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OCTREE NEUTROSOPHIC SETS USING ROUGH MATRIX

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Abstract : The octree is a hierarchical tree data structure commonly employed in computer graphics, spatial indexing, and three-dimensional data representation. Traditional octree methodologies generally categorise regions as black (completely shaded), white (unshaded), or grey (partially shaded), aligning with a binary (0–1) logic, with the intermediate state managed in an ad hoc manner. This article extends binary logic to a three-valued logic octree, facilitating a more nuanced and systematic depiction of spatial uncertainty. We present a neutrosophic quadtree framework that extends the traditional quadtree by explicitly representing the components of truth, indeterminacy, and falsity for each node, in accordance with neutrosophic set theory. We propose a consolidated methodology for expressing imprecise spatial data by integrating these two approaches, wherein each node conveys either three-valued (true, false, intermediate) or neutrosophic (T, I, F) information. Additionally, we illustrate how both structures can be represented and examined using rough matrices, employing the concepts of lower, higher, and boundary approximations to measure certainty and indeterminacy inside hierarchical divisions. This comprehensive perspective amplifies the descriptive capability of tree-based spatial models and paves the way for advancements in image reconstruction, feature extraction, and uncertainty management in multidimensional datasets.

Keywords: Rough sets, *Rough membership function* , *Rough Approximations*, *Information Systems,* Rough matrices, quadrough matrix, octroughmatrix, neutrosophic sets.

- Introduction

The Challenge of Vagueness in Spatial Data Representation in real-world spatial data, encompassing diverse domains from medical imaging to environmental scans and point clouds, frequently exhibits inherent vagueness, imprecision, and incompleteness. Traditional set theory, which relies on a binary classification where an element is either definitively a member or not, proves inadequate for accurately representing such uncertainties. This limitation becomes particularly apparent when dealing with blurred object edges in images, ill-defined geological boundaries, or noisy sensor readings in three-dimensional (3D) environments.

To address these challenges, Rough Set Theory (RST), introduced by Z. Pawlak in 1982, offers a robust mathematical framework specifically designed to handle vagueness and incompleteness without necessitating probabilistic assumptions. RST achieves this by defining sets not through precise boundaries, but through their lower and upper approximations. This approach explicitly characterizes a "boundary region" for imprecise concepts, comprising elements that cannot be definitively classified as belonging to or not belonging to a set. If this boundary region is empty, the set is considered crisp; otherwise, it is deemed rough or inexact. The fundamental concept of rough sets, centered on indiscernibility and approximation, provides a robust theoretical foundation for representing imprecise spatial boundaries. This is crucial because spatial data often exhibits inherent fuzziness, making crisp representations insufficient. By explicitly modelling a "boundary region" (elements that are neither clearly in nor clearly out), rough sets offer a more faithful representation of such real-world spatial phenomena. This capability implies that applying rough sets to spatial data structures can capture a richer, more nuanced understanding of spatial relationships than traditional binary approaches, directly motivating the development of structures like the Quad Rough Matrix (QRM) and, by extension, the proposed Octree Rough Matrix (ORM).

Hierarchical data structures are indispensable tools for efficiently organizing, accessing, and manipulating spatial information. They achieve this efficiency by recursively partitioning space into smaller, more manageable regions. Quadtrees, for instance, are two-dimensional (2D) hierarchical structures that recursively subdivide a square region into four smaller quadrants.[1] They are widely employed in 2D image processing, geographic information systems (GIS), and 2D collision detection. Octrees are the natural three-dimensional counterparts of quadtrees. They recursively subdivide a cubic space into eight octants.[3] Both quadtrees and octrees are extensively applied in fields such as computer graphics, image processing, GIS, and game development for tasks like scene management, collision detection, and spatial querying.[2] The recursive subdivision nature of both quadtrees and octrees inherently supports a multi-resolution representation of spatial data. This structural property aligns seamlessly with the multi-granular nature of rough set approximations, where vagueness can be resolved or refined at deeper levels of subdivision. If a spatial region's membership is uncertain (i.e., its rough membership value is between 0 and 1), subdividing it allows for a more precise determination of its "shaded" (occupied) or "unshaded" (empty) components at a finer granularity. This inherent multi-resolution capability of tree structures makes them exceptionally well-suited for integration with rough set theory to represent and analyze spatial vagueness across different scales.

The remainder of this paper is organized as follows: Section 2 provides preliminaries on rough set theory, rough matrices, quadtrees, and a detailed review of QRM. Section 3 introduces the conceptual framework for the Octree Rough Matrix (ORM), including adaptations of key QRM components for 3D. Section 4 outlines the step-by-step methodology for constructing an ORM. Section 5 discusses potential applications, advantages, and limitations. Finally, Section 6 concludes the paper and suggests avenues for future research.

