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Application of Multi Vague Set in Pattern Recognition

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Abstract

The selection of appropriate building materials often involves handling uncertain, vague and incomplete information arising from diverse criteria such as cost, durability, strength and thermal insulation. To address this challenge, we propose a novel framework based on Multi vague sets, which extend traditional vague set theory by incorporating Multi dimensional truth, falsity and hesitation values. This enriched representation captures the imprecise nature of real-world material data more effectively. We define and utilize multiple distance measures tailored for Multi vague environments to compare and classify materials based on their attributes. A case study involving five dimensional evaluations of building materials demonstrates the efficacy of this method in recognizing patterns and assisting in material recommendation. The proposed approach enhances decision making by offering a structured, scalable and interpretable model for vague and hesitant information in construction related applications.

Keywords: Vague set, Multi vague set, distance measure, pattern recognition. Mathematical Subject

Classification (2000): 08A72, 20N25, 03E72.

1 Introduction

Pattern recognition plays a vital role in various fields such as computer vision, medical diagnosis and data mining where accurate classification and decision making are essential despite the presence of uncertainty and imprecision (Zadeh, 1965) [7]. Traditional approaches often struggle to handle ambiguous or vague data effectively. To address these challenges, fuzzy set theory has been widely utilized; however, it sometimes fails to capture the full extent of uncertainty, especially when multiple degrees of vagueness coexist.

Multi vague sets extend the classical vague and fuzzy set concepts by incorporating membership function and non- membership function that simultaneously represent truth and falsity degrees (Atanassov, 1986) [6]. This enriched framework allows for a more nuanced representation of complex and uncertain information, which is particularly useful in pattern recognition tasks where data ambiguity is inherent.

Distance measures between such fuzzy structures are essential tools for pattern recognition, classification and decision making, as they quantify the similarity or dissimilarity between uncertain elements. Szmidt and Kacprzyk (1997) [2] made significant contributions by developing and analyzing distance metrics for intuitionistic fuzzy sets, which have become foundation in the field. Their work demonstrated the effectiveness of distance-based methods in various applications including medical diagnosis and career determination.

In this paper, we propose a novel distance-based approach utilizing Multi vague sets to enhance pattern recognition. By defining and computing appropriate distance measures between Multi vague sets, we enable more effective discrimination and classification of patterns characterized by Multi-dimensional vagueness (N. Ramakrishna, 2020) [11]. The proposed methodology not only captures the inherent uncertainty in data but also integrates hesitation parameters, improving robustness and decision accuracy.
Our work builds upon and extends existing theories in fuzzy and vague set literature, contributing to the ongoing development of intelligent systems capable of handling uncertain and incomplete information. Experimental results demonstrate the effectiveness of the proposed method in real world applications, highlighting its potential for advancing pattern recognition techniques.

2 Preliminaries

Definition 2.1 A Vague set A in the universe of discourse G is a pair (t_A, f_A) where $t_A: G \to [0, 1]$, $f_A: G \to [0, 1]$, are the mappings such that $t_A(x) + f_A(x) \leq 1$, for all $x \in G$. The functions $t_A(x)$ and $f_A(x)$ are true and false membership functions respectively.

Definition 2.2 Let G be a non-empty set. A vague set $A = (t_A, f_A)$ where $t_A(x) = (t_{1A}(x), t_{2A}(x), ..., t_{kA}(x))$ and $f_A(x) = (f_{1A}(x), f_{2A}(x), ..., f_{kA}(x))$ and $t_{iA}: G \to [0, 1]$, $f_{iA}: G \to [0, 1]$, are mappings such that $t_{iA}(x) + f_{iA}(x) \leq 1$, for all $x \in G$, for i =1,2,3,....................................... , k., is called Multi vague set of G with dimension **k**. Here $t_{1A}(x) \geq t_{2A}(x) \geq ......t_{kA}(x)$, for all $x \in G$.

Note: We arranged the true membership sequence is decreasing order, then the corresponding false membership sequence need not be in decreasing or increasing order.

According to fuzzy set theory, if the membership degree of an element is $t_A(x)$, if non- membership degree of an element x is $f_A(x)$. Furthermore, we have $\pi_A(x) = 1 - t_A(x) - f_A(x)$ called the vague set index or hesitation on margin of x in A. $\pi_A(x)$ is the degree of indeterminacy of $x \in G$ to the vague set A and i.e., $\pi_A(x)$
$\in [0, 1]$ for every $x \in G$, $\pi_A(x)$ expresses the lack of knowledge of whether x belongs to the vague set A or not.

Definition 2.3 Let G be non-empty group and A, B, C are vague sets in G.
The distance measure d between vague sets A and B is a mapping $d: G \quad G \quad [0, 1]; \to$ if d (A, B) satisfies the following axioms.

A_1) $0 \leq d(A,B) \leq 1$
A_2) d (A, B) = 0 iff A = B
A_3) d (A, B) = d (B, A)
A_4) d (A, C) + d (C, B) ≥ d (A, B)
A_5) if $A \subseteq B \subseteq C$, then d (A, C) ≥ d (A, B) and d (A, C) ≥ d (B, C)

Distance measure is a term that describes the difference between vague sets and can be considered as a dual concept of similarity measures between vague sets proposed by Szmidt, E., & Kacprzyk, J.

Definition 2.4 Let $A = \{\langle x, t_A(x), f_A(x), \pi_A(x)\rangle \mid x \in G\}$ and
$B = \{\langle x, t_B(x), f_B(x), \pi_B(x) \rangle \mid x \in G\}$ be two vague sets in $G = \{x_1, x_2, ..., x_n\}$.
Based on the geometric interpretation of vague set, Szmidt, E and Kacprzyk, J, proposed the following four distance measures between A and B.

Let $A = \{\langle x_1, t_A(x_1), f_A(x_1), \pi_A(x_1)\rangle$,

$\langle x_2, t_A(x_2), f_A(x_2), \pi_A(x_2)\rangle,$
$\langle ....., ..., ..., \quad \rangle,$
$\langle ....., ..., ..., \quad \rangle,$
$\langle x_n, t_A(x_n), f_A(x_n), \pi_A(x_n)\rangle\}$

$B = \{\langle x_1, t_B(x_1), f_B(x_1), \pi_B(x_1)\rangle,$
$\langle x_2, t_B(x_2), f_B(x_2), \pi_B(x_2)\rangle,$
$\langle ....., ..., ..., \quad \rangle,$
$\langle ....., ..., ..., \quad \rangle,$
$\langle x_n, t_B(x_n), f_B(x_n), \pi_B(x_n)\rangle\}$, then

1) The Hamming distance.

$$d_H(A,B) = \frac{1}{2}\sum_{i=1}^{n}\left(|t_A(x_i) - t_B(x_i)| + |f_A(x_i) - f_B(x_i)| + |\pi_A(x_i) - \pi_B(x_i)|\right)$$

2) The Euclidean distance.

$$d_E(A,B) = \sqrt{\frac{1}{2}\sum_{i=1}^{n}\left(|t_A(x_i) - t_B(x_i)|^2 + |f_A(x_i) - f_B(x_i)|^2 + |\pi_A(x_i) - \pi_B(x_i)|^2\right)}$$

3) The Normalized Euclidean distance.

$$d_E(A,B) = \sqrt{\frac{1}{2n}\sum_{i=1}^{n}(|t_A(x_i) - t_B(x_i)|^2 + |f_A(x_i) - f_B(x_i)|^2 + |\pi_A(x_i) - \pi_B(x_i)|^2)}$$

4) The Normalized Hamming distance.

$$d_H(A,B) = \frac{1}{2n}\sum_{i=1}^{n}(|t_A(x_i) - t_B(x_i)| + |f_A(x_i) - f_B(x_i)| + |\pi_A(x_i) - \pi_B(x_i)|)$$

