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$H_k -$ Cordial Labeling of Barycentric Subdivision of Graphs

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Abstract : A graph $G = (V, E)$ is called $H_k -$ cordial if for each edge e and each vertex v of G have the label $1 \leq |f(e)| \leq k$, $1 \leq |f(v)| \leq k$ and $\left|v_f(i) - v_f(-i)\right| \leq 1$, $\left|e_f(i) - e_f(-i)\right| \leq 1$ for each i with $1 \leq i \leq k$. In this paper we investigate $H_k -$ cordial labeling of Barycentric Subdivision of Triangular Snake Graph, Double Triangular Snake Graph, Quadrilateral Snake Graph, Double Quadrilateral Snake Graph, Comb, $C_n \odot K_1$.

***Keywords* : H −cordial labeling, H_k − cordial labeling, Barycentric Subdivision of graphs.**

I. INTRODUCTION

In this paper we consider only finite, simple and undirected graph $G = (V, E)$ where E is a set of edges of G and V is a set of vertices of G.

Definition 1.1 Let $G = (V, E)$ be a graph. A mapping $f : E \rightarrow \{1, -1\}$ is called $H -$ cordial, if there exists a positive constant k, such that for each vertex v, $|f(v)| = k$ with vertex labeling $f(v) = \sum_{e \in I(v)} f(e)$, where $I(v)$ is the set of all edges incident to vertex v and the following two conditions are satisfied $\left|e_f(1) - e_f(-1)\right| \leq 1$ and $\left|v_f(k) - v_f(-k)\right| \leq 1$. A graph admits $H -$ cordial labeling is called $H -$ cordial graph. Following lemma gives important relation between vertex labeling and edge labeling. [5]

Lemma 1.2 If f is assignment of integer numbers to the vertices and edges of graph G such that for each vertex v, labeling $f(v) = \sum_{e \in I(v)} f(e)$, where $I(v)$ is the set of all edges incident to vertex v then $\sum_{v \in V(G)} f(v) = 2 \sum_{e \in E(G)} f(e)$.[5]

Definition 1.3 An assignment f of integer labels to the edges of a graph is called H_k – cordial labeling, if for each edge e and each vertex v of graph we have $1 \leq |f(e)| \leq k$ and $1 \leq |f(v)| \leq k$ with vertex labeling $f(v) = \sum_{e \in I(v)} f(e)$, where $I(v)$ is the set of all edges incident to vertex v and for each i with $1 \leq i \leq k$ we have $\left|e_f(\mathrm{i}) - e_f(-\mathrm{i})\right| \leq 1$ and $\left|v_f(i) - v_f(-i)\right| \leq 1$. A graph is called H_k – cordial if it admits a H_k – cordial labeling.

It is clear from definition that if graph admits $H -$ cordial labeling then it is $H_2 -$ cordial labeling graph. Also if graph is $H_k -$ cordial then it is $H_{k+1} -$ cordial labeling, but converse is not true. [5]

Definition 1.4 Let G be a graph with p vertices and q edges. The barycentric subdivision of G denoted by $S(G)$ is obtained by subdividing every edge of G with a vertex exactly once. [9]

Definition 1.5 A Triangular Snake Graph T_n is obtained from a path $u_1, u_2, \ldots., u_n$ by joining u_i and u_{i+1} to a new vertex v_i for $1 \leq i \leq n$. that is every edge of a path is replaced by a triangle. [10]

Definition 1.6 Double Triangular Snake Graph $D(T_n)$ consists of two Triangular snakes that have a common path. [6]

Definition 1.7 A Quadrilateral Snake Graph Q_n is obtained from a path $u_1, u_2, \ldots., u_n$ by joining u_i and u_{i+1} to a new vertex v_i and v_i^1 for $1 \leq i \leq n$. i.e. every edge of a path is replaced by a quadrilateral.[6]

Definition 1.8 Double Quadrilateral Snake Graph $D(Q_n)$ consists of two Quadrilateral Snake that have a common path.[6]

Definition 1.9 The graph $P_n \odot K_1$ is called a comb.[8]

Definition 1.10 Let G and H be two graphs with $|V(G)| = n, |V(H)| = m$, corona product of G and H is the graph obtained by taking n copies of H and attaching each such copy of H to every vertex of G. It is denoted by $G \odot H$.[8]

Parmar D. and Joshi J.[10] proved the following results:
1. Triangular Snake Graph T_n is $H -$ cordial if n is even.
2. Triangular Snake Graph T_{n} is $H_3 -$ cordial.
3. Double Triangular Snake Graph $D(T_{\mathrm{n}})$ is $H_3 -$ cordial.