2 . PRELIMINARIES

Rough Set Theory (RST), introduced by Z. Pawlak in 1982[1], provides a formal framework for dealing with imprecise, incomplete, and vague information without requiring prior knowledge of probability distributions. It operates on a universe of objects, U, and an indiscernibility relation, R, which represents our inability to distinguish between certain objects based on available attributes. Pawlak assumed that R is an equivalence relation, partitioning the universe U into elementary sets called indiscernibility classes. Pawlak

[1] developed rough set theory, which demonstrates vagueness not through membership but through the boundary region of a set. If a set's boundary region is void, it is termed crisp; otherwise, it is regarded rough (imprecise). In 1999, Frege and A.Nakamura [2] articulated about Conflict Logic with Degrees, Rough Fuzzy Hybridization . Pawlak [1] proposed that given a set of objects U, known as the universe, and an indiscernible relationship, this represents our ignorance about the elements of U. Pawlak proposed that R represents an equivalence relation. Rough set theory is a novel mathematical approach to analysing incorrect information. The framework has been applied in a variety of sectors, including decision support, engineering, environmental research, banking, medicine, and others. We can establish the rough set concept by applying topological operations such as interior and closure.

Yiyu Yao [3] published an article in 2010 regarding rough set approximations and their associated measures. The study advances fundamental knowledge in rough set theory—elucidating the methods for defining and measuring approximations to facilitate tasks such as rule induction, decision analysis, and knowledge discovery. In 1984, Hanan Samet published an article [4] on quadtrees, stating that a quadtree is a tree data structure formed by breaking the original parent node into four child nodes. Each node has four children. The divided region can be any shape, such as a square, rectangle, or another shape. A quadtree is a data organisation approach that starts with a single square and then divides it into four smaller squares. A quadtree is a hierarchical data structure that operates by regularly dividing space into smaller chunks. There are numerous approaches for grouping them. (i) Data presentation style (ii) Breakdown concept. (iii) The resolution, which is changeable or not. Recent studies emphasise the increasing amalgamation of rough set theory, multi-criteria decision-making (MCDM) methodologies, and neutrosophic algebraic frameworks to tackle uncertainty and indeterminacy in intricate decision-making scenarios.

Vijayabalaji and Balaji [10] proposed a practical rough-MCDM framework combined with an assignment model to discover the Best'11 strategy in cricket, illustrating the efficiency of rough matrices in dealing with imprecise and conflicting performance criteria under multi-attribute evaluation. From a theoretical standpoint, Sivaramakrishnan, Vijayabalaji, and Balaji [11] expanded neutrosophic theory by developing interval-valued anti-fuzzy linear spaces, which provide a generalised algebraic structure capable of modelling truth, indeterminacy, and falsity in one framework. Sivaramakrishnan et al. [12] expanded on this line of research by developing interval-valued neutrosophic fuzzy M-semigroups, which establish critical operational properties that strengthen the mathematical foundation for neutrosophic-based decision and optimisation models. Collectively, these investigations show a clear evolution from applied rough-MCDM models to robust neutrosophic algebraic frameworks, highlighting their appropriateness for intelligent decision-support systems operating in unpredictable environments.

Decision-making under uncertainty remains a major difficulty in modern scientific and engineering problems, especially in situations characterised by ambiguity, limited knowledge, and competing criteria. Classical mathematical models, which rely on precise numerical data, are frequently insufficient to reflect real-world processes that contain ambiguity and indeterminacy. To solve this constraint, soft computing paradigms such as fuzzy sets, soft sets, and neutrosophic sets have been extensively developed and utilised in a wide range of fields, including environmental management, intelligent systems, and multi-criteria decision-making (MCDM).

Soft set theory, which was developed as a parameterised mathematical framework for dealing with uncertainty, has received a lot of attention because of its flexibility and lack of reliance on additional assumptions. Based on this basis, Vijayabalaji et al. introduced the notion of sigmoid valued fuzzy soft sets, which improve classical fuzzy soft models by introducing sigmoid membership functions to capture smooth transitions between membership degrees. This formulation has proven especially useful in environmental decision-support applications, such as haze management, where uncertainty grows gradually rather than abruptly and decision variables behave nonlinearly [14]. The sigmoid valuation allows for improved modelling of threshold-based phenomena and human perception in complicated systems.

Simultaneously, quantitative comparison of soft parameterised objects has arisen as an essential prerequisite for ranking and selection tasks. To this purpose, new distance and similarity measures for soft parameter sets have been created to aid in systematic review and prioritisation in MCDM situations. Vijayabalaji et al. proposed unique measures that maintain mathematical consistency while boosting discrimination skill among options, hence strengthening decision-making in soft contexts [15]. Such measurements play an important role in converting qualitative ambiguity into actionable quantitative information.

Advances in abstract structures such as cubic and n-inner product spaces, which allow generalised algebraic and geometric interpretations for uncertainty modelling, also contribute to the mathematical robustness of these decision frameworks. The construction of cubic n-inner product spaces provides a theoretical foundation for analysing multidimensional interactions among uncertain entities, broadening the scope of soft and fuzzy techniques to higher-order analytical contexts [16]. These structures allow for consistent aggregation, comparison, and optimisation in complex decision spaces.