3 Distance between Multi vague sets

Now, we extend these distances to Multi vague sets.

i.e., $A = \{< x_1, (t_{1A}(x_1), \ldots\ldots, t_{kA}(x_1)), (f_{1A}(x_1), \ldots\ldots, f_{kA}(x_1)), (\pi_{1A}(x_1), \ldots\ldots, \pi_{kA}(x_1))>$

$< .., (t_{1A}(..), \ldots\ldots, t_{kA}(..)), (f_{1A}(..), \ldots\ldots, f_{kA}(..)), (\pi_{1A}(..), \ldots\ldots, \pi_{kA}(..))>$

$< x_1, (t_{1A}(x_j), \ldots\ldots, t_{kA}(x_j)), (f_{1A}(x_j), \ldots\ldots, f_{kA}(x_j)), (\pi_{1A}(x_j), \ldots\ldots, \pi_{kA}(x_j))>$

$< .., (t_{1A}(..), \ldots\ldots, t_{kA}(..)), (f_{1A}(..), \ldots\ldots, f_{kA}(..)), (\pi_{1A}(..), \ldots\ldots, \pi_{kA}(..))>$

$< x_1, (t_{1A}(x_n), \ldots\ldots, t_{kA}(x_n)), (f_{1A}(x_n), \ldots\ldots, f_{kA}(x_n)), (\pi_{1A}(x_n), \ldots\ldots, \pi_{kA}(x_n))>\}$

$B = \{< x_1, (t_{1B}(x_1), \ldots\ldots, t_{kB}(x_1)), (f_{1B}(x_1), \ldots\ldots, f_{kB}(x_1)), (\pi_{1B}(x_1), \ldots\ldots, \pi_{kB}(x_1))>$

$< .., (t_{1B}(..), \ldots\ldots, t_{kB}(..)), (f_{1B}(..), \ldots\ldots, f_{kB}(..)), (\pi_{1B}(..), \ldots\ldots, \pi_{kB}(..))>$

$< x_1, (t_{1B}(x_j), \ldots\ldots, t_{kB}(x_j)), (f_{1B}(x_j), \ldots\ldots, f_{kB}(x_j)), (\pi_{1B}(x_j), \ldots\ldots, \pi_{kB}(x_j))>$

$< .., (t_{1B}(..), \ldots\ldots, t_{kB}(..)), (f_{1B}(..), \ldots\ldots, f_{kB}(..)), (\pi_{1B}(..), \ldots\ldots, \pi_{kB}(..))>$

$< x_1, (t_{1B}(x_n), \ldots\ldots, t_{kB}(x_n)), (f_{1B}(x_n), \ldots\ldots, f_{kB}(x_n)), (\pi_{1B}(x_n), \ldots\ldots, \pi_{kB}(x_n))>\}$

Here A and B are Multi vague sets with dimension **k**, and having n elements.
and $t_{1A} \geq t_{2A} \geq \cdots \geq t_{kA},\ t_{iA}: G \to [0,1],\ f_{iA}: G \to [0,1]\ and\ \pi_{iA}: G \to [0,1]$ are membership, non- membership and hesitant functions respectively. Also
$t_{iA} + f_{iA} + \pi_{iA} = 1, \forall i = 1,2,..,k.$ And j=1,2,…,n.

1) The Manhattan distance.

$$d_{Man}(A,B) = \frac{1}{n}\sum_{j=1}^{n}\sum_{i=1}^{k}\left(\left|t_{iA}(x_j) - t_{iB}(x_j)\right| + \left|f_{iA}(x_j) - f_{iB}(x_j)\right| + \left|\pi_{iA}(x_j) - \pi_{iB}(x_j)\right|\right)$$

2) The Euclidean distance.

$$d_E(A,B) = \sqrt{\frac{1}{n}\sum_{j=1}^{n}\sum_{i=1}^{k}(|t_A(x_i) - t_B(x_i)|^2 + |f_A(x_i) - f_B(x_i)|^2 + |\pi_A(x_i) - \pi_B(x_i)|^2)}$$

3) The Normalized Euclidean distance.

$$d_{n-E}(A,B) = \sqrt{\frac{1}{nk}\sum_{j=1}^{n}\sum_{i=1}^{k}\left(\left|t_{iA}(x_j) - t_{iB}(x_j)\right|^2 + \left|f_{iA}(x_j) - f_{iB}(x_j)\right|^2 + \left|\pi_{iA}(x_j) - \pi_{iB}(x_j)\right|^2\right)}$$

4) The Normalized Manhattan distance.

$$d_{n-Man}(A,B) = \frac{1}{nk}\sum_{j=1}^{n}\sum_{i=1}^{k}\left(\left|t_{iA}(x_j) - t_{iB}(x_j)\right| + \left|f_{iA}(x_j) - f_{iB}(x_j)\right| + \left|\pi_{iA}(x_j) - \pi_{iB}(x_j)\right|\right)$$

Note: Hamming distance counts the number of positions at which two vectors (or strings) of equal length differ. It's typically used for categorical, binary or discrete data, while Manhattan measures the sum of absolute differences between coordinates in a Multi dimensional space. It's a numeric measure for continuous or real-valued vectors. So, we replacing Hamming distance with Manhattan distance.

Example 3.1 Let G be a nonempty group. A and B are Multi vague sets of G with dimension 4, with 3 elements. Let x, y and $z \in G$

A = {⟨x, (0.9, 0.8, 0.7, 0.6), (0, 0.1, 0.1, 0.3)⟩,
⟨y, (0.8, 0.7, 0.7, 0.5), (0, 0, 0.2, 0.3)⟩,
⟨z, (0.7, 0.6, 0.6, 0.4), (0.2, 0.3, 0.3, 0.4)⟩}

B = {⟨x, (0.7, 0.6, 0.5, 0.5), (0.1, 0.1, 0.2, 0.2)⟩,
⟨y, (0.6, 0.5, 0.4, 0.4), (0.2, 0.3, 0.3, 0.2)⟩,
⟨z, (0.6, 0.5, 0.3, 0.1), (0.2, 0.3, 0.6, 0.3)⟩}

Now we find distance measure between Multi vague sets A and B.
Given (G, .) is a group and x, y and z G ∈
we find hesitant values, include in A and B, then

A = {⟨x, (0.9, 0.8, 0.7, 0.6), (0, 0.1, 0.1, 0.3), (0.1, 0.1, 0.2, 0.1)⟩,
⟨y, (0.8, 0.7, 0.7, 0.5), (0, 0, 0.2, 0.3), (0.2, 0.3, 0.1, 0.2)⟩,
⟨z, (0.7, 0.6, 0.6, 0.4), (0.2, 0.3, 0.3, 0.4), (0.1, 0.1, 0.1, 0.2)⟩}

B = {⟨x, (0.7, 0.6, 0.5, 0.5), (0.1, 0.1, 0.2, 0.2), (0.2, 0.3, 0.3, 0.3)⟩,
⟨y, (0.6, 0.5, 0.4, 0.4), (0.2, 0.3, 0.3, 0.2), (0.2, 0.2, 0.3, 0.4)⟩,
⟨z, (0.6, 0.5, 0.3, 0.1), (0.2, 0.3, 0.6, 0.3), (0.2, 0.2, 0.1, 0.6)⟩}

1) The Manhattan distance.

$$d_{Man}(A,B) = \frac{1}{n}\sum_{j=1}^{n}\sum_{i=1}^{k}\left(\left|t_{iA}(x_j) - t_{iB}(x_j)\right| + \left|f_{iA}(x_j) - f_{iB}(x_j)\right| + \left|\pi_{iA}(x_j) - \pi_{iB}(x_j)\right|\right)$$

