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Energy of Pyramid Fibonacci and Ripples Hamiltonian Graphs

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Abstract: The graph energy is a graph spectrum based quantity introduced in 1970's and the Laplacian spectrum has been applied to Machine Learning. Laplacian Energy (LE), LE like invariant, Skew LE, Normalized LE, Signless LE and Signless Laplacian Resolvent Energy are studied and some of these energies are extended to a special (4,2) graph. Basic facts of some of these energies are discussed and finally, significant difference between Energy and Laplacian Energies are analyzed. Further from Pyramid Fibonacci graph Ripples Hamiltonian graphs are considered and studied.

Keywords: Energy, Fibonacci, Golden Ratio, Hamiltonian, Ripples.

1. INTRODUCTION

A pictorial representation of a graph is very convenient for a visual study, other representations are better for computer processing. A matrix is a convenient and useful way of representing a graph to a computer. Matrices lend themselves easily to mechanical manipulations. Besides many known results of matrix algebra can be readily applied to study the structural properties of graphs from an algebraic point of view. In many applications of Graph theory, such as in electrical network analysis and operations research, matrices also turn out to be the natural way of expressing the problem. In the Mathematical field of Graph Theory, the adjacency matrix of a graph provides a non-pictorial representation of a graph and this raises the natural question of what the highly developed theory of matrices can tell us about graphs. Since, many graphical properties are invariant, under a relabeling of the vertices. We are particularly motivated to study those matrix concepts which are invariant under the simultaneous reordering of rows and columns. Furthermore, Eigen values of a matrix are one such concept [1]. Graph theory is a fundamental powerful mathematical tool for designing and analyzing interconnection networks, since the topological structure of an interconnection networks is a graph. When a graph is use as an interconnection networks, it should contain certain subgraph structures since existence of these structures has special importance for executing certain algorithms. A highly symmetric network is desirable since it is advantage to construction and simulation of some algorithms. Motivated by the algorithms about these topological structure of networks in this paper various Laplacian energies of the initial part of a special graph is examined. In section 2 basic definition and preliminaries are given, in section 3 Energy of $G(4, 3)$ is equivalent to twice the Golden Ratio is shown and in section 4 various Laplacian energies of G are obtained. Also two infinite Graphs studied.

2. Basic Definitions and Preliminaries

In this section, we recall some definitions and theorem [1,2]. The Laplacian matrix which also called as Admittance matrix, Kirchhoff matrix or discrete Laplacian, is a matrix representation of a Graph [2]. The Matrix $L(G) = D(G) - A(G)$ is called Laplacian matrix where $D(G)$ is Diagonal matrix and $A(G)$ is Adjacency matrix and their spectrum is called Laplacian spectrum of a Graph.− The sequence of Eigen values is called the spectrum of G

The Laplacian Energy of a Graph G as put forward by I. Gutman has noteworthy chemical applications and its mathematical aspect is well developed and is been defined as [2,3]

$$LE(G) = \sum_{i=1}^{n} |\mu_i - \frac{2m}{n}|$$

More explicitly, if G is a simple graph with n vertices its Laplacian Matrix $L_{n\times n}$ is defined as $L(G) = D(G)\text{-}A(G)$ Where $D(G)$ is the degree matrix and $A(G)$ is the adjacency matrix of the graph. Since G is a simple graph, A only contains $1's$ or $0's$ and its diagonal elements are all $0's$. In the case of directed graphs, either the in degree or out degree might be used, depending on the application. The elements of L are given by

$$L = \begin{cases} \deg(v_i) & if\ i = j \\ -1 & if\ i \neq j\ and\ v_i\ is\ adjacent\ to\ v_j \\ 0 & otherwise \end{cases}$$

where $\deg(v_i)$ is the degree of the vertex (v_j)

The sum of the square roots of the Laplacian Eigen values. From its name, one could get the impression that the properties of LEL are similar to those of the Laplacian Energy LE.