Joshi J. and Parmar D.[6] [8]proved the following results:
1. Quadrilateral Snake Graph Q_{n} $(n \geq 2)$ is $H_3 -$ cordial.
2. A Double Quadrilateral Snake Graph $D(Q_{\mathrm{n}})(n \geq 3)$ is $H_3 -$ cordial.
3. The Comb ($P_n \odot K_1$) $(n \geq 2\,)$is $H_3 -$ cordial.
4. The crown $C_n \odot K_1$ is $H -$ cordial.

II. MAIN RESULT

Theorem 2.1 The barycentric subdivision graph of a Comb $S(P_n \odot K_1)$ $(n \geq 2\,)$ is $H_2 -$ cordial if n is odd and $H_3 -$ cordial if n is even.

***Proof*:** Let G be barycentric subdivision of a comb $S(P_n \odot K_1)$. Let vertex set and edge set of G be
$V = \{u_i, u'_i, x_i; 1 \leq i \leq n\} \cup \{v_i; 1 \leq i \leq n-1\}$ and
$E = \{u_i v_i, v_i u_{i+1}; 1 \leq i \leq n-1\} \cup \{u_i x_i, x_i u'_i\,; 1 \leq i \leq n\}$
Consider a function $f: E \to \{-2, -1, 1, 2\}$ defined as
$f(u_i v_i) = (-1)^{i+1}\,; 1 \leq i \leq n-1$
$f(v_i u_{i+1}) = (-1)^i.2\,; 1 \leq i \leq n-1$
$f(u_1 x_1) = f(x_1 u'_1) = 1$,
$f(u_n x_n) = f(x_n u'_n) = -1$
$f(u_i x_i) = (-1)^i.2;\ 2 \leq i \leq n\text{-}1$
$f(x_i u'_i) = (-1)^{i+1};\ 2 \leq i \leq n\text{-}1$

$n \geq 2$	Edge Condition	Vertex Condition
n is even	$e_f(1) = n+1, e_f(-1) = n$ $e_f(2) = n-2, e_f(-2) = n-1$	$v_f(1) = 2n-3, v_f(-1) = 2n-2$ $v_f(2) = 2, v_f(-2) = 1$ $v_f(3) = 1, v_f(-3) = 0$
n is odd	$e_f(1) = n, e_f(-1) = n+1$ $e_f(2) = n-1, e_f(-2) = n-2$	$v_f(1) = 2n-2 = v_f(-1)$ $v_f(2) = 2, v_f(-2) = 1$

In each case, the graph satisfies the condition $|e_f(\mathrm{i}) - e_f(-\mathrm{i})| \leq 1$ and $|v_f(i) - v_f(-i)| \leq 1$.
Hence, $S(P_n \odot K_1)$ is $H_3 -$ cordial.

Example 2.2 $S(P_5 \odot K_1)$ is $H_2 -$ cordial shown in Figure 1.

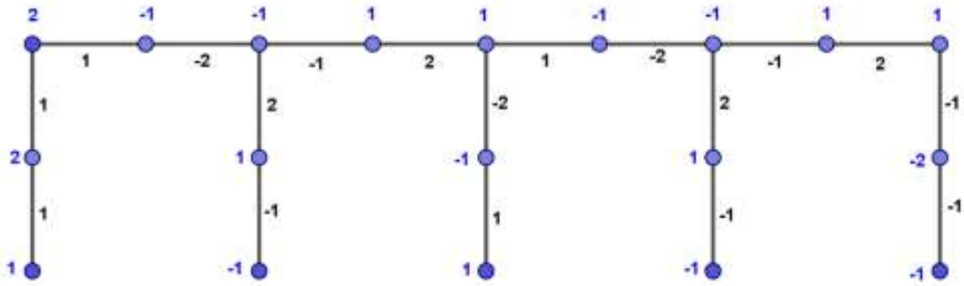

Figure 1 $S(P_5 \odot K_1)$

Theorem 2.3 The barycentric subdivision of $C_n \odot K_1$ graph is $H_2 -$ cordial $(n \geq 3)$.