Here dimension = k = 4 and number of elements = n = 3

$d_{Man}(A,B) = \frac{1}{3}[\{|0.9-0.7| + |0.8-0.6| + |0.7-0.5| + |0.6-0.5| +$
$|0-0.1| + |0.1-0.1| + |0.1-0.2| + |0.3-0.2| +$
$|0.1-0.2| + |0.1-0.3| + |0.2-0.3| + |0.1-0.3|\} +$
$\{|0.8-0.6| + |0.7-0.5| + |0.7-0.4| + |0.5-0.4| +$
$|0-0.2| + |0-0.3| + |0.2-0.3| + |0.3-0.2| +$
$|0.2-0.2| + |0.3-0.2| + |0.1-0.3| + |0.2-0.4|\} +$
$\{|0.7-0.6| + |0.6-0.5| + |0.6-0.3| + |0.4-0.1| +$
$|0.2-0.2| + |0.3-0.3| + |0.3-0.6| + |0.4-0.3| +$
$|0.1-0.2| + |0.1-0.2| + |0.1-0.1| + |0.2-0.6|\}]$

$= \frac{1}{3}[\{(0.2+0.2+0.2+0.1) + (0.1+0+0.1+0.1) + (0.1+0.2+0.1+0.2)\}$
$+ \{(0.2+0.2+0.3+0.1) + (0.2+0.3+0.1+0.1) + (0+0.1+0.2+0.2)\}$
$+ \{(0.1+0.1+0.3+0.3) + (0+0+0.3+0.1) + (0.1+0.1+0+0.4)\}]$

$= \frac{1}{3}[(0.9+0.3+0.6) + (0.8+0.7+0.5) + (0.8+0.4+0.6)]$

$\therefore d_{Man}(A,B) = \frac{5.6}{3} = 1.86$

1) The Normalized Manhattan distance.

$$d_{n-Man}(A,B) = \frac{1}{nk}\sum_{j=1}^{n}\sum_{i=1}^{k}\left(\left|t_{iA}(x_j) - t_{iB}(x_j)\right| + \left|f_{iA}(x_j) - f_{iB}(x_j)\right| + \left|\pi_{iA}(x_j) - \pi_{iB}(x_j)\right|\right)$$

2) The Euclidean distance.

$$d_E(A,B) = \sqrt{\frac{1}{n}\sum_{j=1}^{n}\sum_{i=1}^{k}\left(|t_A(x_i) - t_B(x_i)|^2 + |f_A(x_i) - f_B(x_i)|^2 + |\pi_A(x_i) - \pi_B(x_i)|^2\right)}$$

3) The Normalized Euclidean distance.

$$d_{n-E}(A,B) = \sqrt{\frac{1}{nk}\sum_{j=1}^{n}\sum_{i=1}^{k}\left(\left|t_{iA}(x_j) - t_{iB}(x_j)\right|^2 + \left|f_{iA}(x_j) - f_{iB}(x_j)\right|^2 + \left|\pi_{iA}(x_j) - \pi_{iB}(x_j)\right|^2\right)}$$

Note: We observed that

1) $0 \le d_H(A,B) \le n$

2) $0 \le d_{n-H}(A,B) \le 1$

3) $0 \le d_H(A,B) \le \sqrt{n}$

4) $0 \le d_{n-E}(A,B) \le 1$

4 Model of vague sets in pattern Recognition

In this process, a set of patterns in (vague in nature) and another unknown pattern called is given (also vague in nature). Both the set of the patterns and that of the pattern and that of the sample are within the same feature space or attributes 'm'. The task is to find the distance between each of the patterns and the sample. The smallest or shortest distance between any of the patterns and the sample shows that, the same belongs to that pattern. This is what pattern recognition.

Assume that there exist m patterns given by

$A_l = \{< x_j, t_{A_l}(x_j), f_{A_l}(x_j), \pi_{A_l}(x_j) \mid x_j \in X \}$, j=1,2,…,n., l=1,2,…m.

Here $t_{iA}: G \to [0,1]$, $f_{iA}: G \to [0,1]$ and $\pi_{iA}: G \to [0,1]$ are membership, non- membership and hesitant fuzzy mappings. And $t_{iA} + f_{iA} + \pi_{iA} = 1$

$B = \{< x_j, t_B x_j, f_B x_j, \pi_B x_j > \mid x_j \in X\}$ be the sample to be tested.

According to E.Smidt, J.Kacprzyk [22, 24], see the following example.

Let $X = \{x_1, x_2, x_3, x_4\}$, n = 4, be the attributes.

Let A_1, A_2, A_3, A_4, A_5, and A_6 are classification of different building materials. B is another kind of unknown building material.

Let x_1 = Compressive strength (CS)

x_2 = Thermal Insulation (TI)

x_3 = Cost Efficiency (CE)

x_4 = Durability (D)

$A_1 = \{\langle t_{A1}(x_1), f_{A1}(x_1), \pi_{A1}(x_1)\rangle, \langle t_{A1}(x_2), f_{A1}(x_2), \pi_{A1}(x_2)\rangle,$
$\langle t_{A1}(x_3), f_{A1}(x_3), \pi_{A1}(x_3)\rangle, \langle t_{A1}(x_4), f_{A1}(x_4), \pi_{A1}(x_4)\rangle\}$

$A_2 = \{\langle t_{A2}(x_1), f_{A2}(x_1), \pi_{A2}(x_1)\rangle, \langle t_{A2}(x_2), f_{A2}(x_2), \pi_{A2}(x_2)\rangle,$
$\langle t_{A2}(x_3), f_{A2}(x_3), \pi_{A2}(x_3)\rangle, \langle t_{A2}(x_4), f_{A2}(x_4), \pi_{A2}(x_4)\rangle\}$

$A_3 = \{\langle t_{A3}(x_1), f_{A3}(x_1), \pi_{A3}(x_1)\rangle, \langle t_{A3}(x_2), f_{A3}(x_2), \pi_{A3}(x_2)\rangle,$
$\langle t_{A3}(x_3), f_{A3}(x_3), \pi_{A3}(x_3)\rangle, \langle t_{A3}(x_4), f_{A3}(x_4), \pi_{A3}(x_4)\rangle\}$

$A_4 = \{\langle t_{A4}(x_1), f_{A4}(x_1), \pi_{A4}(x_1)\rangle, \langle t_{A4}(x_2), f_{A4}(x_2), \pi_{A4}(x_2)\rangle,$
$\langle t_{A4}(x_3), f_{A4}(x_3), \pi_{A4}(x_3)\rangle, \langle t_{A4}(x_4), f_{A4}(x_4), \pi_{A4}(x_4)\rangle\}$

$A_5 = \{\langle t_{A5}(x_1), f_{A5}(x_1), \pi_{A5}(x_1)\rangle, \langle t_{A5}(x_2), f_{A5}(x_2), \pi_{A5}(x_2)\rangle,$
$\langle t_{A5}(x_3), f_{A5}(x_3), \pi_{A5}(x_3)\rangle, \langle t_{A5}(x_4), f_{A5}(x_4), \pi_{A5}(x_4)\rangle\}$

$A_6 = \{\langle t_{A6}(x_1), f_{A6}(x_1), \pi_{A6}(x_1)\rangle, \langle t_{A6}(x_2), f_{A6}(x_2), \pi_{A6}(x_2)\rangle,$
$\langle t_{A6}(x_3), f_{A6}(x_3), \pi_{A6}(x_3)\rangle, \langle t_{A6}(x_4), f_{A6}(x_4), \pi_{A6}(x_4)\rangle\}$

Equivalently,

$A_l = \{\langle t_{Al}(x_1), f_{Al}(x_1), \pi_{Al}(x_1)\rangle, \langle t_{Al}(x_2), f_{Al}(x_2), \pi_{Al}(x_2)\rangle,$
$\langle t_{Al}(x_3), f_{Al}(x_3), \pi_{Al}(x_3)\rangle, \langle t_{Al}(x_4), f_{Al}(x_4), \pi_{Al}(x_4)\rangle\}$

where l = 1, 2, 3, 4, 5 and 6.

