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Color Class Dominating Sets on Regular graphs of degree 3 and 4

Dr. A. Vijayalekshmi,
Associate Professor, Department of Mathematics,
S.T. Hindu College, Nagercoil – 629002,
Tamil Nadu, India
(Affiliated to Manonmaniam Sundaranar University, Tirunelveli – 627012)

S. G. Vidhya,
Research Scholar, Reg.No: 19223152092017,
Department of Mathematics,
S.T. Hindu College, Nagercoil – 629002,
Tamil Nadu, India
(Affiliated to Manonmaniam Sundaranar University, Tirunelveli – 627012)

Abstract

Let $G=(V,E)$ be a graph. A color class dominating set of G is a proper coloring $\mathcal{C}$ of G with extra property that every color class in $\mathcal{C}$ is dominated by a vertex in G. A color class dominating set is said to be minimal color class dominating set if no proper subset of $\mathcal{C}$ is a color class dominating set of G. The color class domination number of G is the minimum cardinality taken over all minimal color class dominating sets of G and is denoted by $\gamma_\chi(G)$. Here we also obtain $\gamma_\chi(G)$ of regular graph degree 3 and degree 4.

Keywords: Chromatic number, Domination number, Color Class Dominating set, Color Class Domination number.

Mathematics subject classification: 05C15, 05C69

1. Introduction

All graphs considered in this paper are finite, undirected graphs and we follow standard definitions of graph theory [2]. Let $G=(V,E)$ be a graph of order p. The open neighborhood $N(v)$ of vertex $v \in V(G)$ consist of the set of all vertices adjacent to v. The closed neighborhood of v is $N[v]=N(v)\cup\{v\}$. For a set $S \subseteq V$, the open neighborhood $N(S)$ is defined to be $\cup_{v\epsilon S} N(v)$, and the closed neighborhood of S is $N[S] = N(S) \cup S$ for any subset H of vertices of G, the induced sub graph $< H >$ is the maximal sub graph of G with vertex set H. A subset S of V is called a dominating set if every vertex in $V - S$ is adjacent to some vertex in S. A dominating set S is called a minimal dominating set if no proper subset of S is a dominating set of G. The domination number $\gamma(G)$ is the minimum cardinality taken over all minimal dominating sets of G. A γ- set is any minimal dominating set with cardinality γ. A proper coloring of G is an assignment of colors to the vertices of G such that adjacent vertices have different colors. The smallest number of colors for which there exists a proper coloring of G is called chromatic number of G and is denoted by $\chi(G)$. A color class dominating set of G is a proper coloring $\mathcal{C}$ of G with the extra property that every color class in $\mathcal{C}$ is dominated by a vertex in G. A color class dominating set is said to be a minimal color class dominating set if no proper subset of $\mathcal{C}$ is a color class dominating set of G. The color class domination number of G is the minimum cardinality taken over all minimal color class dominating set of G and is denoted by $\gamma_\chi(G)$. This concept was introduced by Vijayalekshmi et al [2]. A graph G is said to be r- regular if degree of each vertex of G is r. A 3-regular graph is also called a cubic graph. In this paper we obtain color class domination of regular graphs of degree 3 and degree 4.

2. Main Results

Theorem 2.1

Let G be a regular graph of degree 3 without triangles, then

$$\gamma_\chi(G) = \begin{cases} \frac{n}{2} & if\ n \equiv 0 (mod\ 4) \\ \left(\frac{n}{2}\right) - 1 & if\ n \equiv 2 (mod 4) \end{cases}$$

Proof:

Let G be the regular graph of degree 3 with order $n = 2p$

Let $V(G) = \{v_1, v_2, v_3 \ldots\ldots\ldots\ldots v_p, v_{p+1} \ldots\ldots\ldots\ldots v_n\}$ where

$N(v_i) = \{v_{i-1}, v_{i+1}, v_{(p+i)-1} \ /i = 2,4 \ldots\ldots\ldots\ldots (p-1)\}$