Proof: Let C_n be a cycle with vertices $u_1, u_2, \ldots u_n$ with $u_{n+1} = u_1$. $C_n \odot K_1$ is obtained from cycle C_n by attaching pendant edge to each vertex. Hence we get new vertices $v_i, 1 \leq i \leq n$ and edges $u_i v_i, 1 \leq i \leq n$. Let $x_1, x_2, \ldots, x_n$ and $y_1, y_2, \ldots, y_n$ be the vertex obtained by subdividing edges $u_i u_{i+1}, u_i v_i, 1 \leq i \leq n$ respectively.

Consider a function $f: E \longrightarrow \{-2, -1, 1, 2\}$ defined as

$$f(u_i x_i) = f(x_i u_{i+1}) = \begin{cases} 1 & ; 1 \leq i \leq \left\lceil \frac{n}{2} \right\rceil \\ -1 & ; Otherwise \end{cases}$$

$$f(v_iy_i) = f(y_iu_i) = \begin{cases} -1 & ; 1 \le i \le \left\lceil \frac{n}{2} \right\rceil \\ 1 & ; Otherwise \end{cases}$$

	Edge Condition	Vertex Condition
$n \ge 3$	$e_f(1) = 2n = e_f(-1)$	$v_f(1) = n = v_f(-1)$ $v_f(2) = n = v_f(-2)$

In each case, the graph satisfies the condition $\left|e_f(\mathrm{i}) - e_f(-\mathrm{i})\right| \le 1$ and $\left|v_f(i) - v_f(-i)\right| \le 1$.
Hence, $S(C_n \odot K_1)$ is $H_2 -$ cordial.

Example 2.4 $S(C_n \odot K_1)$ is $H_2 -$ cordial shown in Figure 2.

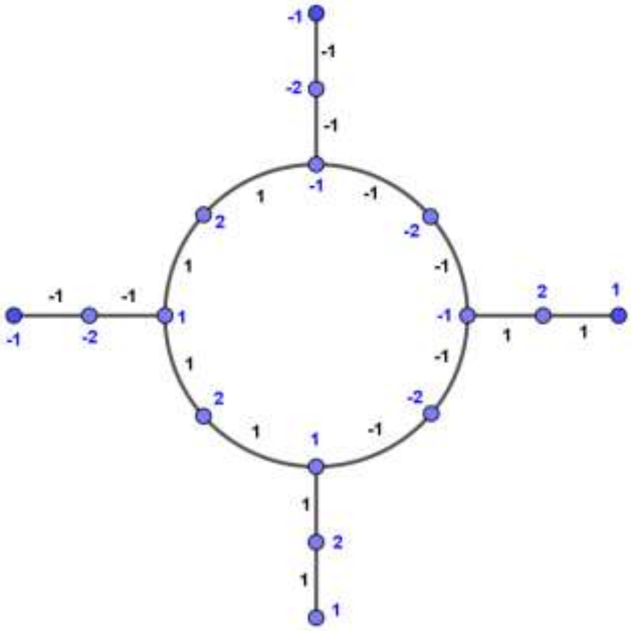

Figure 2 $S(C_n \odot K_1)$

Theorem 2.5 The barycentric subdivision of Triangular snake graph $S(T_n), n \ge 3$ is $H_3 -$ cordial.
Proof: Let x_i, r_i and t_i be the vertex obtained by subdividing edges u_iu_{i+1}, u_iv_i and $v_iu_{i+1}; 1 \le i \le n - 1$ respectively.
Consider a function $f: E \longrightarrow \{-2, -1, 1, 2\}$ defined as
Case 1 If n is odd

$$f(u_ir_i) = f(r_iv_i) = f(v_it_i) = f(t_iu_{i+1}) = (-1)^{i+1}; 1 \le i \le n - 1$$

$$f(u_ix_i) = \begin{cases} 1 & ; 1 \le i \le \frac{n-1}{2} \\ 2 & ; i = \frac{n+1}{2} \\ -1 & ; Otherwise \end{cases}$$

$$f(x_iu_{i+1}) = \begin{cases} 1 & ; 1 \le i \le \frac{n-1}{2} \\ -1 & ; Otherwise \end{cases}$$