$$LEL(G) = \sum_{i=1}^{n} \sqrt{\mu_i} \qquad (2)$$

Let S (G) be the skew adjacency matrix of a simple digraph G. Then the skew Laplacian energy of the digraph G is defined as

$$E_{SL}(G) = \sum_{i=1}^{n} \lambda_i^2 \qquad (3)$$

Where n is the order of G and $_1,\lambda_2,\lambda_3,\ldots.,\lambda_n$ are the eigenvalues of the Laplacian matrix
L (G) = D (G)-S (G) of the digraph G
The normalized Laplacian matrix of a graph G, denoted by L, is defined to be the matrix with entries $L(x, y)$

$$L(x,y) = \begin{cases} 1 & if\ x = y\ and\ d_y \neq 0 \\ \dfrac{-1}{\sqrt{d_x d_y}} & if\ x\ and\ y\ are\ adjacent\ in\ G \\ 0 & otherwise \end{cases}$$

Note that L has the following relationship to A and D.

$$L = 1 - D^{-1/2} AD^{-1/2} \qquad (4)$$

Let $D(G)$ = diag ($d_1, d_2, \ldots, d_n$) be the diagonal matrix associated to G, where $d_i = d(v_i) = deg(v_i)$ for all $i = 1, 2, \ldots\ldots\ldots n$. The matrix

$$L(G) = D(G) + A(G) \qquad (5)$$

is called the Signless Laplacian Matrix.

The Signless Laplacian Resolvent Energy RQ (G) of a graph G may be defined similarly by means of the formula

$$RQ(G) = \sum_{i=1}^{n} \frac{1}{2n - 1 - q_i}$$

Where $q_1, q_2, q_3, \ldots, q_n$ are Signless Laplacian eigen values of G.

A cutpoint of a graph G is a point whose removal increases the number of components.

A Bridge of a graph G is a line whose removal increases the number of components.

A connected non-trivial graph having no cut point is a Block.

Let D be a graph with p vertices. The reachability matrix $R = (r_{ij})$ is the pxp matrix with $r_{ij} = 1$ if v_j is reachable from v_i and 0 otherwise. We assume that, each vertex is reachable from itself. The distance matrix is the $p \times p$ matrix whose $(i, j)^{th}$ entry gives the distance from the point v_i to the point v_j and is infinity if there is no path from v_i to v_j. The detour matrix is the $p \times p$ matrix whose $(i, j)^{th}$ entry is the length of any longest $v_i - v_j$ path and is ∞ if there is no such path

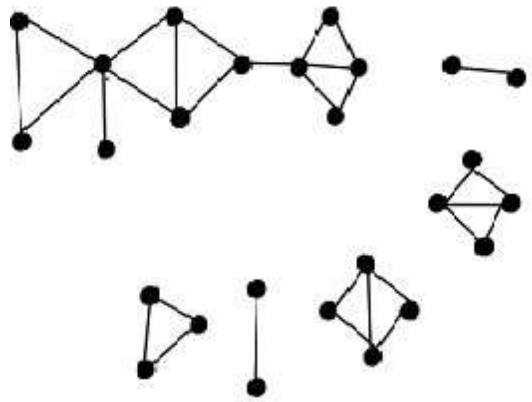

Figure 1. Blocks of G

If G is a connected graph with at least three points then following statements are equivalent:

1. G is a block
2. Any two points of G lie on a common cycle.
3. Any point and any line of G lie on a common cycle.
4. Any two lines of G lie on a common cycle [1].