Despite these gains, real-world decision issues frequently entail not only uncertainty, but also indeterminacy and inconsistency, which fuzzy and soft frameworks cannot represent adequately. Smarandache's neutrosophic theory bridges this gap by explicitly adding truth, indeterminacy, and falsity components into set representation [18]. Interval-valued neutrosophic models improve expressiveness by allowing for variable boundaries on these components. The research of interval-valued neutrosophic graphs shows how effective this approach is at representing uncertain relational structures, especially in networked and multi-criteria situations [17].

Motivated by these discoveries, this study addresses complicated decision-making scenarios by combining fuzzy soft modelling, distance-similarity analysis, and neutrosophic theory. The proposed system attempts to provide a more comprehensive and trustworthy decision-support mechanism by combining sigmoid valued fuzzy representations, mathematically sound similarity measurements, and neutrosophic uncertainty modelling. This integrated method is especially appropriate for engineering, environmental, and intelligent system applications involving uncertainty, nonlinearity, and indeterminacy.

Further in this section we recall some basic definitions and results which will be needed for this paper.

Definition 2.1.[3]

The set X can be divided according to the basic sets of R, namely a lower approximation and upper approximation set. Approximation is used to represent the roughness of the knowledge. Suppose a set X contained in U represents a vague concept, The R-lower and R-upper approximations of X are defined by the equations.

- R-lower approximation of X

$$R_*(X) = \bigcup_{x \in U} \{R(x): R(x) \subseteq X\}$$

- R-upper approximation of X

$$R^*(X) = \bigcup_{x \in U} \{R(x): R(x) \cap X \neq \emptyset\}$$

- R-boundary region of X

$$RN_R(X) = R^*(X) - R_*(X)$$

Also we can define

- R-positive of X

$$POS_R(X) = R_*(X)$$

- R-negative of X

$$NEG_R(X) = U - R^*(X)$$

Rough sets are defined by approximations [1].

Definition 2.2.[1]

Rough membership function defined by Pawlak[1] is given by $\mu_X^R : U \rightarrow [0,1]$
where

$$\mu_X^R = \frac{|X \cap R(X)|}{|R(X)|}$$

and |X| denotes cardinality of X.

Definition 2.3[2]

The rough membership function can be used to define approximations and the boundary region of a set by Frege and Nakamura [2], as shown below;

$$R_*(X) = \{x \in u : \mu_X^R = 1\}$$
$$R^*(X) = \{x \in u : \mu_X^R = 0\}$$
$$RN_R(X) = \{x \in u : 0 < \mu_X^R < 1\}$$

Definition 2.4.[4]

Quadtree is defined as follows. Without loss of generality, assume that the given binary image is a 2n × 2n array of unit square "pixels". If the image does not cover the entire array, then we subdivide the array into quadrants, sub-quadrants,………., until we obtain blocks (possibly single pixels) that are entirely contained in the region or entirely disjoint from it.

Definition 2.5.[5]

The term image is use to refer the original array of pixels. If its elements are either BLACK or WHITE then it is said to be binary. If shades of GRAY are possible, then the image is said to be gray-scale image. Two pixels are said to be 4 – adjacent if they are adjacent to each other in the horizontal or vertical directions. If the concept of adjacency also includes adjacency at a corner (i.e., diagonal adjacencies), then the pixels are said to be 8 – adjacent.

Definition 2.6. [7]

We can define a Rough matrix as follows

$$R_M = [r_{ij}]_{m \times n} = \begin{bmatrix} r_{11} & \cdots & r_{1n} \\ \vdots & \ddots & \vdots \\ r_{m1} & \cdots & r_{mn} \end{bmatrix}$$

where

$$r_{ij} \in \mu_X^R$$

Definition 2.7.[8]

Values of Rough Matrix are defined as follows.

$$R_M = \begin{cases} 1 & : if\ x \in R_*(X) \\ (0,1) & : if\ x \in RN_R(X) \\ 0 & : if\ x \in U \setminus R_*(X) \end{cases}$$

Definition 2.9.[9]

Let (U, A) be a rough set over any approximations, consider subset of U × A is uniquely defined by

$$R_A = \{(u,b) : u \in R_A(b), b \in A\}$$

which is called as indiscernibility relation form (U, A). R_A is a function

$$\emptyset R_A : U \times A \rightarrow [0,1]$$

is defined by

$$\emptyset R_A = \begin{cases} 1 & : if\ x \in R_*(X) \\ (0,1) & : if\ x \in RN_R(X) \\ 0 & : if\ x \in U \setminus R_*(X) \end{cases}$$

then

$$R_M = \emptyset R_A(u_i, a_i) = [r_{ij}]_{m \times n} = \begin{bmatrix} r_{11} & \cdots & r_{1n} \\ \vdots & \ddots & \vdots \\ r_{m1} & \cdots & r_{mn} \end{bmatrix}$$

is called a m × n rough matrix over approximations. The set of all m × n rough matrices over approximations space will be denoted by $RM_{m\times n}$.

