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# Some Properties of Leaf Graph

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***Abstract :***

In this paper, a new concept of Leaf Graph is introduced. A Graph is said be a Leaf Graph if it has a Leaf Structure some theorems on the leaf Graph are discussed. The Leaf Graph and some Theorems on Leaf Graphs and also newly defined.

Keywords:

Leaf Graph, Connected Graph, Complete Graph, Degree of the Graph, Tree, Cyclic Graph.

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## 1. INTRODUCTION

Graph theory is a powerful branch of mathematics used to study the relationships between objects through vertices and edges. A Leaf is a vertex with degree one, often representing an end point in a tree or network. Graphs that emphasize such vertices are called Leaf Graphs. These structures are commonly found in computer science, Biology, and Communication networks. Leaf notes often represent clients, terminals or endpoints in real-world applications.

Studying the Leaf Graph helps us understand how these boundaries vertices affect connectivity and structure. This project focuses on identifying and proving key properties of Leaf Graphs. We examine how Leaf distribution impacts the connectedness of a Graph. One important aspect is whether the Graph remains connected in the absence of end vertices. We also study how having multiple leaves in a component can ensure overall connectivity

## II. Preliminaries

### 2.1. Definition:

A *Graph* G = (V, E) is a mathematical structure consisting of a set of vertices V and a set of edges E, where each edge connects two vertices.

### 2.2 Definition:

A *Vertex* is a fundamental unit or point in a graph where edges meet. It represent the relationship or connection between them.

**2.3 Definition:**

An *edge* is a line that connects two vertices in a graph. It represents the relationship or connection between them.

**2.4 Definition:**

The *degree* of a vertex is the number of edges incident (connected) to it.

If deg(v) = 1, then v is called a leaf or end vertex.

**2.5 Definition:**

A Graph is said to be a *Leaf Graph* if, except for the end vertices, all the vertices degree is three.

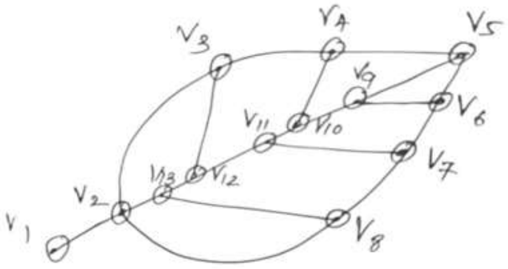


Fig:2.1

**2.6 Definition:**

A *Component* of a graph is a maximal connected subgraph. If a is not connected, It has more than one component.

**2.7 Definition:**

A Leaf graph is said to be *connected* if there is a path between any pair of vertices in the graph.

**Theorem 2.1:**

Every Leaf Graph is not complete.

**Proof:**

Given that: G is a leaf graph

Prove that: G is not a complete graph.

Suppose that: G is complete.

G is complete, then every pair of vertices is joined by an edge. G is a leaf graph; we have the end vertices. Every complete graph has more than one vertex. But the end vertex's degree is one.

Hence, every leaf graph G is not a complete graph.

**Example:**

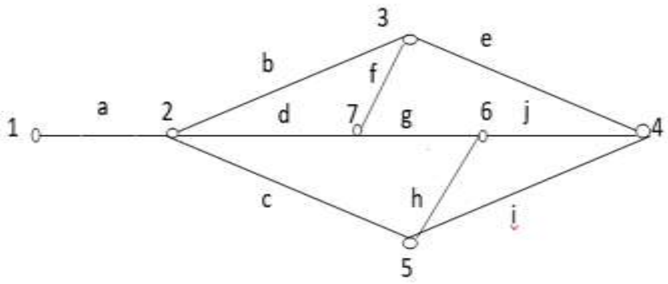


Fig : 2.2

n(p) = 7 , n(q) = 9, deg(3,4,5,6,7) = 3 , deg(1) = 1, deg(2) = 4

Hence, The Graph is not complete.

Hence The Proof

**Theorem:2.2**

A leaf graph is connected IFF at least it have two components.

**Proof:**

Assume that: The Leaf graph is connected.

Show that: Atleast two leaves lie in the same component.

If the graph is connected, there must be a path between any two vertices. Suppose that each

The leaf belongs to a different component.

Then, there is no path between them. Which is a contradiction. The Graph is connected. Therefore, at least two leaves must be in the same component.

Conversely,

Assume that: At least two leaves lie in the same component.

Let those two leaves be V1and V2. Since they lie in the same component. There exists a path connecting V1 and V2. If every vertex is connected path in this component, the graph is connected.

Hence, the graph is connected

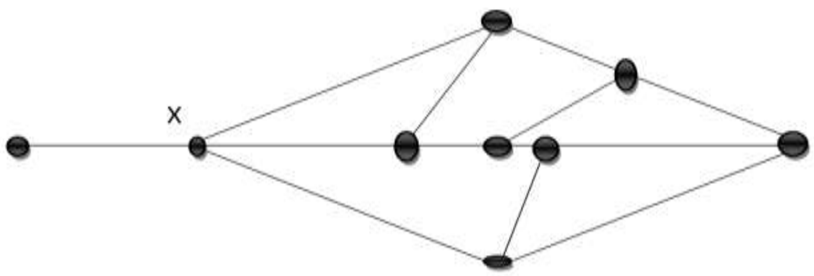


Fig : 2.3

**Theorem: 2.3**

If a leaf graph does not contain any end vertices (no vertex of degree 1), then the graph is connected.

**Proof:**

Assume that: The graph does not contain any end vertices. That is, every vertex has a degree greater than or equal to 2. This means that no vertex is isolated.

Let's suppose that the graph is not connected. Then it has two or more components. In such disconnected components, at least one component will have fewer edges. This introduces a leaf ( degree 1). But we assumed no leaves exist, which is a contradiction.

Therefore, the graph must be connected.

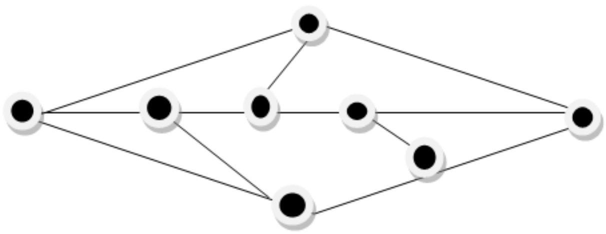

Fig : 2.4

## 3. Conclusion:

This paper focused on the concept of the Leaf Graph and its unique properties. We studied their structure, degree, and how leaf nodes are connected. Leaf graphs are useful in areas like computer science and network design. Understanding them helps solve real-world problems more efficiently. This study can be expanded further for advanced research and applications.

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