3. Spectrum of a Laplacian Matrix

In this section, a graph *G* with four vertices with three edges (Figure 2) is considered and whose energy is equivalent to twice the Golden Ratio is established.

$$A(G) = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$= 3.236$$

The spectrum set for the below graph is $\{-1, -0.618, 0.618, 1\}$. Generally, the Eigen values of Adjacency matrix is denoted by $\{\lambda_1, \lambda_2, \ldots, \lambda_n\}$.

$$E(G) = 2\left\{\frac{f_{i+1}}{f_i}\right\}$$

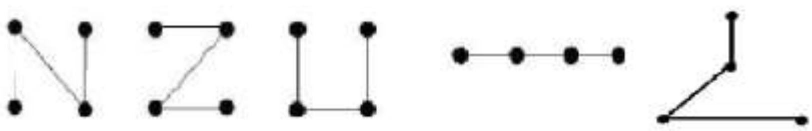

Figure 2. Types of (4,3) graphs G

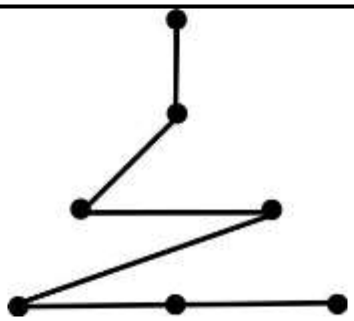

FIGURE 3. G(7,6)

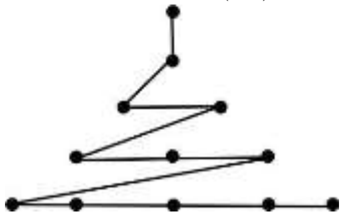

FIGURE 4. G(12,11)

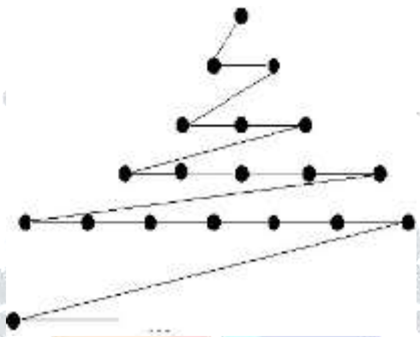

FIGURE 5. Infinite Graph

Where $\{f_i\}_{i=1}^{\infty}$ is the famous Fibonacci sequence. Since $(G)<2(n-1)$. G is called NonHyperenergetic graph. Since, $A(G)$ is a real symmetric matrix and therefore all the eigen values are real. Since, all eigen values are real more formally we can write $AV = \lambda\ V\ \ v : V \to R$ or $v \in R^v$ where $\{\lambda_i\}^4_{i=1}$ forms an orthonormal basis.

$$G = \bigcup_{i=1}^{n} G_i$$

$G_1 = (4, 3)$; $G_2 = (3, 3)$; $G_3 = (5, 5)$; G4 = (8, 8)

It is interesting to note that, for G (7, 6) $\{(v_3, v_5)\}$ are cut points and edges {(3, 4), (5, 6), (8, 9)} are bridges. This graph can be extended to a general graph G (n, n-1) where $\{v_{3,5} \ldots v_{n-3}\}$ are cut points and { (3, 4), (5, 6), (8, 9), ...} In a similar manner we can construct an Infinite graph with n vertices and m edges where $n > 2$, $m > 1$. In future some of these energies may be extended to the Infinite graph. Furthermore three types of matrices of G (7, 6) are given below

$$\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix} \quad \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 0 & 1 & 2 & 3 & 4 & 5 \\ 2 & 1 & 0 & 1 & 2 & 3 & 4 \\ 3 & 2 & 1 & 0 & 1 & 2 & 3 \\ 4 & 3 & 2 & 1 & 0 & 1 & 2 \\ 5 & 4 & 3 & 2 & 1 & 0 & 1 \\ 6 & 5 & 4 & 3 & 2 & 1 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 0 & 1 & 2 & 3 & 4 & 5 \\ 2 & 1 & 0 & 1 & 2 & 3 & 4 \\ 3 & 2 & 1 & 0 & 1 & 2 & 3 \\ 4 & 3 & 2 & 1 & 0 & 1 & 2 \\ 5 & 4 & 3 & 2 & 1 & 0 & 1 \\ 6 & 5 & 4 & 3 & 2 & 1 & 0 \end{pmatrix}$$

Reachability Matrix Distance Matrix Detour Matrix

This concept may also be extended to the infinite array and it may have significant application in Machine Learning.