Definition 2.10. [9]

Rough sets can be partitioned into three non-empty subsets such that we can visualize completely shaded nodes as upper approximations, completely unshaded nodes as lower approximation and nodes in between that as boundary region. We can provide the membership value for upper approximation as 1, lower approximation as 0, boundary region in between (0, 1). This analogues the structure of rough matrix. If all are either 0 or 1, then there is no necessity to subdivide that region further. Let us call that as saturated nodes. If the values in between (0, 1) we need to subdivide the region further until we ends with either 0 or 1.

Definition 2.11. [9]

The definition of QRM is as follows. In this set we are defining all the entries of the matrix is either 1, 0 or in (0, 1). If all are completely shaded region, then we assign the value 1 for that, which is none other than upper approximation. If none is shaded, then we assign the value 0 for that, which is none other than lower approximation. If something is in neither 1 nor 0 then we need to give the membership value according to the pixels shaded in that region.

$$QRM = \frac{Number\ of\ shaded\ pixels}{Total\ number\ of\ pixels\ in\ the\ region}$$

2.2 From Rough Matrix to Quad Rough Matrix (QRM)

The concept of a Rough Matrix (RM) was introduced to provide a structured, algebraic representation of rough sets. Its entries are rough membership values, quantifying the degree of membership of elements within rough sets.[1] Building upon this, Vijayabalaji and Balaji coined the Quad Rough Matrix (QRM) specifically to apply rough matrix theory to 2D image data within a quadtree structure. QRM entries reflect the "roughness" of image regions, with values of 1 for completely shaded areas (representing the upper approximation), 0 for completely unshaded areas (representing the lower approximation of the complement), and values in the open interval (0,1) for partially shaded boundary regions.

The development of QRM signifies a crucial step towards bridging abstract rough set theory with concrete spatial data structures. It provides a quantitative measure (a membership value) for the "roughness" of image regions, moving beyond qualitative descriptions. By assigning a numerical value (from 0 to 1) to each node based on its "shadedness" or density, QRM transforms the qualitative concept of rough approximations (lower, upper, boundary regions) into a computable, quantitative metric. This is a significant advancement because it allows for algorithmic processing and analysis of vague spatial information, paving the way for applications like image compression, segmentation, and feature extraction where traditional binary methods might lose critical information or be overly rigid. It provides a concrete, measurable representation of uncertainty within spatial data.

3. Conceptual framework for the Octree Rough Matrix (ORM)

Motivation for Octree Rough Matrix (ORM) is increasing volume and complexity of 3D spatial data, such as medical scans, 3D models derived from LiDAR, and point clouds, necessitate advanced data structures that can effectively represent and process their inherent uncertainties. While QRM successfully addresses 2D vagueness, a direct 3D analog is crucial to extend the benefits of rough set theory to volumetric data.

Octrees are explicitly defined as the "three-dimensional analog of quadtrees". Their existing applications in 3D graphics, medical imaging, and spatial indexing [5] underscore the critical need for robust 3D spatial data handling. Traditional spatial data structures, including basic octrees, often struggle to efficiently handle tasks in high-dimensional spaces, especially with large numbers of objects or when data is noisy or incomplete. Medical imaging, for instance, benefits significantly from octrees for data fidelity and compression while preserving 3D content

This paper proposes the "Octree Rough Matrix" (ORM) as a novel extension, building directly upon the principles and calculation methodology of QRM, adapted for the 3D octree structure. As 3D data becomes increasingly ubiquitous (e.g., in autonomous vehicles, advanced medical diagnostics, virtual/augmented reality, and complex scientific simulations), the ability to represent and process its inherent uncertainties becomes paramount. ORM, by integrating rough set theory with octrees, offers a novel and theoretically grounded way to handle this. This is not just about scaling up a 2D concept; it is about providing a more expressive, computationally efficient, and robust model for 3D data that is often inherently noisy, incomplete, or vaguely defined. The broader implication is the potential for ORM to become a foundational data structure for next-generation 3D spatial analysis, particularly in fields where precise yet flexible representation of spatial occupancy and its uncertainty is critical. This extension aims to provide a powerful framework for representing and analyzing vague 3D spatial information, enhancing applications in fields where 3D uncertainty is prevalent.

3.1. Defining the Octree Rough Matrix (ORM)

The Octree Rough Matrix (ORM) is proposed as a novel data structure designed to represent rough approximations of 3D spatial data within an octree framework. It directly extends the principles of the Quad Rough Matrix (QRM) to the volumetric domain. Similar to QRM, ORM entries will be values within the interval . These values indicate the degree of "occupancy," "density," or "membership" of a given octant within the 3D space:

- A value of **1** signifies an octant that is entirely "occupied" or "shaded" (analogous to the R-lower approximation or completely shaded nodes), representing full membership.
- A value of **0** signifies an octant that is entirely "empty" or "unshaded" (analogous to the R-negative region or completely unshaded nodes), representing non-membership.
- Values **between (0,1)** signify an octant that is partially "occupied" or "rough" (analogous to the R-boundary region or "grey" nodes), indicating uncertainty or partial membership within the defined 3D concept.