Let's see the values,

A_1 = {(1, 0, 0), (0.8, 0, 0.2), (0.6, 0.2, 0.2), (0.5, 0.2, 0.3)}

A_2 = {(0.7, 0.1, 0.2), (0.9, 0.1, 0), (0.8, 0.1, 0.1), (0.6, 0.2, 0.2)}

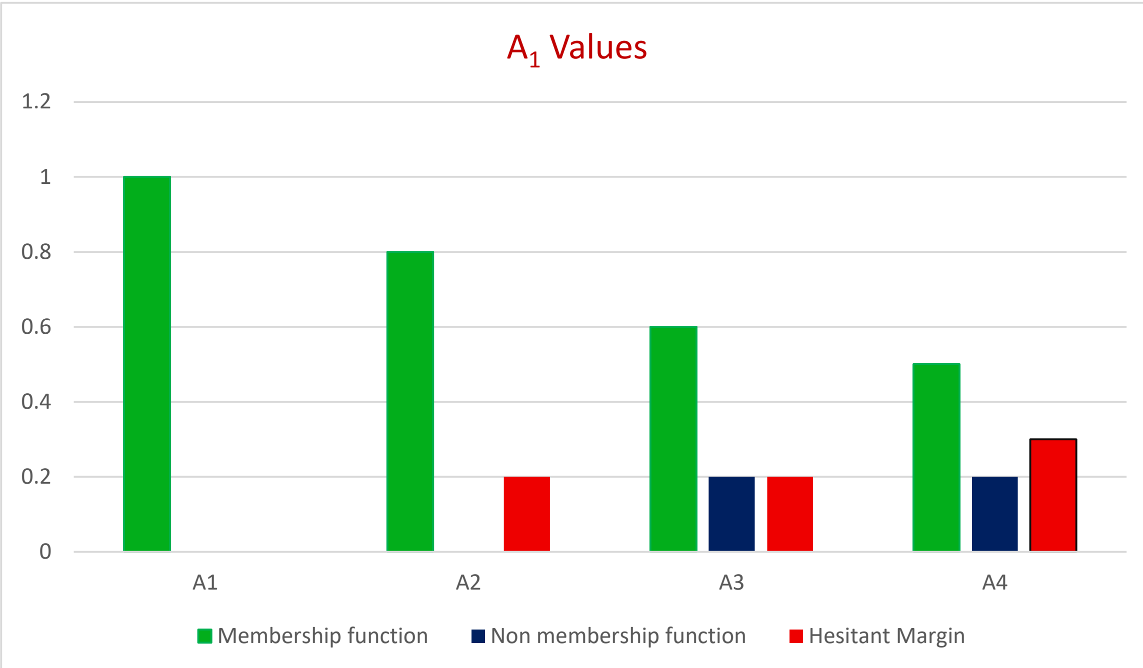
A_3 = {(0.6, 0.3, 0.1), (0.7, 0.1, 0.2), (1, 0, 0), (0.9, 0.1, 0)}

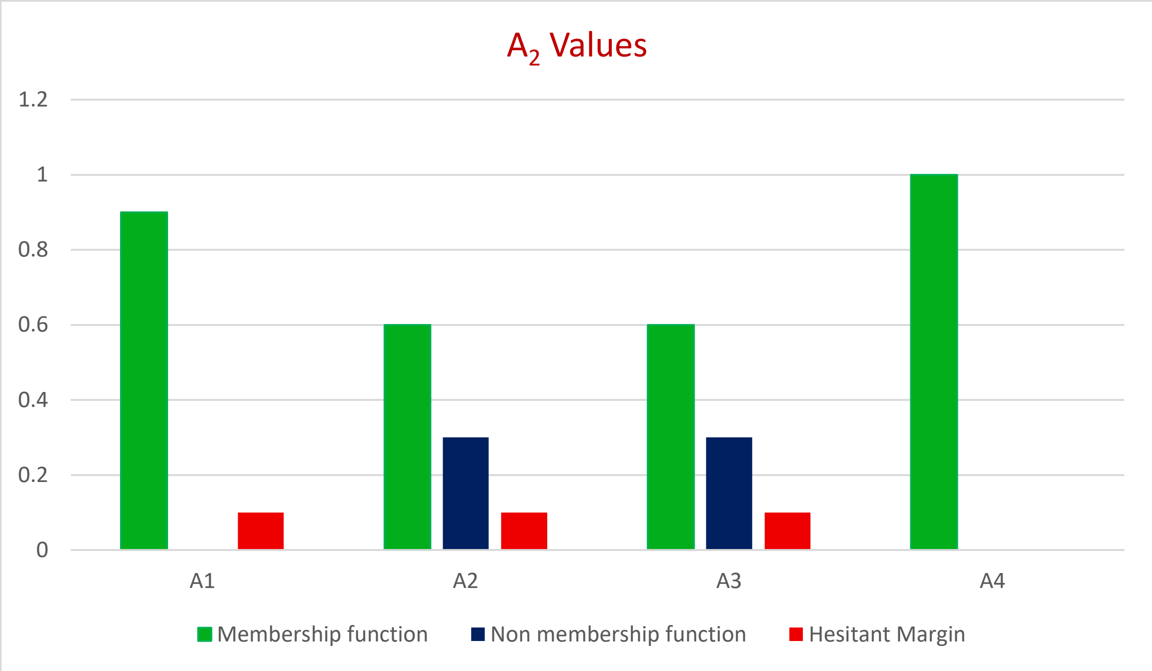
A_4 = {(0.9, 0, 0.1), (0.6, 0.3, 0.1), (0.6, 0.3, 0.1), (1, 0, 0)}

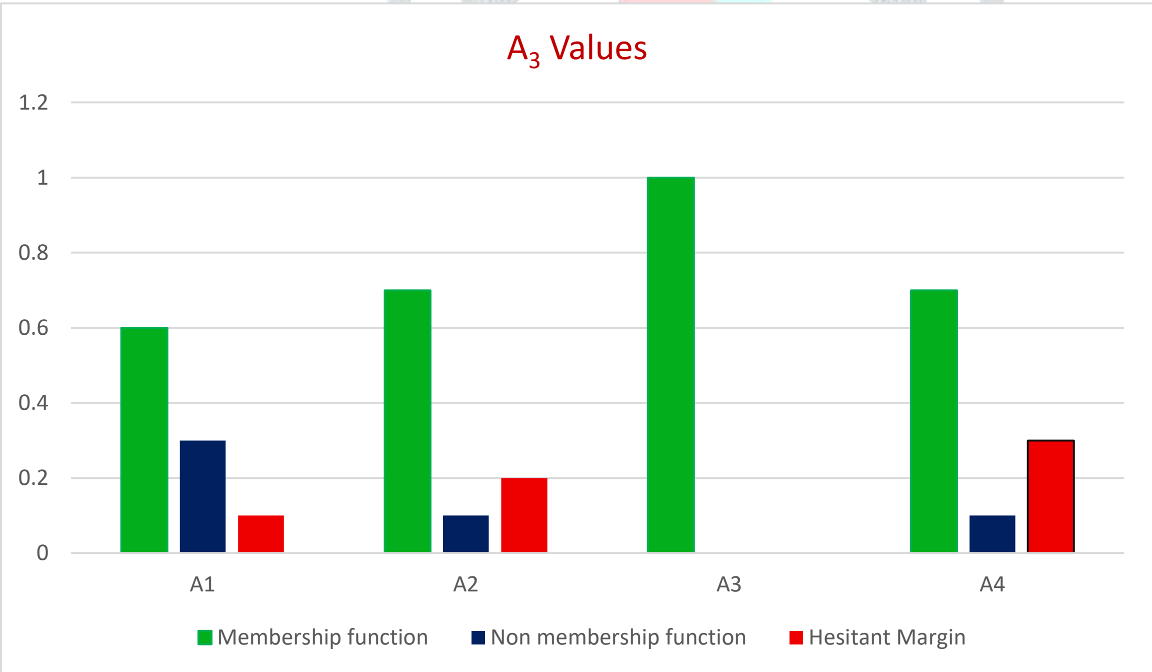
A_5 = {(0.5, 0.3, 0.2), (0.9, 0, 0.1), (1, 0, 0), (0.7, 0.1, 0.2)}