$N(v_i) = \{v_{i-1}, v_{i+1}, v_{(p+i)+1} \ /i = 3,5 \ldots\ldots\ldots\ldots (p-1)\}$

$N(v_1) = \{v_2, v_p, v_{p+2}\}$

$N(v_p) = \{v_{p-1}, v_1, v_{(n-1)}\}$

$N(v_{p+1}) = \{v_2, v_{p+2}, v_n\}$ and $N(v_n) = \{v_{n-1}, v_{p-1}, v_{p+1}\}$

$N(v_i) = \{v_{i-1}, v_{i+1}, v_{i-p-1} \ /i = (p+2), (p+4) \ldots\ldots\ldots\ldots\ldots (n-1)\}$

$N(v_i) = \{v_{i-1}, v_{i+1}, v_{i-p+1} \ /i = (p+3), (p+5) \ldots\ldots\ldots\ldots\ldots (n-1)\}$

We consider 2 cases

Case (i) $n \equiv 0(mod4)$

Assign distinct colors say $i(i = 2,4 \ldots \ldots \ldots \ldots \ldots (p-2))$ and p to the vertices $\{v_i,\ v_{p+i}\}$ and $\{v_n\}$ respectively. Also assign distinct colors, say, $i(i = 5,7 \ldots \ldots \ldots \ldots \ldots (p-3), (p-1))$ to the vertices $\{v_i,\ v_{p+i+2}\}$ and $\{v_{p-1}\}$ respectively. Assign colors 1 and 3 to the vertices $\{v_1, v_3, v_{p+1}\}$ and $\{v_{p+3}, v_{p+5}\}$ respectively, we get $\gamma\ (G) = {}^{n}{}_{\chi}2$ –

Illustration: (G_{20})

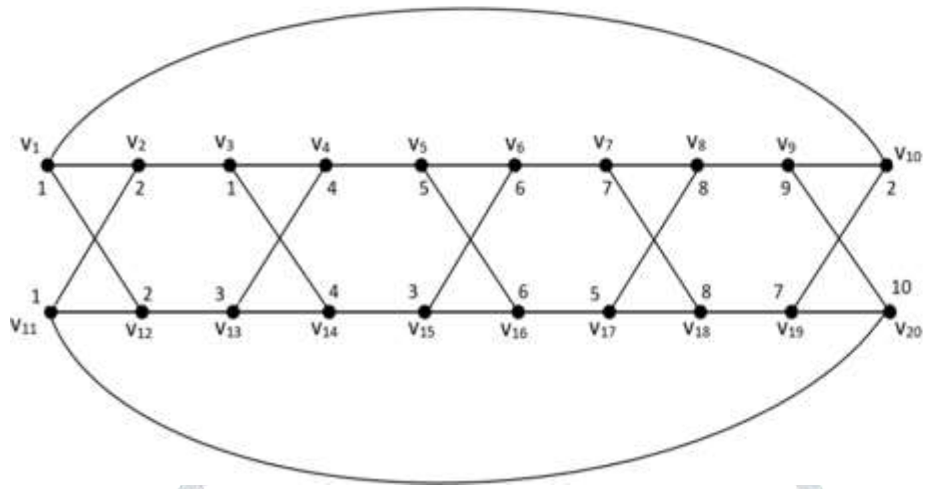

Figure 1

$\gamma_{\chi}(G_{20}) = 10$

Case (ii)

When $n \equiv 2(mod 4)$

Assign colors 1 and 3 to the vertices $\{v_1, v_3, v_{p+1}\}$ and $\{v_{p+3}, v_{p+5}\}$ respectively. Assign distinct colors, say $2i\ (1 \leq i \leq {}^{p-1})$ to the vertices, say,2