This definition directly parallels QRM's definition and the broader rough matrix concept [9 & 10], which explicitly links rough matrix entries to rough set approximations and the concept of "saturated nodes." The ORM, by explicitly representing values in (0,1), moves beyond the traditional "black/white" or "occupied/empty" paradigm of octrees. While a traditional octree's "mixed" state simply indicates that further subdivision is required, it does not quantify the mixed or uncertain that region is. ORM, by assigning a specific numerical value within (0,1), provides a *degree* of mixedness or uncertainty. This means that even at leaf nodes, if they are not entirely full or empty, they can still hold a rough membership value, providing richer information than a simple binary classification (e.g., "this voxel is 60% occupied"). This is particularly useful for representing objects with ill-defined boundaries, handling noisy sensor data, or for data compression strategies where some loss of crispness is acceptable for memory efficiency, as hinted by applications in medical imaging.[9] ORM aims to capture the inherent vagueness and imprecision often present in 3D objects or regions, providing a more nuanced and informative representation compared to traditional binary (occupied/empty) octrees.

3.2. Adapting the Dense Factor for 3D Octree Subdivisions

The "dense factor" concept, crucial for normalizing QRM entries in 2D, must be adapted for the 3D volumetric context of octrees. Drawing from the QRM's definition of dense factor , which is area-based (squared width), the 3D Dense Factor is defined as the smallest individual *volume* contribution of a subdivision to the total volume of the 3D image or spatial domain.

The proposed formula for the 3D Dense Factor is:

$$3D\ Dense\ Factor\ = \left\{\frac{Edge\ Length\ of\ the\ Image}{Number\ of\ Subdivisions\ in\ one\ dimension}\right\}^3$$

Here, "Number of Subdivisions in one dimension" is determined by the octree's depth, specifically 2^n, where 'n' is the number of levels in the octree (similar to 2^n for quadtrees in 1). This ensures the factor correctly reflects a volumetric unit.
For example, consider a 3D image with an edge length of 100 voxels (i.e., a 100x100x100 voxel volume). If the octree has 3 levels, the number of subdivisions in one dimension would be $2^3 = 8$.

$$3D\ Dense\ Factor\ = \left(\frac{100}{8}\right)^3\ =\ (12.5)^3\ =\ 1953.125$$

This calculated 3D Dense Factor (1953.125 in this example) represents the individual volume weight contribution of a single smallest subdivision unit to the total volume of the 3D image. The cubic nature of the 3D Dense Factor (power of 3 instead of 2) is a direct consequence of the dimensional increase. This seemingly subtle change has significant implications for how "density" is perceived and normalized in 3D, ensuring volumetric consistency and accurate representation of rough membership in a 3D context. The dense factor is more than a simple scaling factor; it represents the minimum unit volume that contributes to the overall "shadedness" of the 3D object. By multiplying the "number of shaded voxels" within an octant by this dense factor, a "raw shaded volume" consistent with the overall image scale is obtained. This "raw shaded volume" is then normalized by the Total number of voxels in the region (which is also a volume). This two-step normalization process (dense factor for unit volume, then division by current octant's volume) ensures that the final ORM value is a consistent, relative measure of "roughness" that can be meaningfully compared across different octants and levels of the tree, accurately reflecting the true proportion of occupied space within that specific region of the original 3D image. This consistency is paramount for robust and interpretable spatial analysis across the hierarchy. In a similar manner we can find the value of each elements value in the ORM matrix.

3.3. Extending Shaded Pixels to Occupied Voxels/Volume

In the context of QRM, the "Number of shaded pixels" is a crucial input, representing the count of occupied 2D units. For ORM, this concept is extended to "Number of shaded voxels" or "Occupied Volume" within a given octant. "Shaded" in 3D refers to voxels (volumetric pixels, the 3D equivalent of 2D pixels) that represent the object or concept of interest within the 3D space. This count would be obtained from the input 3D image, point cloud, or volumetric dataset. For instance, in a medical CT scan, "shaded voxels" might correspond to tissue types above a certain density threshold.

The transition from "pixels" to "voxels" is more than just a change in terminology; it implies a fundamental shift in the underlying data acquisition, representation, and processing. This highlights the need for ORM to handle diverse 3D data formats and the complexities associated with defining "occupancy" in volumetric data. Pixels are 2D area units, while voxels are 3D volumetric units. Therefore, for ORM, the "shaded" count must refer to the number of occupied voxels within a given octant. This implies that the input data for

ORM must be a volumetric representation (e.g., a 3D grid of binary or grayscale voxels, or a point cloud that can be voxelized). The concept of "shadedness" now refers to the proportion of the volume within an octant that is occupied by the object of interest. This aligns with the idea of rough membership for a 3D spatial concept, where the degree of membership is tied to the volumetric density or occupancy rather than just area density. This also raises considerations about how "shaded" is determined for non-binary 3D data, such as grayscale CT scans, where a thresholding operation might be necessary.