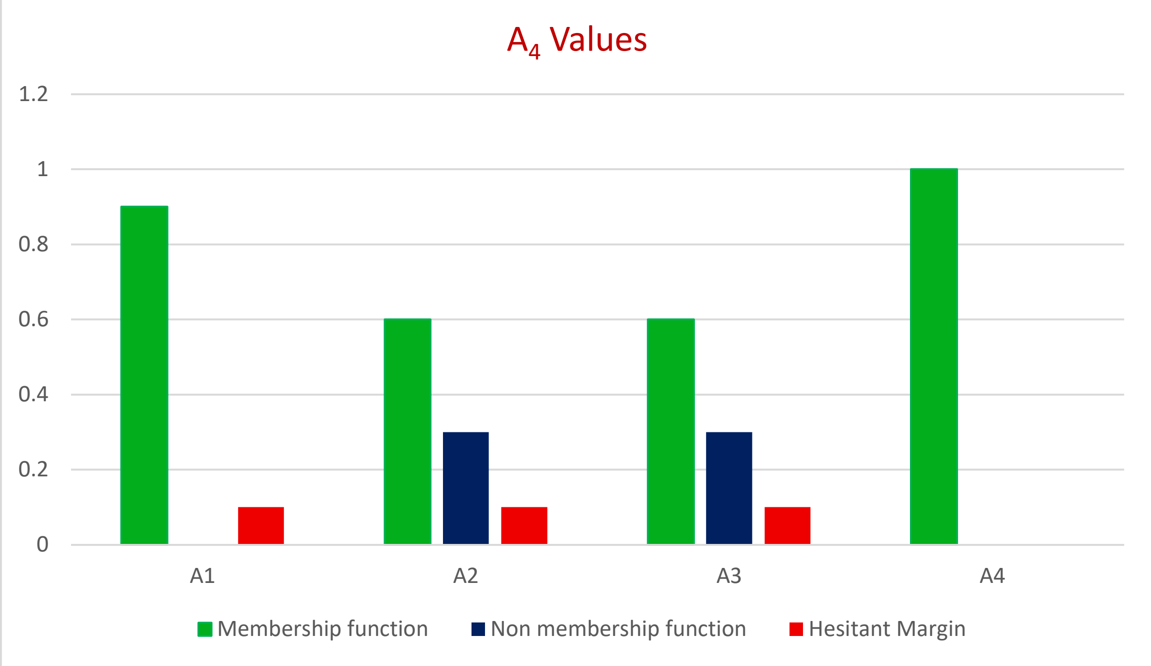
A_6 = {(0.8, 0, 0.2), (0.7, 0.2, 0.1), (0.7, 0.1, 0.2), (0.4, 0.3, 0.3)}

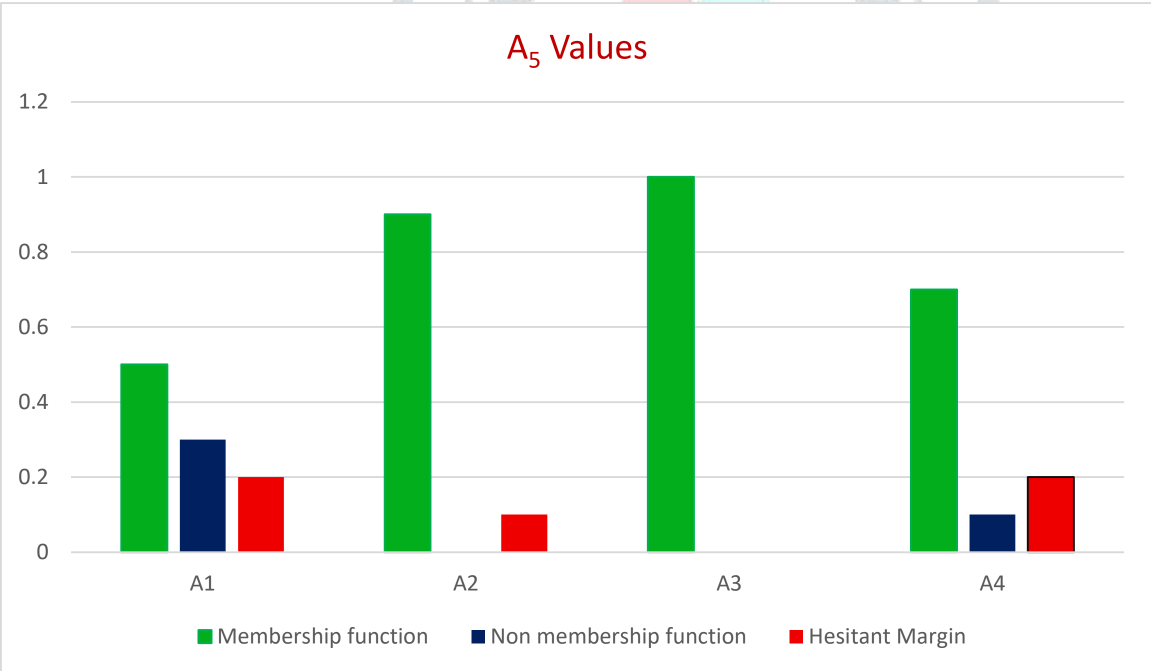
and B = {(0.6, 0.3, 0.1), (0.5, 0.3, 0.2), (0.9, 0.1, 0), (0.8, 0.2, 0)}

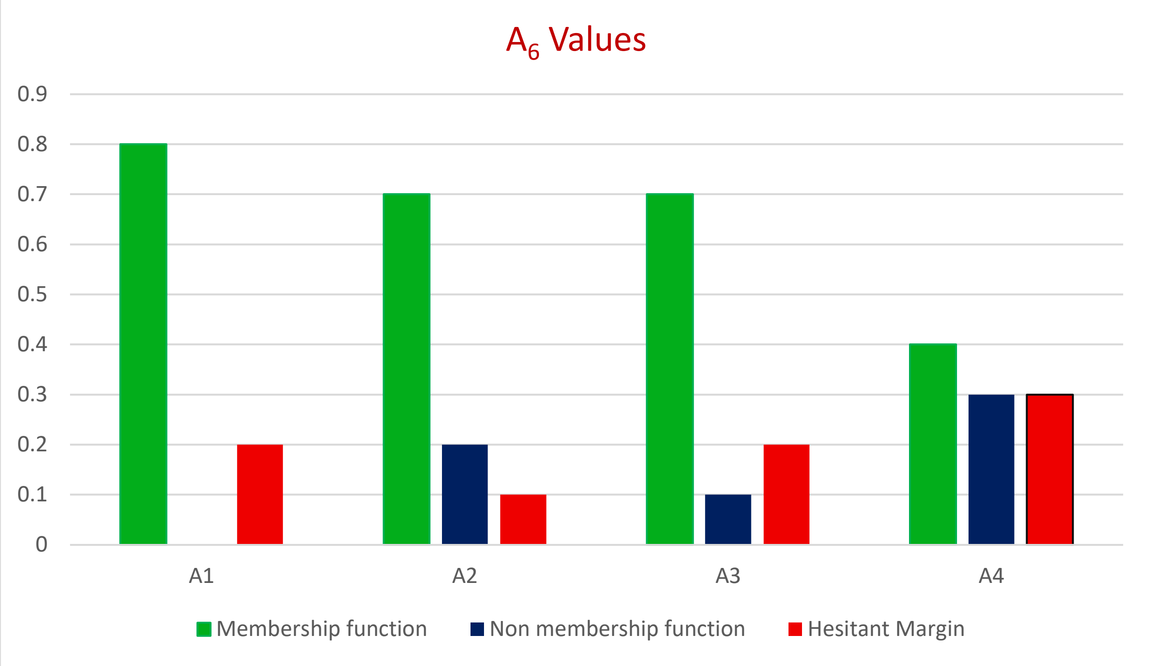

A_1 Values
1.2
1
0.8
0.6
0.4
0.2
0
A1
A2
A3
A4
Membership function
Non membership function
Hesitant Margin