$n \ge 3$	Edge Condition	Vertex Condition
n is odd	$e_f(1) = 3n - 3, e_f(-1) = 3n - 4$ $e_f(2) = 1, e_f(-2) = 0$	$v_f(1) = 1, v_f(-1) = 0$ $v_f(2) = \frac{5n-5}{2}, v_f(-2) = \frac{5n-7}{2}$ $v_f(3) = 1, v_f(-3) = 0$

Case 2 If n is even

$$f(u_ir_i) = f(r_iv_i) = f(v_it_i) = f(t_iu_{i+1}) = (-1)^{i+1}; 1 \le i \le n - 2$$

$$f(u_ix_i) = \begin{cases} 1 & ; 1 \le i \le \frac{n}{2} \\ -1 & ; Otherwise \end{cases}$$

$$f(x_iu_{i+1}) = \begin{cases} 1 & ; 1 \le i \le \dfrac{n-2}{2} \\ 2 & ; i = \dfrac{n}{2} \\ -1 & ; Otherwise \end{cases}$$

$f(u_{n-1}r_{n-1}) = 1, f(r_{n-1}v_{n-1}) = -2, f(v_{n-1}t_{n-1}) = f(t_{n-1}u_n) = -1.$

$n \ge 3$	Edge Condition	Vertex Condition
n is even	$e_f(1) = 3n - 4 = e_f(-1)$ $e_f(2) = 1 = e_f(-2)$	$v_f(1) = 1 = v_f(-1)$ $v_f(2) = \dfrac{5n-8}{2} = v_f(-2)$ $v_f(3) = 1 = v_f(-3)$

In each case, the graph satisfies the condition $|e_f(\mathrm{i}) - e_f(-\mathrm{i})| \le 1$ and $|v_f(i) - v_f(-i)| \le 1$. Hence, $S(T_n)$ is $H_3 -$ cordial.

Example 2.6 $S(T_5)$is $H_3 -$ cordial shown in Figure 3.

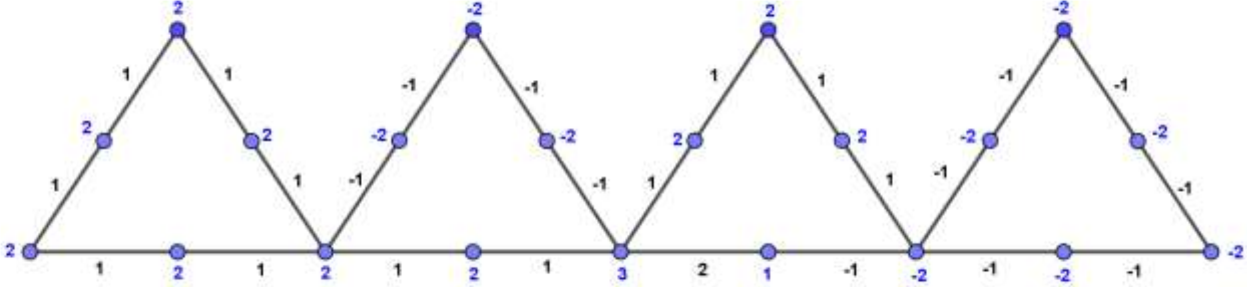

Figure 3 $S(T_5)$

Theorem 2.7 The barycentric subdivision of Quadrilateral snake graph $S(Q_n), n \ge 3$ is $H_3 -$ cordial.

Proof: Let x_i, y_i, r_i and t_i be the vertex obtained by subdividing edges $u_iu_{i+1}, v_iw_i, u_iv_i$ and $w_iu_{i+1}; 1 \le i \le n-1$ respectively.

Consider a function $f: E \longrightarrow \{-2,-1,1,2\}$ defined as

Case 1 If n is odd

$f(u_ir_i) = f(r_iv_i) = f(v_iy_i) = f(y_iw_i) = f(w_it_i) = f(t_iu_{i+1}) = (-1)^{i+1}; 1 \le i \le n-1$

$$f(u_ix_i) = \begin{cases} 1 & ; 1 \le i \le \dfrac{n-1}{2} \\ 2 & ; i = \dfrac{n+1}{2} \\ -1 & ; Otherwise \end{cases}$$

$$f(x_iu_{i+1}) = \begin{cases} 1 & ; 1 \le i \le \dfrac{n-1}{2} \\ -1 & ; Otherwise \end{cases}$$