3.4. Formulating the ORM Entry Calculation

Based on the adaptations discussed, the refined formula for calculating ORM entries for each octant is proposed, directly analogous to the QRM formula [1]:

$$ORM = \frac{Number\ of\ shaded\ voxels\ *\ 3D\ Dense\ Factor}{Total\ number\ of\ voxels\ in\ the\ region}$$

The Total number of voxels in the region for an octant at a specific level 'n' (where 'n' is the level of the octant, starting from 1 for children of the root) is given by:

$$Total\ number\ of\ voxels\ in\ the\ region = \frac{Total\ number\ of\ voxels\ in\ the\ 3D\ image}{2^{3n}}$$

The calculation process will iterate through each level of the octree, from level 1 up to (n-1), where 'n' is the maximum number of levels. The entries for the 8 octants forming a subdivision will be organized based on a defined 3D traversal order (e.g., a standard octant numbering convention or Morton order). The consistent scaling factor of 2^{3n} for "total number of voxels in the region" is crucial for maintaining the hierarchical consistency of rough approximations across different octree levels. This factor precisely accounts for the volumetric reduction at each level of subdivision in an octree. If the original 3D image has $L \times L \times L$ voxels, and at level n each octant has an edge length of $\frac{L}{2^n}$, then its volume is $\left(\frac{L}{2n}\right)^3 = L^3/(2n)^3$ =L^3/2^3n. This ensures that the Total number of voxels in the region accurately represents the volume of the current octant relative to the total image volume. This consistency is paramount for reliable spatial analysis, as it guarantees that the rough membership value calculated for any octant, at any level, is a true proportional representation of its occupied volume within the overall 3D space, enabling meaningful comparisons across the entire octree hierarchy.

4. Construction Methodology for Octree Rough Matrix

The construction of an Octree Rough Matrix (ORM) from 3D spatial data involves a systematic three-step process, directly mirroring the methodology established for the Quad Rough Matrix (QRM) but adapted for the volumetric domain.

4.1. Step 1: Construction of Octree from 3D Image/Spatial Data

The initial step for constructing an ORM involves building a standard octree from the given 3D input data. This data can manifest in various forms, including a binary voxel grid, a grayscale volumetric image (such as a Computed Tomography (CT) or Magnetic Resonance Imaging (MRI) scan), or a point cloud that can be converted into a voxelized representation. The process follows the recursive subdivision principle inherent to octrees.[3] Starting from a root node that encompasses the entire 3D space, this volume is recursively subdivided into eight equally sized octants. This subdivision continues until a predefined termination condition is met.

Common termination conditions that dictate the granularity and depth of the octree include:

- **Maximum depth:** The tree reaches a specified maximum number of levels or subdivisions.[3] This limits the computational complexity and memory footprint.
- **Homogeneity:** An octant is determined to be entirely "occupied" (all shaded voxels) or entirely

"empty" (no shaded voxels). In such cases, further subdivision is unnecessary, as the rough membership value would be either 1 or 0, respectively.

- **Minimum voxel count:** An octant contains fewer than a given number of "shaded voxels".[2] This condition is useful for filtering out sparse noise or very small features.
- **Minimum size/volume:** An octant reaches a minimum spatial dimension or volume.[2] This prevents infinite recursion in continuous spaces or overly fine subdivisions that may not be meaningful.
- **Intensity tolerance:** Specifically for grayscale volumetric data, such as medical images, subdivision may cease if the intensity variation within an octant falls below a certain threshold.[9] This allows for controlled compression and data fidelity, preserving essential information while reducing data volume.

The construction process determines the total "Number of levels in the octree" (n), which is a crucial parameter for subsequent calculations of the 3D Dense Factor and ORM entries. The choice of termination condition is a critical design parameter for ORM, directly impacting the level of detail, computational cost, and the "roughness" captured. If the subdivision stops prematurely (e.g., a shallow tree due to a low maximum depth), many nodes might remain "rough" (i.e., their ORM values will be between 0 and 1), representing coarser approximations. Conversely, if the tree is allowed to go too deep (e.g., until every octant is perfectly crisp), it might over-segment the data, potentially losing the benefits of rough representation and becoming computationally expensive, closer to a traditional crisp voxel grid. The "intensity tolerance" [9] is particularly relevant as it allows for controlled compression and fidelity, implying that ORM can be tailored for specific application needs where a certain level of "acceptable roughness" is desired for efficiency or to filter out noise. This highlights a crucial design trade-off between the precision of rough approximation and the computational resources (memory, time) required.