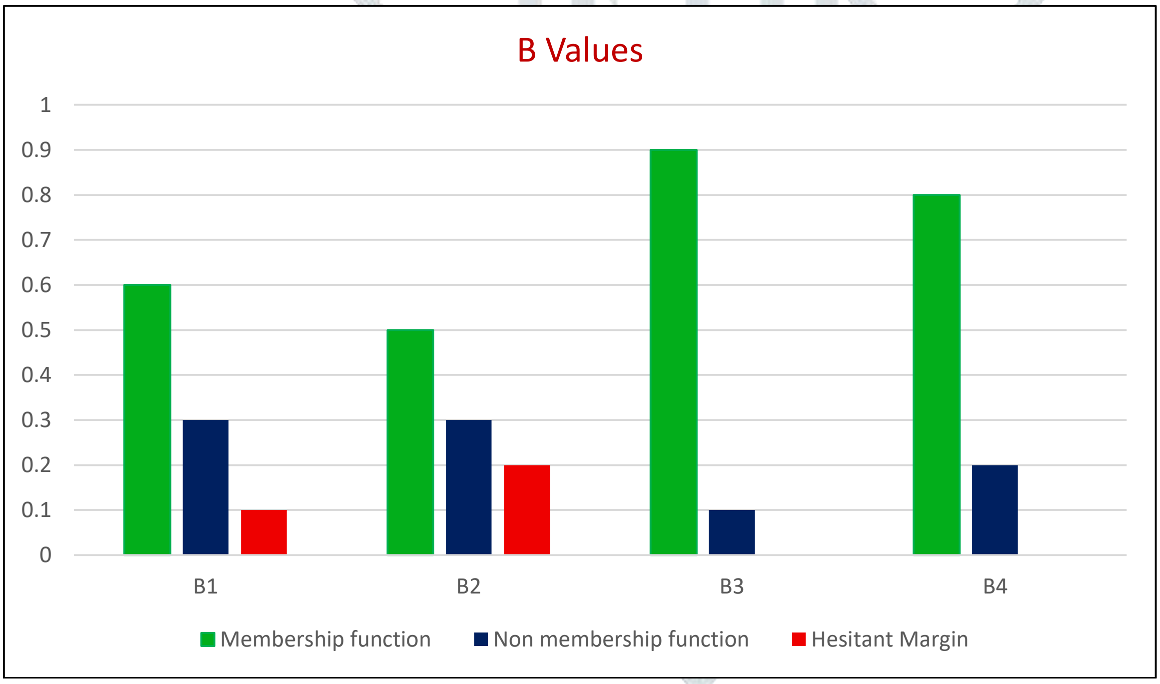

A2 Values
1.2
1
0.8
0.6
0.4
0.2
0
A1
A2
A3
A4
Membership function
Non membership function
Hesitant Margin


A3 Values
1.2
1
0.8
0.6
0.4
0.2
0
A1
A2
A3
A4
Membership function
Non membership function
Hesitant Margin


A_4 Values
1.2
1
0.8
0.6
0.4
0.2
0
A1
A2
A3
A4
Membership function
Non membership function
Hesitant Margin


A_5 Values
1.2
1
0.8
0.6
0.4
0.2
0
A1
A2
A3
A4
Membership function
Non membership function
Hesitant Margin





Here $t_{A1}(x_1)$ represents the membership value of compressive strength of A_1 pattern. $f_{A1}(x_1)$ represents the non- membership value of compressive strength of A_1 pattern. $\pi_{A1}(x_1)$ represents the indeterminacy value of compressive strength of A_1 pattern.

$t_{A1}(x_2)$ represents the membership value of thermal insulation of A_1 pattern. $f_{A1}(x_2)$ represents the non- membership value of thermal insulation of A_1 pattern. $\pi_{A1}(x_2)$ represents the indeterminacy value of thermal insulation of A_1 pattern.

$t_{A1}(x_3)$ represents the membership value of cost efficiency of A_1 pattern. $f_{A1}(x_3)$ represents the non-membership value of cost efficiency of A_l pattern. π_{Al} (x_3) represents the indeterminacy value of cost efficiency of A_l pattern.

and t_{Al} (x_4) represents the membership value of durability of A_l pattern. f_{Al} (x_4) represents the non-membership value of durability of A_l pattern. π_{Al} (x_4) represents the indeterminacy value of durability of A_l pattern.

Here l = 1, 2, 3, 4, 5 and 6

Now, we check which pattern will be closer to unknown pattern B. for that we use normalized Euclidean distance formula,

$$d_{n-E}(A,B) = \sqrt{\frac{1}{2n}\sum_{i=1}^{n}(|t_A(x_i) - t_B(x_i)|^2 + |f_A(x_i) - f_B(x_i)|^2 + |\pi_A(x_i) - \pi_B(x_i)|^2)}$$

Here X= {x_1, x_2, x_3, x_4}

$$d_{n-E}(A,B) = \sqrt{\frac{1}{8}\sum_{i=1}^{4}(|t_A(x_i) - t_B(x_i)|^2 + |f_A(x_i) - f_B(x_i)|^2 + |\pi_A(x_i) - \pi_B(x_i)|^2)}$$

$d_{n-E}(A,B)$ = square root of $\{\frac{1}{8}(|t_A(x_1) - t_B(x_1)|^2 + |f_A(x_1) - f_B(x_1)|^2 + |\pi_A(x_1) - \pi_B(x_1)|^2) + (|t_A(x_2) - t_B(x_2)|^2 + |f_A(x_2) - f_B(x_2)|^2 + |\pi_A(x_2) - \pi_B(x_2)|^2) + (|t_A(x_3) - t_B(x_3)|^2 + |f_A(x_3) - f_B(x_3)|^2 + |\pi_A(x_3) - \pi_A(x_3)|^2) + (|t_A(x_4) - t_B(x_4)|^2 + |f_A(x_4) - t_B(x_4)|^2 + |\pi_A(x_4) - \pi_B(x_4)|^2)\}$

And A is replaced by A_1 and A_6 every time.

