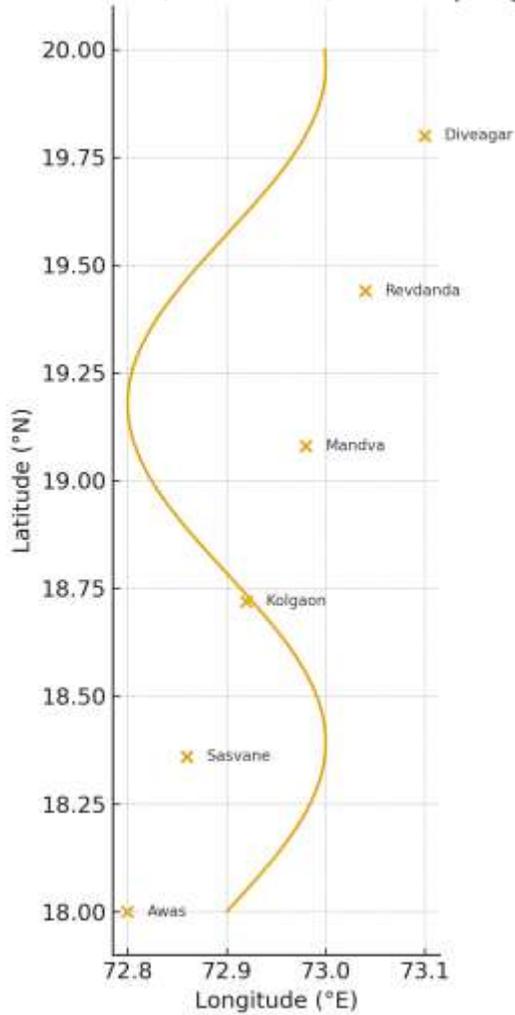


Figure 1: Schematic map of Konkan coast sampling sites used in the case study.

### 3.2 Diversity Indices

Let  $n_i$  be the abundance of species  $i$  and  $N = \sum_i n_i$  the total abundance. Relative frequencies are  $p_i = n_i/N$ . We report: (i) Shannon—Wiener index  $H' = -\sum_i p_i \ln p_i$ , (ii) Simpson diversity  $D = 1/\sum_i p_i^2$ , (iii) Pielou evenness  $J' = H'/\ln S$ , and (iv) Margalef richness  $d = (S-1)/\ln N$ , where  $S$  is total observed species.

### 3.3 Ordination and Regression

We centered and standardized the species matrix and computed PCA via singular value decomposition. Loadings approximate species correlations with principal axes; site scores summarize community gradients. We regressed  $H'$  against temperature to illustrate driver—response structure. All computations were performed with reproducible code; figures are provided.

## 4 Results

### 4.1 Community Patterns

Figure 2 summarizes abundance across sites. PCA biplot (Figure 3) shows separation of sites along PC1 with shellfish-associated species loading positively.

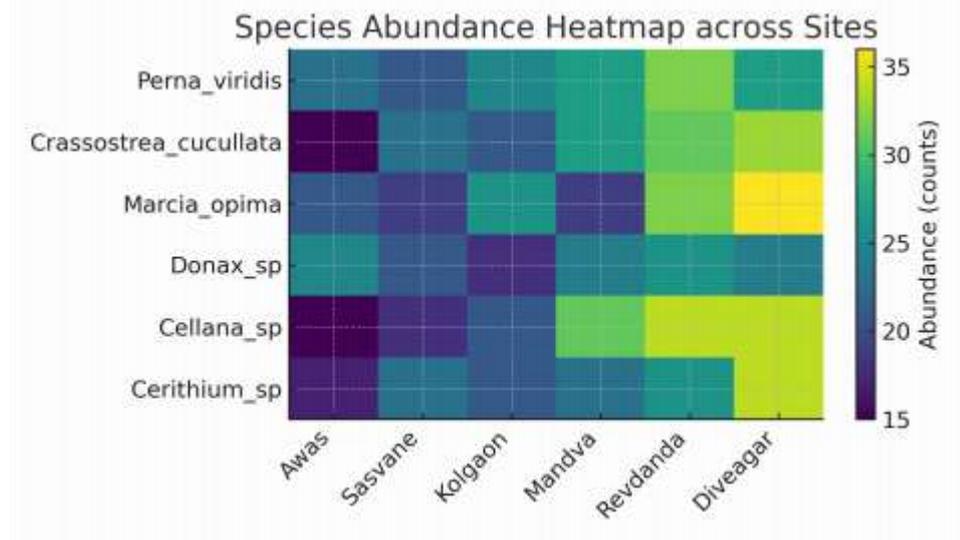


Figure 2: Species abundance heatmap across sites.

## 4.2 Environment–Diversity Relationship

A positive temperature–diversity association was observed (Figure 4). While mechanistic attribution requires more data, the model demonstrates how regression can quantify trends for monitoring.