4.2. Step 2: Determining the 3D Dense Factor

Once the octree structure is established, the 3D Dense Factor is calculated. This factor represents the volumetric contribution of the smallest possible subdivision unit to the overall 3D image or space. Adapting the formula from QRM , the 3D Dense Factor is calculated as:

$$3D\ Dense\ Factor\ = \left\{\frac{Edge\ length\ of\ the\ image}{Number\ of\ Subdivisions\ in\ one\ dimension}\right\}^3$$

4.3. Step 3: Calculation of ORM Entries

The final step involves calculating the ORM entries for each octant within the tree. This is performed iteratively for each level of the octree, typically starting from level 1 (the children of the root) up to level (n-1), where 'n' is the total number of levels in the octree, analogous to the QRM calculation.

For each octant within a given subdivision:

- **Determine the Number of shaded voxels:** This count represents the total number of occupied voxels within the specific octant being evaluated. This value is obtained from the processed 3D input data, which could involve counting binary voxels or applying a threshold to grayscale volumetric data.
- Calculate Total number of voxels in the region: This represents the total volumetric capacity of the current octant. For an octant at level 'n', this is calculated as: Total number of voxels in the region = (Total number of voxels in the 3D image) / 2^(3n) For our running example (a 100x100x100 voxel image, with a total of 1,000,000 voxels), for level one (n=1):
$$Total\ number\ of\ voxels\ in\ the\ region\ =\ 1{,}000{,}000\ /\ 2^{\wedge}(3*1)\ =\ 1{,}000{,}000\ /\ 8\ =\ 125{,}000$$
The consistent scaling factor of 23n for "total number of voxels in the region" is crucial for maintaining the hierarchical consistency of rough approximations across different octree levels. This factor precisely accounts for the volumetric reduction at each level of subdivision in an octree. If the original

3D image has L×L×L voxels, and at level n each octant has an edge length of L/2n, then its volume is (L/2n)3=L^3/(2n)^3=L^3/2^3n. This ensures that the Total number of voxels in the region accurately represents the volume of the current octant relative to the total image volume. This consistency is paramount for reliable spatial analysis.

- Apply the ORM formula: Using the calculated Number of shaded voxels, the 3D Dense Factor, and Total number of voxels in the region, the ORM entry for that specific octant is computed: ORM = (Number of shaded voxels * 3D Dense Factor) / Total number of voxels in the region

The entries for the 8 octants are typically organized into a conceptual 3D matrix or an ordered list corresponding to a defined traversal order (e.g., Z-order curve or Morton order) of the octree nodes. This process is repeated for all relevant levels of the octree, yielding a multi-resolution representation of the 3D spatial data's rough approximations.

5. Applications, Advantages, and Limitations of ORM

5.1. Potential Applications

The Octree Rough Matrix (ORM) provides a robust framework for representing and analyzing 3D spatial data with inherent vagueness, opening up several promising application areas:

- **3D Image Processing and Analysis:** ORM can enhance tasks such as volumetric image segmentation, where boundaries between different tissues or objects are often fuzzy (e.g., in medical CT or MRI scans [9]). It can also contribute to 3D data compression by representing regions with a rough membership value rather than requiring full binary detail, potentially reducing memory footprint while preserving critical boundary information. Feature extraction from complex 3D geological or environmental datasets could also benefit from ORM's ability to quantify spatial uncertainty.
- **Computer Graphics and Game Development:** In dynamic 3D environments, ORM could be utilized for advanced Level of Detail (LOD) management, where objects at a distance can be represented with coarser rough approximations, reducing rendering overhead. It also offers a more nuanced approach to collision detection for objects with deformable or ill-defined boundaries, allowing for "soft" collisions based on rough overlap rather than rigid binary intersections.
- **Spatial Databases and Geographic Information Systems (GIS):** ORM provides a powerful mechanism for indexing and querying vague 3D spatial objects, such as uncertain geological formations, atmospheric phenomena, or urban models with imprecise boundaries. It enables spatial queries that can retrieve regions based on their degree of "roughness" or occupancy, facilitating more flexible and realistic spatial analysis.
- **Robotics and Autonomous Systems:** For robots navigating complex environments or autonomous vehicles interpreting sensor data (e.g., LiDAR point clouds), ORM can represent the environment with explicit uncertainty. This allows for more robust path planning and obstacle avoidance in situations where sensor readings are noisy or incomplete, providing a more reliable understanding of the spatial context.

5.2. Advantages

The proposed Octree Rough Matrix offers several distinct advantages over traditional 3D spatial data structures:

- **Explicit Representation of Vagueness:** Unlike binary octrees that force crisp classifications, ORM explicitly quantifies and represents the inherent vagueness and uncertainty in 3D spatial data through its rough membership values (0, 1, and (0,1)). This provides a more faithful and informative model of real-world phenomena.
- **Multi-resolution Analysis:** The hierarchical nature of the octree combined with rough set approximations enables multi-resolution analysis of 3D rough data. This allows for examining spatial

vagueness at different levels of granularity, from coarse overall approximations to fine-grained details, without losing the context of uncertainty.