1) $d_{n-E}(A_1,B)$ = square root of $\{\frac{1}{8}[(|\ 1 - 0.6|^2 + |0 - 0.3\ |^2 + |0 - 0.1\ |^2) + (|0.8 - 0.5\ |^2 + |0.2 - 0.3\ |^2 + |0 - 0.2\ |^2) + (|0.6 - 0.9\ |^2 + |0.2 - 0.1\ |^2 + |0.2 - 0\ |^2) + (0.5 - 0.8|\ |^2 + |0.2 - 0.2\ |^2 + |0.3 - 0\ |^2)]\}$

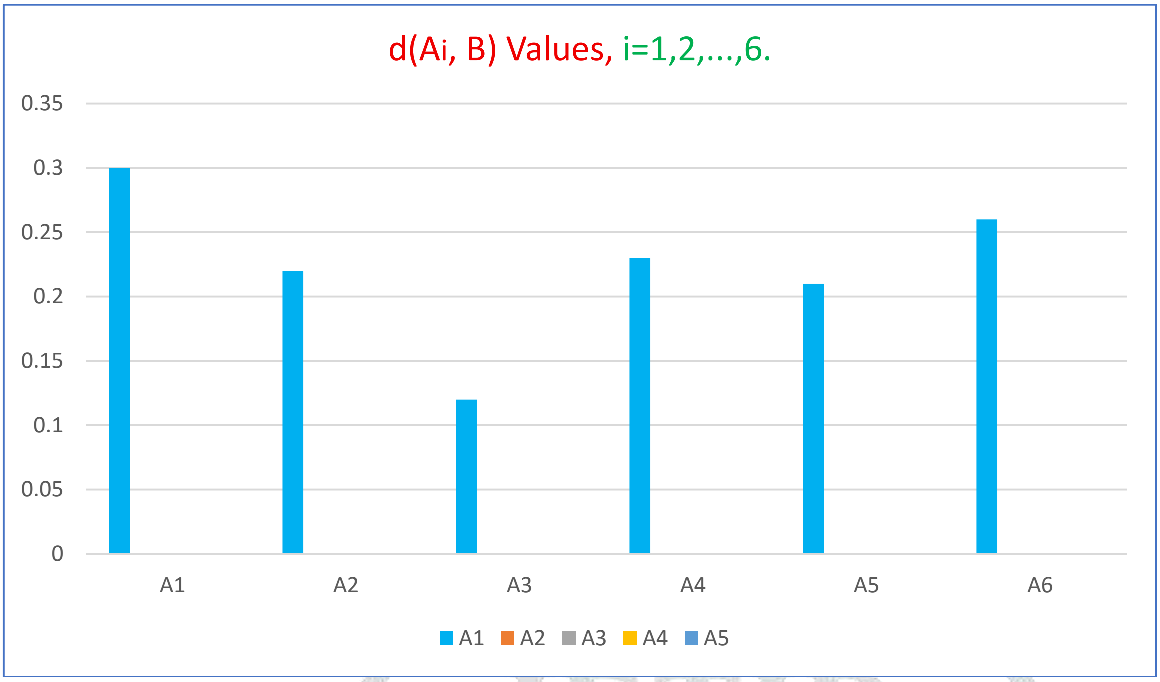
= square root of $\{\frac{1}{8}[(0.16 + 0.09 + 0.01) + (0.09 + 0.01 + 0.04) + (0.09 + 0.01 + 0.04) + (0.09 + 0 + 0.09)$

$= \sqrt{\frac{1}{8}}(0.26 + 0.14 + 0.18) = \sqrt{\frac{1}{8}(0.72)} = \sqrt{0.09} = 0.3$

$$d_{n-E}(A_1,B) = 0.3$$

similarly calculating remaining values, we have

1	$d_{n-E}(A_1,B)$	0.30
2	$d_{n-E}(A_2,B)$	0.22
3	$d_{n-E}(A_3,B)$	0.12
4	$d_{n-E}(A_4,B)$	0.23
5	$d_{n-E}(A_5,B)$	0.21
6	$d_{n-E}(A_6,B)$	0.26



Since the shortest distance between pattern A_3 is much closer to pattern B, hence pattern B belongs to pattern A_3.

5 Model of Multi vague sets in pattern Recognition

Now, we introduce the Multi vague set concept in each attribute, by introducing these values, the distance will be more accurate and precise, more over considering all parameters in minute level.

$A_l = \left\{< t_{A_l}(x_j), f_{A_l}(x_j), \pi_{A_j}(x_j) > \middle|\ x_j \in X \right\}$ becomes

$A_l = \left\{< t_{A_{il}}(x_j)\}_{i=1}^{k}, f_{A_{il}}(x_j)\}_{i=1}^{k}, \pi_{A_{ij}}(x_j)\}_{i=1}^{k} > \middle|\ x_j \in X \right\}_{j=1}^{n}$ and l = 1,2..., m. Here $t_{iA_l}: X \to [0,1]$, $f_{iA_l}: X \to [0,1]$ and $\pi_{iA_l}: X \to [0,1]$ membership, non- membership and hesitant margin respectively. So, we modify the above example with multi dimension k = 5, l= 1, 2,3,4,5 and 6, also j = 1,2,3 and 4.

i.e., $A_l = \{[< t_{1A_l}(x_j), t_{2A_l}(x_j), t_{3A_l}(x_j), t_{4A_l}(x_j), t_{5A_l}(x_j) >,$

$< f_{1A_l}(x_j), f_{2A_l}(x_j), f_{3A_l}(x_j), f_{4A_l}(x_j), f_{5A_l}(x_j) >,$

$< \pi_{1A_l}(x_j), \pi_{2A_l}(x_j), \pi_{3A_l}(x_j), \pi_{4A_l}(x_j), \pi_{5A_l}(x_j) >] |\ x_j \in X \}_{j=1}^{4}$

or $A_l = \{[< t_{1A_l}(x_1), t_{2A_l}(x_1), t_{3A_l}(x_1), t_{4A_l}(x_1), t_{5A_l}(x_1) >,$

$< f_{1A_l}(x_1), f_{2A_l}(x_1), f_{3A_l}(x_1), f_{4A_l}(x_1), f_{5A_l}(x_1) >,$

$< \pi_{1A_l}(x_1), \pi_{2A_l}(x_1), \pi_{3A_l}(x_1), \pi_{4A_l}(x_1), \pi_{5A_l}(x_1) >]$

$[< t_{1A_l}(x_2), t_{2A_l}(x_2), t_{3A_l}(x_2), t_{4A_l}(x_2), t_{5A_l}(x_2) >,$

$< f_{1A_l}(x_2), f_{2A_l}(x_2), f_{3A_l}(x_2), f_{4A_l}(x_2), f_{5A_l}(x_2) >,$

$< \pi_{1A_l}(x_2), \pi_{2A_l}(x_2), \pi_{3A_l}(x_2), \pi_{4A_l}(x_2), \pi_{5A_l}(x_2) >]$

$[< t_{1A_l}(x_3), t_{2A_l}(x_3), t_{3A_l}(x_3), t_{4A_l}(x_3), t_{5A_l}(x_3) >,$

$$< f_{1A_l}(x_3), f_{2A_l}(x_3), f_{3A_l}(x_3), f_{4A_l}(x_3), f_{5A_l}(x_3) >,$$

$$< \pi_{1A_l}(x_3), \pi_{2A_l}(x_3), \pi_{3A_l}(x_3), \pi_{4A_l}(x_3), \pi_{5A_l}(x_3) >]$$

$$[< t_{1A_l}(x_4), t_{2A_l}(x_4), t_{3A_l}(x_4), t_{4A_l}(x_4), t_{5A_l}(x_4) >,$$

$$< f_{1A_l}(x_4), f_{2A_l}(x_4), f_{3A_l}(x_4), f_{4A_l}(x_4), f_{5A_l}(x_4) >,$$

$$< \pi_{1A_l}(x_4), \pi_{2A_l}(x_4), \pi_{3A_l}(x_4), \pi_{4A_l}(x_4), \pi_{5A_l}(x_4) >]$$

$$[< t_{1A_l}(x_5), t_{2A_l}(x_5), t_{3A_l}(x_5), t_{4A_l}(x_5), t_{5A_l}(x_5) >,$$

$$< f_{1A_l}(x_5), f_{2A_l}(x_5), f_{3A_l}(x_5), f_{4A_l}(x_5), f_{5A_l}(x_5) >,$$

$$< \pi_{1A_l}(x_5), \pi_{2A_l}(x_5), \pi_{3A_l}(x_5), \pi_{4A_l}(x_5), \pi_{5A_l}(x_5) >]\}$$

where l = 1,2,3,5 and 6. different patterns given. we subdivide each attribute as follows,