4. Various Laplacian Energies of G

The Laplacian spectrum has been applied to Machine Learning. The Laplacian energy found applications in Image analysis and it is also attempted in Medical investigation of Brain activity (EEG). An electroencephalogram (EEG) is a test that detects electrical activity of brain using small, metal discs (electrodes) attached to scalp. Brain cells communicate via electrical impulses and are active all the time, even when asleep. This activity shows up as wavy lines on an EEG recording. Various Laplacian energies are calculated for $G(4, 3)$.

4.1 Laplacian Energy

$$SL(G) = D(G) - A(G)$$

$$D(G) = \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{pmatrix} A(G) = \begin{pmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \\ b_{31} & b_{32} & b_{33} & b_{34} \\ b_{41} & b_{42} & b_{43} & b_{44} \end{pmatrix}$$

$$D(G) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad A(G) = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$D(G) = \begin{pmatrix} 1 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 1 \end{pmatrix}$$

$$L(G) = \sum_{i=1}^{n} \left|\mu_i - \frac{2m}{n}\right|, \text{ where } \{\mu_i\}_{i=1}^{n} \text{ is the spectrum of } L(G)$$

$$= 4.5 \qquad (7)$$

4.2 Laplacian like Invariant LEL describes well the properties which are accounted by the majority of molecular descriptors: motor octane number, entropy, molar volume, molar refraction, particularly the Acentric factor AF parameter, but also more difficult proper- ties like boiling point, melting point and partition coefficient Log P. In a set of polycyclic aromatic hydrocarbons, LEL was proved.

$$LEL(G) = \sum_{i=1}^{n} \sqrt{\mu_i}$$

$$\mu_i(G) = \sqrt{2} + 2, 0, 2, 2 - \sqrt{2}$$

$$LEL(G) = \sqrt{\sqrt{2} + 2 + 0 + 2 + 2 - \sqrt{-2}}$$

$$= \sqrt{6}$$

$$= 2.449 \qquad (8)$$

4.3 Signless Laplacian Energy

$$SL(G) = D(G) + A(G)$$

$$SL(G) = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix}$$

$$= 4.2 \qquad (9)$$

4.4 Skew Laplacian Energy

Let $S(G)$ be the skew adjacency matrix of a simple digraph G. Then the skew Laplacian energy of the digraph G is defined as

$$E_{SL}(G) = \sum_{i=1}^{n} \lambda_i^2$$

Where n is the order of G and $\lambda_1, \lambda_2 ... \lambda_n$ are the eigenvalues of the Laplacian matrix $L(G) = D(G) - S(G)$ of the digraph G. According to the Definition skew laplacian energy can be defined only in the Directed − Graph.

4.5 Normalized Laplacian Energy

$$L(x,y) = \begin{cases} 1 & if\ x = y\ and\ d_y \neq 0 \\ \frac{-1}{\sqrt{d_x d_y}} & if\ x\ and\ y\ are\ adjacent\ in\ G \\ 0 & otherwise \end{cases}$$

$$L(G) = \begin{pmatrix} 1 & \frac{-1}{\sqrt{2}} & 0 & 0 \\ \frac{-1}{\sqrt{2}} & 1 & \frac{-1}{\sqrt{2}} & 0 \\ 0 & \frac{-1}{\sqrt{2}} & 1 & \frac{-1}{\sqrt{3}} \\ 0 & 0 & \frac{-1}{\sqrt{3}} & 1 \end{pmatrix}$$

Normalized Laplacian Energy (G) = 4.01 (10)