- **Potential for Data Compression:** By allowing nodes to hold rough membership values rather than being fully subdivided until crisp, ORM can achieve data compression. This is particularly beneficial for large 3D datasets where some level of "acceptable roughness" can significantly reduce memory and storage requirements without discarding all boundary information, as seen in medical imaging applications.[9]
- **Integration with Rough Set Theory:** ORM firmly grounds 3D spatial representation within the formal mathematical framework of rough set theory. This integration opens avenues for applying established rough set operations and analytical techniques directly to 3D spatial data, facilitating more rigorous and theoretically sound spatial reasoning.

5.3. Limitations and Challenges

Despite its advantages, the implementation and application of ORM also present certain limitations and challenges:

- **Computational Cost:** For extremely large 3D datasets or when constructing very deep octrees to achieve high precision, the computational cost of building and traversing the ORM can be substantial. This necessitates efficient algorithms and potentially parallel processing techniques.
- **Defining "Shaded Voxels":** For non-binary volumetric data (e.g., grayscale CT scans or continuous sensor readings), the definition of "shaded voxels" requires a clear thresholding or classification mechanism. The choice of this threshold can significantly impact the resulting rough approximations and the ORM values.
- **Choice of Optimal Termination Conditions:** Selecting the most appropriate termination conditions for octree construction (e.g., maximum depth, homogeneity threshold, minimum voxel count, intensity tolerance) is crucial. An improper choice can lead to over-segmentation (excessive detail, high cost) or under-segmentation (insufficient detail, loss of critical rough information).
- **Complexity of 3D Traversal and Indexing:** While octrees provide efficient spatial indexing, operations like neighbor searches, range queries, and updates in 3D are inherently more complex than their 2D quadtree counterparts, requiring careful algorithmic design.

6. COMPARATIVE ANALYSIS:

To clearly delineate the foundation for extending QRM to ORM, a comparative analysis of quadtree and octree properties is presented in Table 1. This comparison highlights the structural and functional similarities and, more importantly, the critical differences between these two hierarchical data structures. This visual comparison clarifies how the QRM concepts (e.g., "number of shaded pixels," "dense factor," "total region size") must be adapted for the 3D octree context. For instance, it makes it evident that "pixels" become "voxels," "squares" become "cubes," and the scaling factor for "total region size" will change from an area-based 22n to a volume-based 23n. This table serves as a foundational bridge, visually setting the stage for the direct and logical adaptation of the QRM methodology to the ORM.

Feature	Quadtree	Octree
Dimensions	2D (planar)	3D (volumetric)
Children per Node	4 (quadrants)	8 (octants)

Subdivision Shape	Square or Rectangle	Cube or Rectangular Prism
Primary Applications	2D Image Processing, GIS, 2D Collision Detection	3D Graphics, Medical Imaging, Point Clouds, 3D Collision Detection, Spatial Indexing
Analogous To	N/A (base hierarchical spatial structure)	Quadtree (3D extension)
Unit of Data	Pixels (2D area units)	Voxels (3D volume units)
Recursive Subdivision Principle	Divide into 4 equally sized quadrants	Divide into 8 equally sized octants

- **Table 1: Comparison of Quadtree and Octree Properties**

7. Conclusion

The conceptualization and methodology for the Octree Rough Matrix (ORM) represent a significant advancement in the representation and analysis of three-dimensional spatial data, particularly where vagueness and imprecision are inherent. By systematically extending the principles of the Quad Rough Matrix (QRM), ORM provides a robust and theoretically grounded framework for integrating rough set theory with hierarchical 3D spatial data structures. The transition from 2D pixels to 3D voxels, and the adaptation of concepts like the "dense factor" to volumetric measures, ensure that ORM accurately quantifies the degree of "occupancy" or "membership" within an octant, reflecting the true rough nature of 3D spatial phenomena.

ORM moves beyond traditional binary spatial representations by explicitly modeling boundary regions with values between 0 and 1, offering a more nuanced and faithful depiction of real-world objects and environments. This multi-resolution capability, coupled with the formal rigor of rough set theory, positions ORM as a powerful tool for applications in diverse fields, including advanced 3D image processing, computer graphics, spatial databases, and robotics, where handling uncertainty is paramount.

Future research should focus on empirical validation of ORM through implementation and application to various real-world 3D datasets, such as medical imagery, geological models, and point clouds. Investigations into performance optimization techniques, including parallel processing and GPU acceleration for ORM construction and query operations, are crucial for handling massive datasets. Further work could also explore adaptive ORM construction methods that dynamically adjust termination conditions based on data characteristics or application requirements. Comparative studies with other uncertainty representation methods, such as fuzzy logic or probabilistic approaches within 3D spatial structures, would also provide valuable insights into ORM's strengths and weaknesses. Finally, the development of specific rough set operations (e.g., union, intersection, complement) directly applicable to ORMs could unlock new analytical capabilities for complex 3D spatial reasoning.

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