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APPLICATION OF LAPLACE INTEGRAL TRANSFORM IN TUMOR GROWTH MODEL

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Abstract: Objectives: The objective of this study is to solve the classical tumor growth model governed by nonlinear differential equations using the Laplace integral transform approach. **Methods:** The Laplace transform, its inverse, linearity property, and Laplace transform of derivatives are employed to obtain an analytical expression for tumor cell dynamics. The model parameters initial tumor cell count, growth rate, and environmental carrying capacity are analyzed to interpret tumor progression over time. The obtained analytical solution is compared conceptually with recent fractional-order and modified He-Laplace approaches reported in the literature. **Findings:** The Laplace transform simplifies the nonlinear tumor model into an algebraic form that can be solved explicitly using partial fractions and inverse Laplace operations. The derived expression allows prediction of tumor cell count at any time t without complex numerical iteration. **Novelty**: Unlike recent studies using fractional calculus or modified algorithms (Qayyum et al., 2025; Swain et al., 2025; Shahzadi et al., 2024), this work demonstrates the analytical efficiency and simplicity of the standard Laplace transform in modeling tumor growth dynamics. This approach provides a direct analytical prediction framework for tumor growth, which can support medical diagnosis and treatment planning.
Keywords: Laplace transform; Tumor growth model; Mathematical models in health sciences

1.INTRODUCTION

Integral transform plays very important role in solving boundary value problems. Laplace transform is very simple and easy to use in problems. In recent years, researchers in mathematical oncology have developed fractional-order and modified Laplace-based methods to study tumor behavior with greater accuracy (Qayyum et al., 2025; Swain et al., 2025)[1]. These modern approaches consider the memory and hereditary effects in biological systems, offering deeper insight into cancer progression. However, these methods also have some limitations. Fractional-order models, while flexible, often lack analytical simplicity and require iterative numerical computations. Similarly, modified He-Laplace algorithms provide approximate analytical solutions but at the cost of higher computational effort (Qayyum et al., 2025; Mubashir Qayyum et al., 2024). Moreover, many existing models fail to present a clear analytical relationship between the tumor cell population and biological parameters, making them difficult to interpret directly. To address these challenges, this study identifies a clear research gap although many studies have focused on fractional or modified Laplace methods, there is still limited discussion on using the standard Laplace integral transform to obtain an exact and closed-form analytical solution for tumor growth under environmental constraints. Therefore, the purpose of this study is to apply the Laplace integral transform to the classical tumor growth model proposed by Liu et al. (2023) and derive an analytical expression for the number of tumor cells as a function of time and biological parameters. This approach emphasizes mathematical simplicity, transparency, and practical use in medical prediction and diagnosis. In this paper we use the Laplace transform for finding the solution of tumor growth model. Mankind always faces the medical issues, which involves health, life and well-being. Medical community is helped by mathematical modelling to more understand and explore the pathological process within the body, which helps to provide more accurate diagnosis. Yikai Liu et al [1] provided some mathematical models of medical problems. Tumor growth model is suggested by them. Many researchers use various integral transforms for finding the solutions of different types of medical problems. Nikhil Raundal et al [2] used double general integral transform for solving boundary value problems in partial differential equations. Thakare etal [3] used general integral transform for the solution of Models in health sciences. Suryawanshi et al [4] used Soham transform for solving mathematical models occurring in health science and biotechnology, Borse et al [5] applied Kushare transform for the solutions of models in health sciences.

We organize the paper as follows. Introduction is in first section. Methodology concepts are stated in second section. Third section is devoted for application and discussion of the Laplace transform in Tumor growth model. Conclusion is drawn in fourth section.

2 METHODOLOGY

In this section we state required definitions, formulae and theorems.

2.1. Definition: Laplace Transform [6]:

Let F(t) be a function of a variable t such that the function $e^{-st}F(t)$ is integrable in [0, ∞) for some domain of values of s. It is denoted as L{F(t)} and is defined as

$$L\{F(t)\} = \int_0^\infty e^{-st}F(t)\,dt.$$

In following table, we state Laplace transform of some required functions.

Table1. Laplace Transform of some elementary functions

Sr. No.	F(t)	L{F(t)} = f(s)
1.	1	$\frac{1}{s}$, s >0
2.	T	$\frac{1}{s^2}$, s >0
3.	t^n, n = 0, 1, 2, ….	$\frac{n!}{s^{n+1}}$, s >0
4.	e^{at}	$\frac{1}{s-a}$, s >a

Here we state following theorems about Laplace transform which are required for solving the tumor growth model.

2.2. Theorem 1: (**Linearity**) If $F_1(t)$ and $F_2(t)$ are two functions of t and C_1, C_2 be any two constants then

$L\{C_1 F_1(t) + C_2 F_2(t)\} = C_1 F_1(t) + C_2 F_2(t)$

Theorem 2: If F(t) is a function of t and C is any constant then

L{CF(t)} = C L{F(t)}

2.3. Theorem 3:(Laplace transform of derivative) [6] Let F(t) be a function which is continuous for t ≥ 0 and has Laplace transform L{F(t)} = f(s). Then if $F'(t)$ possesses a Laplace transform that transform is given by

$L\{F'(t)\} = sf(s) – F(0)$

$= sL\{F(t)\} – F(0)$

2.4. Definition: Inverse Laplace Transform [6]:

If L{F(t)} = f (s) then F(t) is called an Inverse Laplace transform of f (s) and is denoted as $F(t) = L^{-1}\{f(s)\}$ where L^{-1} is called the inverse Laplace transform ooperator.