4.6 .Signless Laplacian Resolvent Energy

$$RQ(G) = \sum_{i=1}^{n} \frac{1}{2n-1-q_i}$$

$$Eigenvalues: \sqrt{2}+2, 0, 2, 2-\sqrt{2}$$

$$= \frac{1}{2(4)-1-\sqrt{2}-2} + \frac{1}{2(4)-1-0} + \frac{1}{2(4)-1-2} + \frac{1}{2(4)-1-\sqrt{2}}$$

$$= \frac{386}{105}$$

=3.676 (11)

Table 1. Values for Energies of Various Laplacian

ENERGIES	VALUES
Laplacian Energy	4.5
Laplacian Like Invariant	2.449
Signless Laplacian Energy	2.7498
Normalized Laplacian Energy	4.01
Signless Laplacian Resolvent Energy	3.676

LEL < SLE < Energy < SLRE < NLE < LE is found

5. Energy of pyramid Fibonacci Graph and Ripples Hamiltonian Graph [8]

In this section pyramid Fibonacci Graph and Ripple Hamiltonian Graph are considered and the Energies of the two graphs are calculated. Both the Graphs are Infinite Graphs and here we considered the basic values.

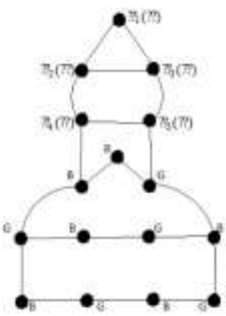

Figure 6: Three colourable $\mathcal{G}_{\mathcal{F}}^{p}$

$\boldsymbol{D(G) - A(G)}$

$$= \begin{pmatrix} 2&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\ 0&3&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&5&0&0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&3&0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&3&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&2&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&3&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&3&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&3&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&2&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&2&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&3&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0&2&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0&0&2&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&2&0\\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&2 \end{pmatrix} - \begin{pmatrix} 0&1&1&0&0&0&0&0&0&0&0&0&0&0&0&0\\ 1&0&1&1&0&0&0&0&0&0&0&0&0&0&0&0\\ 1&1&0&0&1&0&0&0&0&0&0&0&0&0&0&0\\ 0&1&0&0&1&0&1&0&0&0&0&0&0&0&0&0\\ 0&0&1&1&0&0&0&1&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&1&1&0&0&0&0&0&0&0&0\\ 0&0&0&1&0&1&0&0&0&1&0&0&0&0&0&0\\ 0&0&0&0&1&1&0&0&0&0&0&1&0&0&0&0\\ 0&0&0&0&0&0&1&0&0&1&0&0&1&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&1&0&1&0&0&0\\ 0&0&0&0&0&0&0&0&0&1&0&1&0&0&0&0\\ 0&0&0&0&0&0&0&1&0&0&1&0&0&0&0&1\\ 0&0&0&0&0&0&0&0&1&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0&1&0&1&0\\ 0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&1\\ 0&0&0&0&0&0&0&0&0&0&0&1&0&0&1&0 \end{pmatrix}$$

$$= \begin{pmatrix} 2&-1&-1&0&0&0&0&0&0&0&0&0&0&0&0&0\\ -1&3&-1&-1&0&0&0&0&0&0&0&0&0&0&0&0\\ -1&-1&5&0&-1&0&0&0&0&0&0&0&0&0&0&0\\ 0&-1&0&3&-1&0&-1&0&0&0&0&0&0&0&0&0\\ 0&0&-1&-1&3&0&0&-1&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&2&-1&-1&0&0&0&0&0&0&0&0\\ 0&0&0&-1&0&-1&3&0&0&-1&0&0&0&0&0&0\\ 0&0&0&0&-1&-1&0&3&0&0&0&-1&0&0&0&0\\ 0&0&0&0&0&0&-1&0&3&-1&0&0&-1&0&0&0\\ 0&0&0&0&0&0&0&0&0&2&-1&0&-1&0&0&0\\ 0&0&0&0&0&0&0&0&0&-1&2&-1&0&0&0&0\\ 0&0&0&0&0&0&0&-1&0&0&-1&3&0&0&0&-1\\ 0&0&0&0&0&0&0&0&-1&0&0&0&2&-1&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0&-1&2&-1&0\\ 0&0&0&0&0&0&0&0&0&0&0&0&0&-1&2&-1\\ 0&0&0&0&0&0&0&0&0&0&0&-1&0&0&-1&2 \end{pmatrix}$$