$n \ge 3$	Edge Condition	Vertex Condition
n is odd	$e_f(1) = 4n - 4, e_f(-1) = 4n - 5$ $e_f(2) = 1, e_f(-2) = 0$	$v_f(1) = 1, v_f(-1) = 0$ $v_f(2) = \dfrac{7n-7}{2}, v_f(-2) = \dfrac{7n-9}{2}$ $v_f(3) = 1, v_f(-3) = 0$

Case 2 If n is even

$f(u_ir_i) = f(r_iv_i) = f(v_iy_i) = f(y_iw_i) = f(w_it_i) = f(t_iu_{i+1}) = (-1)^{i+1}; 1 \le i \le n-2$

$$f(u_ix_i) = \begin{cases} 1 & ; 1 \le i \le \dfrac{n}{2} \\ -1 & ; Otherwise \end{cases}$$

$$f(x_iu_{i+1}) = \begin{cases} 1 & ; 1 \le i \le \dfrac{n-2}{2} \\ 2 & ; i = \dfrac{n}{2} \\ -1 & ; Otherwise \end{cases}$$

$f(u_{n-1}r_{n-1}) = f(r_{n-1}v_{n-1}) = 1, f(v_{n-1}y_{n-1}) = -2$
$, f(y_{n-1}w_{n-1}) = f(w_{n-1}t_{n-1}) = f(t_{n-1}u_n) = -1.$

$n \geq 3$	Edge Condition	Vertex Condition
n is even	$e_f(1) = 4n - 5 = e_f(-1)$ $e_f(2) = 1 = e_f(-2)$	$v_f(1) = 1 = v_f(-1)$ $v_f(2) = \frac{7n-10}{2} = v_f(-2)$ $v_f(3) = 1 = v_f(-3)$

In each case, the graph satisfies the condition $\left|e_f(\mathrm{i}) - e_f(-\mathrm{i})\right| \leq 1$ and $\left|v_f(i) - v_f(-i)\right| \leq 1$. Hence, $S(Q_n)$ is $H_3 -$ cordial.

Example 2.8 $S(Q_4)$is $H_3 -$ cordial shown in Figure 4.

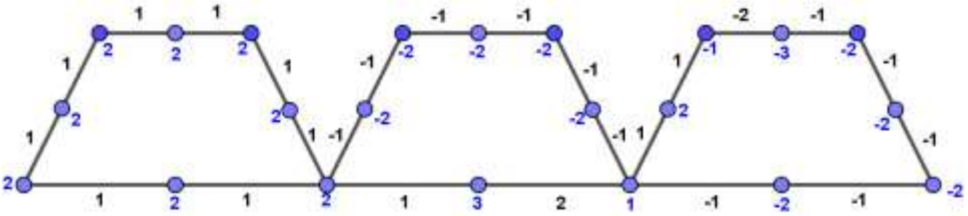

Figure 4 $S(Q_4)$

Theorem 2.9 The barycentric subdivision of Double Triangular snake graph $S(DT_n), n \geq 3$ is $H_3 -$ cordial.
Proof: Let x_i, r_i and t_i be the vertex obtained by subdividing edges $u_i u_{i+1}, u_i v_i$ and $v_i u_{i+1}; 1 \leq i \leq n - 1$ respectively. r'_i and t'_i be the vertex obtained by subdividing edges $u_i v'_i$ and $v'_i u_{i+1}; 1 \leq i \leq n - 1$ respectively.
Consider a function $f: E \longrightarrow \{-2, -1, 1, 2\}$ defined as
Case 1 If n is odd

$$f(u_i r_i) = f(r_i v_i) = f(v_i t_i) = f(t_i u_{i+1}) = 1; 1 \leq i \leq n - 1$$
$$f(u_i r'_i) = f(r'_i v'_i) = f(v'_i t'_i) = f(t'_i u_{i+1}) = -1; 1 \leq i \leq n - 1$$
$$f(u_i x_i) = \begin{cases} 1 & ; 1 \leq i \leq \frac{n-1}{2} \\ 2 & ; i = \frac{n+1}{2} \\ -1 & ; Otherwise \end{cases}$$
$$f(x_i u_{i+1}) = \begin{cases} 1 & ; 1 \leq i \leq \frac{n-1}{2} \\ -1 & ; Otherwise \end{cases}$$