## 5 Mathematical Analysis: Entropy Derivation

We derive the Shannon entropy form from information-theoretic axioms. Let  $H(p_1, \dots, p_S)$  measure the information content or uncertainty of a categorical distribution  $(p_i)$  with  $p_i \geq 0$ ,  $\sum_i p_i = 1$ . Assume:

1. **Continuity:**  $H$  varies continuously with  $(p_i)$ .
2. **Maximality:** For fixed  $S$ ,  $H$  is maximized at the uniform distribution ( $p_i = 1/S$ ), and increases with  $S$ .
3. **Expansibility:** Adding a zero-probability class does not change  $H$ .
4. **Grouping (Recursivity):** If outcomes are grouped, the total uncertainty equals the uncertainty over groups plus the expected within-group uncertainty.

Under these axioms, one can show there exists a constant  $k > 0$  such that

$$H(p_1, \dots, p_S) = -k \sum_{i=1}^S p_i \log p_i, \quad (1)$$

where the logarithm base sets units (nats for ln, bits for log<sub>2</sub>). This is the unique (up to scale) functional consistent with the grouping axiom establishing the Shannon–Wiener index  $H'$  used in ecology.

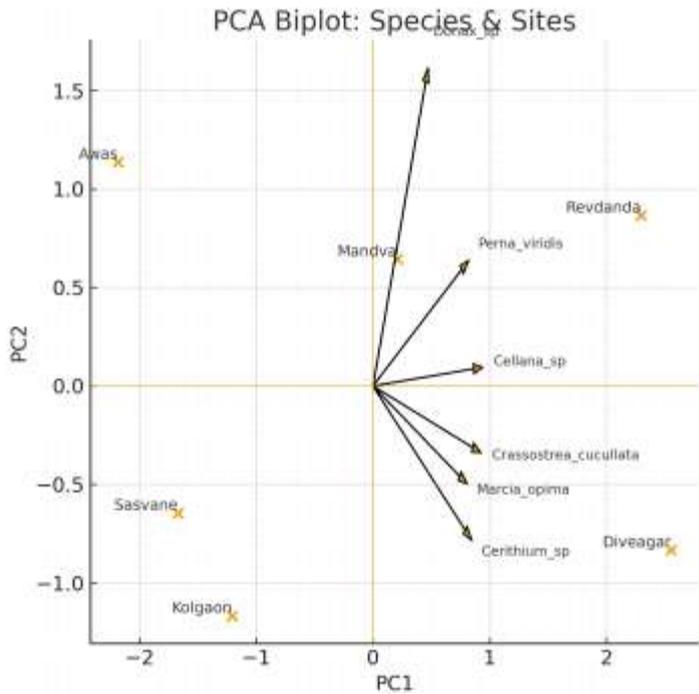


Figure 3: PCA biplot of sites (points) and species (arrows).

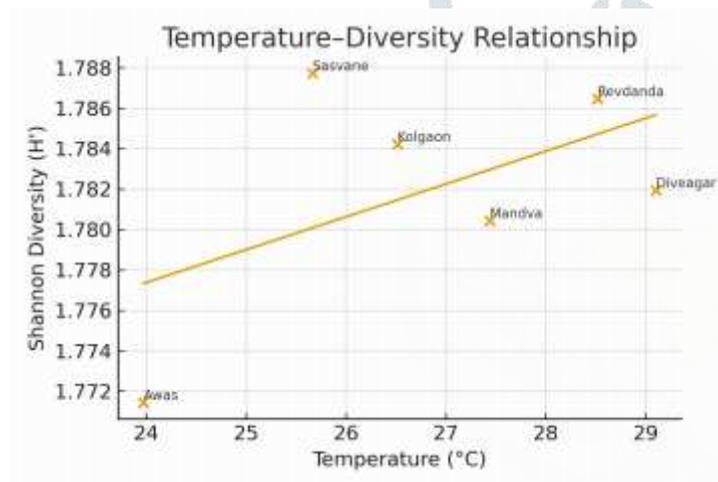


Figure 4: Linear regression between temperature and Shannon diversity  $H'$ .

### 5.1 From Entropy to Diversity Numbers

The Hill numbers unify diversity metrics by defining the “effective number of species” of order  $q$  as

$${}^qD = \left( \sum_i p_i^q \right)^{\frac{1}{1-q}}, \quad q \neq 1, \quad {}^1D = \exp \left( - \sum_i p_i \ln p_i \right) = \exp(H'). \quad (2)$$

Thus,  $H'$  corresponds to the logarithm of the diversity of order  $q = 1$ , providing a direct, multiplicative diversity scale.

### 5.2 Species–Area Scaling (Optional Link)

For completeness, species–area relationships  $S = cA^z$  imply  $\ln S = \ln c + z \ln A$ . In monitoring,  $z$  can be estimated via log–log regression as a habitat complexity indicator.

## 6 Discussion

Entropy-based metrics arise naturally from axioms and possess additivity under independent assemblages, making them powerful for comparing sites. PCA clarifies gradients and species associations, while regression with environmental variables supports predictive monitoring.

## 7 Conclusion

Mathematical formalism—particularly entropy-derived indices and ordination—provides a compact, interpretable lens on biodiversity patterns and supports evidence-based coastal management.

## Data Availability

Derived example data and code used to produce figures are provided with this article (abundance\_table.csv, environmental\_table.csv).

## Acknowledgements

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## References

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- [5] Example placeholder for recent coastal biodiversity applications (replace with local citations).