$$Energy\ of\ a\ Graph\ E(G) = \sum_{i=1}^{n} |\lambda_i|$$

λ_i = 2, 0.041, 0.387, 0.646, 0.763, 0.964, 1.852, 2.197, 2.581, 3. 397, 3.3495, 3.672, 4.235, 4.792, 4.935, 6.042.

= 41.999

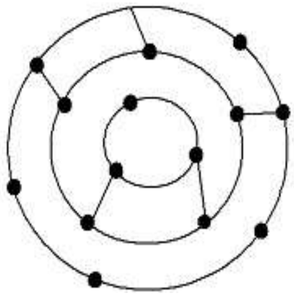

Figure 7: Nested H- Factor for RHG

$\boldsymbol{D(G) - A(G)}$

$$= \begin{pmatrix} 2&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&3&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&3&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&3&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&3&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&3&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&3&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&2&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&3&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&2&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&3&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&2&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&2&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&2&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&2 \end{pmatrix} - \begin{pmatrix} 0&1&1&0&0&0&0&0&0&0&0&0&0&0&0 \\ 1&0&1&0&0&0&1&0&0&0&0&0&0&0&0 \\ 1&1&0&0&0&1&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&1&0&0&1&1&0&0&0&0&0&0 \\ 0&0&0&1&0&1&0&0&0&0&1&0&0&0&0 \\ 0&0&1&0&1&0&1&0&0&0&0&0&0&0&0 \\ 0&1&0&0&0&1&0&1&0&0&0&0&0&0&0 \\ 0&0&0&1&0&0&1&0&0&0&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0&0&1&0&0&0&0&1 \\ 0&0&0&0&0&0&0&0&1&0&1&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0&1&0&1&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&1&0&1&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&1&0&1&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&1&0&1 \\ 0&0&0&0&0&0&0&0&1&0&0&0&0&1&0 \end{pmatrix}$$

$$= \begin{pmatrix} 2&-1&-1&0&0&0&0&0&0&0&0&0&0&0&0 \\ -1&3&-1&0&0&0&-1&0&0&0&0&0&0&0&0 \\ -1&-1&3&0&-1&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&3&-1&0&0&-1&-1&0&0&0&0&0&0 \\ 0&0&0&-1&3&-1&0&0&0&0&-1&0&0&0&0 \\ 0&0&-1&0&-1&3&-1&0&0&0&0&0&0&0&0 \\ 0&-1&0&0&0&-1&3&-1&0&0&0&0&0&0&0 \\ 0&0&0&-1&0&0&-1&2&0&0&0&0&0&0&0 \\ 0&0&0&-1&0&0&0&0&3&-1&0&0&0&0&-1 \\ 0&0&0&0&0&0&0&0&-1&2&-1&0&0&0&0 \\ 0&0&0&0&-1&0&0&0&0&-1&3&-1&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&-1&2&-1&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&-1&2&-1&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&-1&2&-1 \\ 0&0&0&0&0&0&0&0&-1&0&0&0&0&-1&2 \end{pmatrix}$$

$$Energy\ of\ a\ Graph\ E(G) = \sum_{i=1}^{n} |\lambda_i|$$

$$\lambda_i = (0, 0.197, 0.682, 0.915, 1.134, 1.860, 2.227, 2.508, 2.804, 3.353, 3.649, 3.874, 4.580, 4.928, 5.290)$$

$= 38.001$

6. Conclusion

In this paper, comparison of various Laplacian Energies and basic facts of some of the energies are discussed and finally, significant difference between energy and Laplacian energies are analyzed. In future some of these energies may be extended to the Infinite graphs. Graph with good energies are calculated and in future this result may be extended to infinite graph.

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