$\{v_{2i}$

$, v_{p+2i}\}$. Also assign distinct colors, say, $(2i-1)$ $(3 \leq i \leq ^{p-1})$ to the_vertices, say2
v
$, v$
$\}$, we obtain γ
coloring of G. Hence γ $(G) = (^{n}) - 1$
$2i-1$
$p+2i+2$ χ χ 2

Illustration: (G_{22})

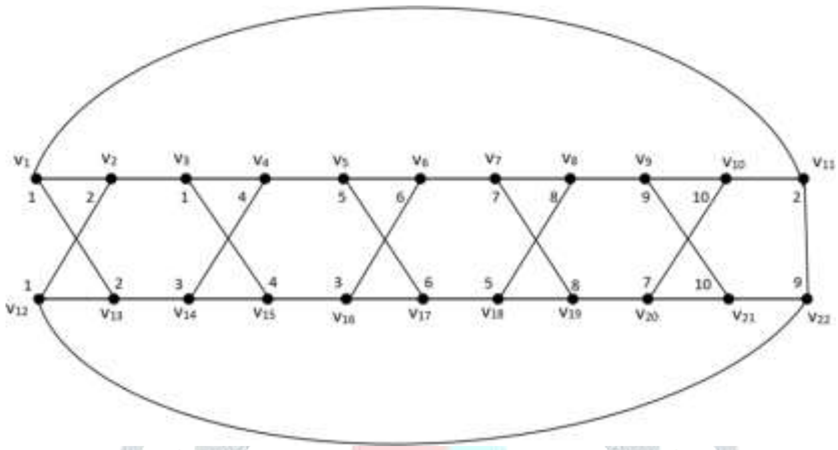

Figure 2

$\gamma_{\chi}(G_{22}) = 10$

Theorem 2.2

^{n} $if\ n\ is\ even$
If G is a regular graph of degree 4 then $\gamma_{\chi}(G) = \{$ ^{2} $-$
$\lceil - \rceil$ $if\ n\ is\ odd$ n
2

Proof

Let G be a regular graph of degree 4 with order $n = 2p$

Let $V(G) = \{v_1, v_2, v_3 \ldots\ldots\ldots\ldots\ldots\ldots\ldots v_n\}$ we consider two cases Case (i) when n is even

Let $N(v_i) = \{v_2, v_p, v_{2p-1}, v_{2p}\}$

$N(v_p) = \{v_1, v_{p-1}, v_{(p+1)}, v_{(p+2)}\}$

$N(v_{p+1}) = \{v_{p-2}, v_{p-1}, v_{p+1}\}$

$N(v_n) = \{v_1, v_2, v_{p+1}, v_{n-1}\}$

Assign distinct colors 1 and 2 to the vertices $\{v_1\}, \{v_{p+1}\}$ and $\{v_p, v_n\}$ respectively. Also assign distinct colors, say 3 $(3 \leq i \leq ^{n})$ to the vertices $\{v$

$_{,}v\}$ we obtain the

2

γ_{χ} coloring of G. Thus $\gamma_{\chi}(G) = ^{n}$

$-12p-(i-2)$

$\overline{2}$

Illustration: (G_{26})