Case 2 If n is even

$$f(u_i r_i) = f(r_i v_i) = f(v_i t_i) = f(t_i u_{i+1}) = 1; 1 \leq i \leq n - 1$$
$$f(u_i r'_i) = f(r'_i v'_i) = f(v'_i t'_i) = f(t'_i u_{i+1}) = -1; 1 \leq i \leq n - 1$$
$$f(u_i x_i) = \begin{cases} 1 & ; 1 \leq i \leq \frac{n}{2} \\ -1 & ; Otherwise \end{cases}$$
$$f(x_i u_{i+1}) = \begin{cases} 1 & ; 1 \leq i \leq \frac{n-2}{2} \\ 2 & ; i = \frac{n}{2} \\ -1 & ; Otherwise \end{cases}$$

$n \geq 3$	Edge Condition	Vertex Condition
n	$e_f(1) = 5n - 5, e_f(-1) = 5n - 6$ $e_f(2) = 1, e_f(-2) = 0$	$v_f(1) = 2, v_f(-1) = 1$ $v_f(2) = 4n - 5, v_f(-2) = 4n - 6$ $v_f(3) = 1, v_f(-3) = 0$

In each case, the graph satisfies the condition $\left|e_f(\mathrm{i}) - e_f(-\mathrm{i})\right| \leq 1$ and $\left|v_f(i) - v_f(-i)\right| \leq 1$. Hence, $S(DT_n)$ is $H_3 -$ cordial.

Example 2.10 $S(DT_5)$is $H_3 -$ cordial shown in Figure 5.

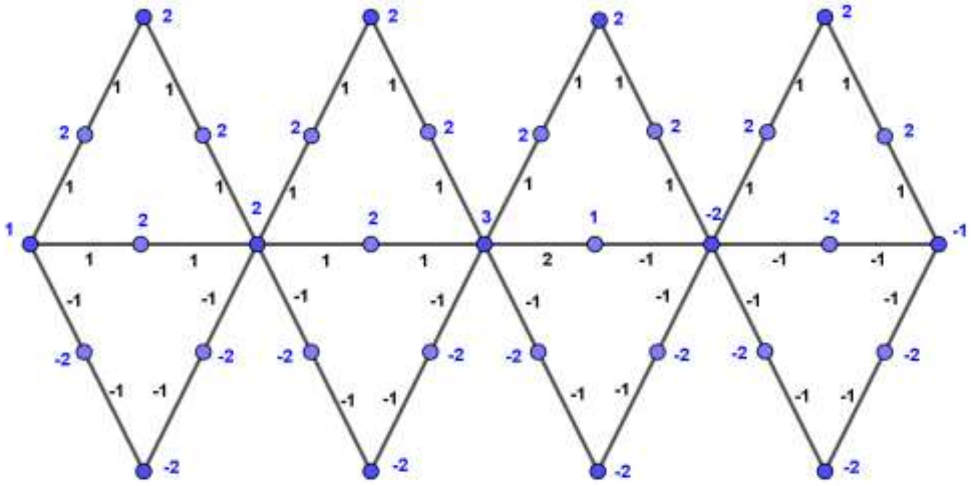

Figure 5 $S(DT_5)$

Theorem 2.11 The barycentric subdivision of Double Quadrilateral snake graph $S(DQ_n), n \geq 3$ is H_3- cordial.

Proof: Let x_i, y_i, r_i and t_i be the vertex obtained by subdividing edges $u_iu_{i+1}, v_iw_i, u_iv_i$ and $w_iu_{i+1}; 1 \leq i \leq n-1$ respectively. Let y'_i, r'_i and t'_i be the vertex obtained by subdividing edges $v'_iw'_i, u_iv'_i$ and $w'_iu_{i+1}; 1 \leq i \leq n-1$ respectively.

Consider a function $f: E \longrightarrow \{-2,-1,1,2\}$ defined as

Case 1 If n is odd

$$f(u_ir_i) = f(r_iv_i) = f(v_iy_i) = f(y_iw_i) = f(w_it_i) = f(t_iu_{i+1}) = 1; 1 \leq i \leq n-1$$

$$f(u_ir'_i) = f(r'_iv'_i) = f(v'_iy'_i) = f(y'_iw'_i) = f(w'_it'_i) = f(t'_iu_{i+1}) = -1; 1 \leq i \leq n-1$$

$$f(u_ix_i) = \begin{cases} 1 & ; 1 \leq i \leq \frac{n-1}{2} \\ 2 & ; i = \frac{n+1}{2} \\ -1 & ; Otherwise \end{cases}$$

$$f(x_iu_{i+1}) = \begin{cases} 1 & ; 1 \leq i \leq \frac{n-1}{2} \\ -1 & ; Otherwise \end{cases}$$