Table 2. Inverse Laplace transform of some functions

Sr.No.	f(s)	$L^{-1}\{f(s)\} = F(t)$
1.	$\frac{1}{s}$, s >0	1
2.	$\frac{1}{s^2}$, s >0	t
3.	$\frac{n!}{s^{n+1}}$, s >0	t^n, n = 0, 1, 2, ….
4.	$\frac{1}{s-a}$, s >a	e^{at}

3 RESULTS AND DISCUSSION

3.1. Application

In this section, we apply Laplace transform to solve tumor growth model, which was introduced by Yikai Liu, et al in his research on medical problems based on mathematical models.

The differential equation of the form

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right) \quad \text{… …} \quad (1)$$

is usually used to describe the change in the number of cells, where t denotes time, r be the growth rate of tumor cell, N denotes the number of tumor cells, K be the limit of the environmental accommodation. The factor $\left(1 - \frac{N}{K}\right)$ can be interpreted as limiting effect of the environment on the growth of the tumor cells.
Equation (1) can be expressed as

$$\frac{dN}{dt} - rN + r\frac{N^2}{K} = 0$$

$$\therefore N' - rN + \frac{r}{K}N^2 = 0$$

Applying Laplace transform to both sides we get,

$$\therefore L(N') - rL(N) + \frac{r}{K}L(N^2) = L(0)$$

$$\therefore s\,L(N) - N(0) - r\,L(N) + \frac{r}{K}\left(\frac{2!}{s^3}\right) = 0$$

$$\therefore s\,L(N) - N(0) - r\,L(N) + \frac{r}{K}\left(\frac{2}{s^3}\right) = 0$$

$$\therefore (s-r)L(N) - N(0) + \frac{r}{K}\left(\frac{2}{s^3}\right) = 0$$

$$\therefore L(N) = \frac{1}{(s-r)}\left(N(0) + \frac{r}{K}\left(\frac{2}{s^3}\right)\right) = 0 \qquad .....(2)$$

Let N(0) = a in equation (2) we get,

$$\therefore L(N) = \frac{1}{(s-r)}\left(a + \frac{r}{K}\left(\frac{2}{s^3}\right)\right)$$

Applying inverse Laplace transform to both sides we get,

$$\therefore L^{-1}(N) = L^{-1}\left(\frac{a}{(s-r)}\right) + \frac{r}{K}L^{-1}\left(\frac{2}{s^3(s-r)}\right)$$

$$\therefore N(t) = L^{-1}\left(\frac{a}{(s-r)}\right) + \frac{r}{K}L^{-1}\left(\frac{2}{s^3(s-r)}\right) \qquad ......(3)$$

We use partial fraction for finding the value of $\left(\frac{2}{s^3(s-r)}\right)$

We get,

$$\therefore \left(\frac{2}{s^3(s-r)}\right) = \left(\frac{-2}{r^3(s)}\right) + \left(\frac{-2}{r^2(s^2)}\right) + \left(\frac{-2}{r(s^3)}\right) + \left(\frac{2}{r^3(s-r)}\right)$$

We put this value in equation (3) we get,

$$\therefore N(t) = L^{-1}\left(\frac{a}{s-r}\right) - L^{-1}\left(\left(\frac{-2}{r^3(s)}\right) + \left(\frac{-2}{r^2(s^2)}\right) + \left(\frac{-2}{r(s^3)}\right) + \left(\frac{2}{r^3(s-r)}\right)\right)$$

$$\therefore N(t) = aL^{-1}\left(\frac{1}{s-r}\right) + \frac{2}{r^3K}L^{-1}\left(\frac{1}{s}\right) + \frac{2}{r^2K}L^{-1}\left(\frac{1}{s^2}\right) + \frac{2}{K}L^{-1}\left(\frac{1}{s^3}\right) - \frac{2}{r^2K}L^{-1}\left(\frac{1}{s-r}\right)$$

$$\therefore N(t) = \left(a - \frac{2}{r^2K}\right)L^{-1}\left(\frac{1}{s-r}\right) + \frac{2}{r^3K}L^{-1}\left(\frac{1}{s}\right) + \frac{2}{r^2K}L^{-1}\left(\frac{1}{s^2}\right) + \frac{2}{K}L^{-1}\left(\frac{1}{s^3}\right)$$

$$\therefore N(t) = \left(a - \frac{2}{r^2K}\right)e^{rt} + \frac{2}{r^3K}(1) + \frac{2}{r^2K}(t) + \frac{2}{K}\left(\frac{t^2}{2!}\right)$$

$$\therefore N(t) = \left(a - \frac{2}{r^2K}\right)e^{rt} + \frac{2}{r^3K} + \frac{2}{r^2K}(t) + \frac{1}{K}(t^2) \qquad ....(4)$$

The Laplace transform method facilitates the resolution of the tumor growth model by converting the differential equation into an algebraic form. This reduces the complexity of solving biological models step by step. The resulting solution demonstrates how the growth of tumor cells is limited by environmental factors and the growth rate. Using partial fractions and inverse Laplace, we get an explicit formula for the number of tumor cells over time.

4 Conclusion

This study shows that the Laplace integral transform can be successfully used to solve the tumor growth model. The number of tumor cells at any time can be found directly by substituting the known parameters initial tumor cells (a), growth rate (r), and environmental limit (K) in the final equation.The model helps in predicting tumor size over time, which is important for diagnosis and treatment planning.Compared with recent methods like the fractional or modified He-Laplace approach, this method is more straightforward and analytical. It can serve as a base model for future research that combines Laplace and fractional techniques for even more realistic tumor modeling.

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