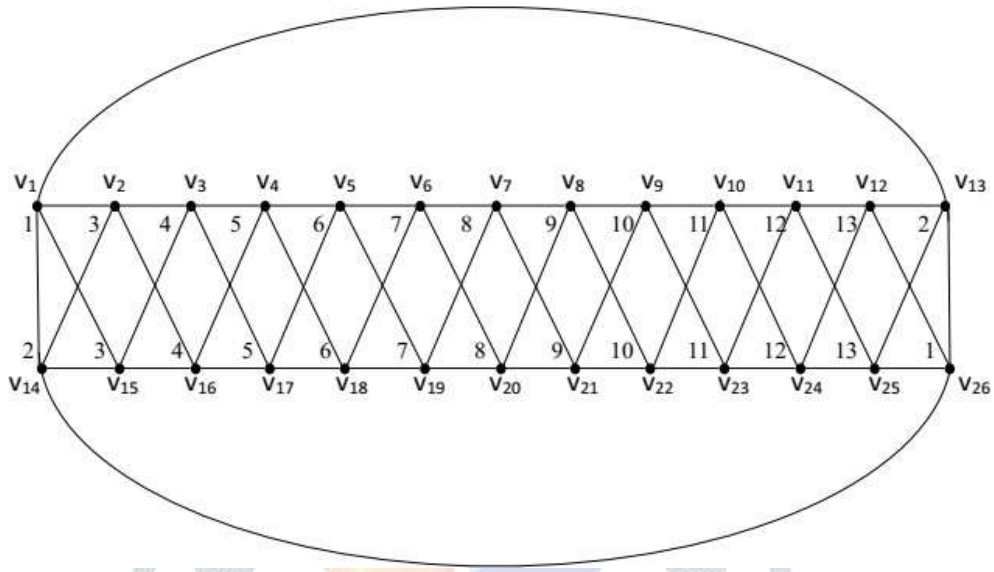

Figure 3

$\gamma_{\chi}(G_{26}) = 13$

Case (ii) when n is odd

Let $N(v_1) = \{v_2, v_3, v_n, v_{n-1}\}$

$N(v_2) = \{v_3, v_4, v_1, v_n\}$

$N(v_{p+1}) = \{v_{p-2}, v_{p-1}, v_{p+1}\}$

$N(v_n) = \{v_1, v_2, v_{n-1}, v_{n-2}\}$

$N(v_{n-1}) = \{v_1, v_n, v_{n-2}, v_{n-3}\}$

$N(v_i) = \{v_{i+1}, v_{i+2}, v_{i-1}, v_{i-2}\}$, $i = 3,4 \ldots\ldots\ldots\ldots (n-2)$ respectively

Assign distinct colors $i(1 \leq i \leq {}^{n}$) and2 $\frac{n}{}$

$\lceil_2\rceil$ to the vertices say $\{v_{i,}$

v_{i+3}

$and\ v_{n-2}\}$

respectively we get γ_χ coloring of G. Thus $\gamma_\chi(G) = \lceil^{n}\rceil$. $\frac{}{2}$

Illustration: (G_{15})

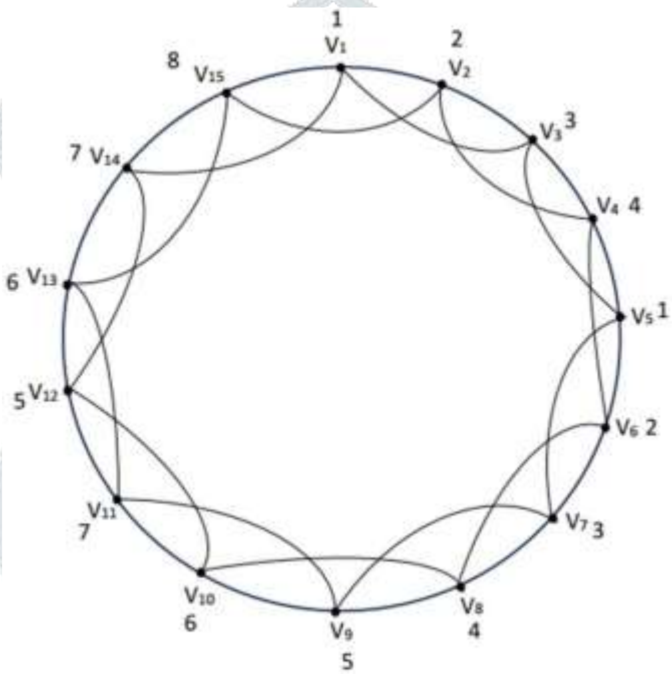

Figure 4

$\gamma_\chi(G_{15}) = 8$

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