Case 2 If n is even

$$f(u_ir_i) = f(r_iv_i) = f(v_iy_i) = f(y_iw_i) = f(w_it_i) = f(t_iu_{i+1}) = 1; 1 \leq i \leq n-1$$

$$f(u_ir'_i) = f(r'_iv'_i) = f(v'_iy'_i) = f(y'_iw'_i) = f(w'_it'_i) = f(t'_iu_{i+1}) = -1; 1 \leq i \leq n-1$$

$$f(u_ix_i) = \begin{cases} 1 & ; 1 \leq i \leq \frac{n}{2} \\ -1 & ; Otherwise \end{cases}$$

$$f(x_iu_{i+1}) = \begin{cases} 1 & ; 1 \leq i \leq \frac{n-2}{2} \\ 2 & ; i = \frac{n}{2} \\ -1 & ; Otherwise \end{cases}$$

$n \geq 3$	Edge Condition	Vertex Condition
n is odd	$e_f(1) = 7n-7, e_f(-1) = 7n-8$ $e_f(2) = 1, e_f(-2) = 0$	$v_f(1) = 2, v_f(-1) = 1$ $v_f(2) = 6n-7, v_f(-2) = 6n-8$ $v_f(3) = 1, v_f(-3) = 0$

In each case, the graph satisfies the condition $\left|e_f(\mathrm{i}) - e_f(-\mathrm{i})\right| \leq 1$ and $\left|v_f(i) - v_f(-i)\right| \leq 1$.
Hence, $S(DQ_n)$ is H_3- cordial.

Example 2.12 $S(DQ_4)$is H_3- cordial shown in Figure 6.

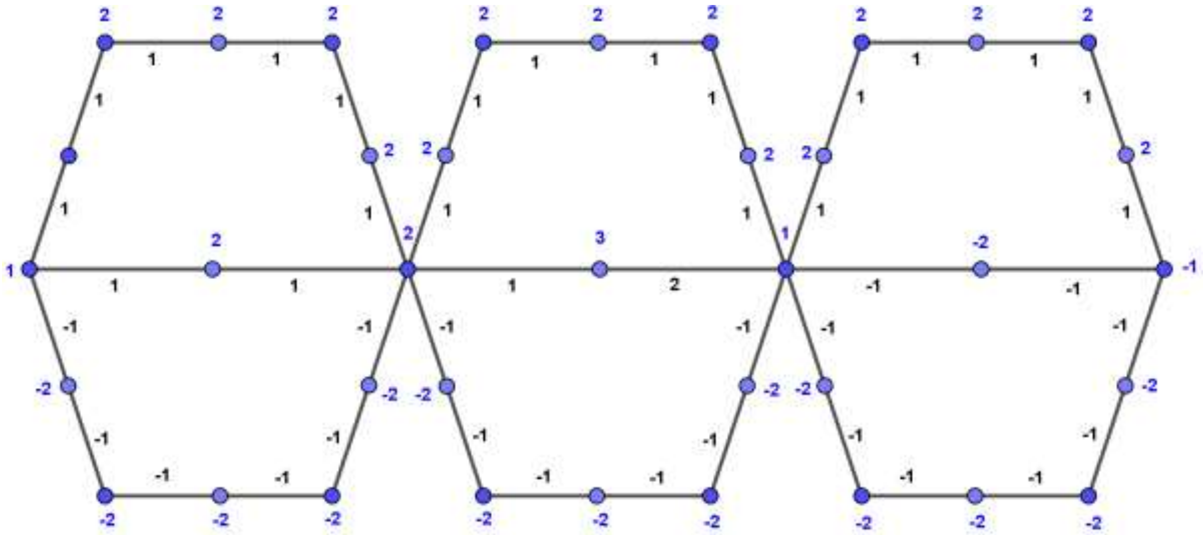

Figure 6 $S(DQ_4)$

III. CONCLUSION

In this paper we have proved that Barycentric Subdivision of Triangular Snake Graph, Double Triangular Snake Graph, Quadrilateral Snake Graph, Double Quadrilateral Snake Graph, Comb, $C_n \odot K_1$ graph are $H_K -$ cordial graphs.

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