1) Compressive Strength (CS)
CS_1: Load bearing under static weight
CS_2: Load bearing under dynamic impact
CS_3: Resistance after water exposure
CS_4: Resistance to cracking under stress
CS_5: Strength retention after aging

2) Thermal Insulation (TI)
TI_1: Heat conduction rate
TI_2: Performance in hot climates
TI_3: Performance in cold climates
TI_4: Thermal mass effectiveness
TI_5: Stability across temperature fluctuations

3) Cost Efficiency (CE)
CE_1: Initial material cost
CE_2: Installation cost
CE_3: Maintenance over time
CE_4: Lifespan to cost ratio
CE_5: Market availability

4) Durability (D)
D_1: Resistance to water damage
D_2: Resistance to chemical exposure
D_3: Wear and tear over time
D_4: Resistance to biological factors (mold, pests)
D_5: Resistance to extreme temperatures

$t_{1A1}(x_1)$, $f_{1A1}(x_1)$, $\pi_{1A1}(x_1)$ are the membership, non- membership and hesitant margin of load bearing under static weight (CS_1).
$t_{2A1}(x_1)$, $f_{2A1}(x_1)$, $\pi_{2A1}(x_1)$ are the membership, non- membership and hesitant margin of load bearing under dynamic impact (CS_2).
$t_{3A1}(x_1)$, $f_{3A1}(x_1)$, $\pi_{3A1}(x_1)$ are the membership, non- membership and hesitant margin of resistance of water exposure (CS_3).
$t_{4A1}(x_1)$, $f_{4A1}(x_1)$, $\pi_{4A1}(x_1)$ are the membership, non- membership and hesitant margin of resistance to cracking under stress (CS_4).
$t_{5A1}(x_1)$, $f_{5A1}(x_1)$, $\pi_{5A1}(x_1)$ are the membership, non- membership and hesitant margin of strength retention after aging (CS_5).

$t_{1A1}(x_2)$, $f_{1A1}(x_2)$, $\pi_{1A1}(x_2)$ are the membership, non- membership and hesitant margin of heat conduction rate (TI_1).
$t_{2A1}(x_2)$, $f_{2A1}(x_2)$, $\pi_{2A1}(x_2)$ are the membership, non- membership and hesitant margin of performance in hot climates (TI_2).
$t_{3A1}(x_2)$, $f_{3A1}(x_2)$, $\pi_{3A1}(x_2)$ are the membership, non- membership and hesitant margin of performance in cold climates rate (TI_3).
$t_{4A1}(x_2)$, $f_{4A1}(x_2)$, $\pi_{4A1}(x_2)$ are the membership, non- membership and hesitant margin of thermal mass effectiveness (TI_4).
$t_{5A1}(x_2)$, $f_{5A1}(x_2)$, $\pi_{5A1}(x_2)$ are the membership, non- membership and hesitant margin of stability across temperature fluctuations (TI_5).

$t_{1A1}(x_3)$, $f_{1A1}(x_3)$, $\pi_{1A1}(x_3)$ are the membership, non- membership and hesitant margin of Initial cost material (CE_1).
$t_{2A1}(x_3)$, $f_{2A1}(x_3)$, $\pi_{2A1}(x_3)$ are the membership, non- membership and hesitant margin of installation cost (CE_2).
$t_{3A1}(x_3)$, $f_{3A1}(x_3)$, $\pi_{3A1}(x_3)$ are the membership, non- membership and hesitant margin of maintenance over time (CE_3).
$t_{4A1}(x_3)$, $f_{4A1}(x_3)$, $\pi_{4A1}(x_3)$ are the membership, non- membership and hesitant margin of life span to cost ratio (CE_4).
$t_{5A1}(x_3)$, $f_{5A1}(x_3)$, $\pi_{5A1}(x_3)$ are the membership, non- membership and hesitant margin of market availability (CE_5).

$t_{1A1}(x_4)$, $f_{1A1}(x_4)$, $\pi_{1A1}(x_4)$ are the membership, non- membership and hesitant margin of resistance to water damage (D_1).
$t_{2A1}(x_4)$, $f_{2A1}(x_4)$, $\pi_{2A1}(x_4)$ are the membership, non-membership and hesitant margin of resistance to chemical exposition. (D_2).
$t_{3A1}(x_4)$, $f_{3A1}(x_4)$, $\pi_{3A1}(x_4)$ are the membership, non- membership and hesitant margin of wear and tear over time. (D_3).
$t_{4A1}(x_4)$, $f_{4A1}(x_4)$, $\pi_{4A1}(x_4)$ are the membership, non- membership and hesitant margin of resistance to biological factors (mold, pests) (D_4).
$t_{5A1}(x_4)$, $f_{5A1}(x_4)$, $\pi_{5A1}(x_4)$ are the membership, non- membership and hesitant margin of resistance to extreme temperatures. (D_5).

Now, we define six known patterns A_l and unknown pattern B.

A_1 = {⟨(0.8, 0.6, 0.4, 0.5, 0.6), (0.1, 0.2, 0.3, 0.2, 0.2), (0.1, 0.2, 0.3, 0.3, 0.2)⟩,
⟨(0.9, 0.7, 0.5, 0.7, 0.5), (0, 0.1, 0.2, 0.1, 0.3), (0.1, 0.2, 0.3, 0.2, 0.2)⟩,
⟨(0.7, 0.6, 0.8, 0.5, 0.7), (0.1, 0.2, 0.1, 0.2, 0.2), (0.2, 0.2, 0.1, 0.3, 0.1)⟩
⟨(0.6, 0.7, 0.7, 0.8, 0.5), (0.3, 0.2, 0.2, 0.1, 0.2), (0.1, 0.1, 0.1, 0.1, 0.3)⟩}

A_2 = {⟨(0.6, 0.5, 0.7, 0.5, 0.7), (0.3, 0.4, 0.1, 0.2, 0.2), (0.1, 0.1, 0.2, 0.3, 0.1)⟩,
⟨(0.6, 0.5, 0.6, 0.4, 0.5), (0.2, 0.3, 0.2, 0.3, 0.3), (0.2, 0.2, 0.2, 0.3, 0.2)⟩,
⟨(0.7, 0.8, 0.5, 0.6, 0.6), (0.1, 0.1, 0.2, 0.2, 0.3), (0.2, 0.1, 0.3, 0.2, 0.1)⟩
⟨(1, 0.7, 1, 0.7, 0.7), (0, 0.2, 0, 0.1, 0), (0, 0.1, 0, 0.2, 0.3)⟩}

A_3 = {⟨(0.5, 0.6, 0.7, 1, 0.9), (0.3, 0.2, 0.1, 0, 0), (0.2, 0.2, 0.2, 0, 0.1)⟩,
⟨(0.8, 0.7, 0.4, 0.3, 0.4), (0.2, 0.1, 0.4, 0.3, 0.5), (0, 0.2, 0.2, 0.4, 0.1)⟩,
⟨(0.3, 0.3, 0.5, 0.4, 0.3), (0.3, 0.3, 0.4, 0.2, 0.5), (0.4, 0.4, 0.1, 0.4, 0.2)⟩
⟨(0.4, 0.5, 0.5, 0.6, 0.3), (0.3, 0.2, 0.3, 0.1, 0.4), (0.3, 0.3, 0.2, 0.3, 0.3)⟩}

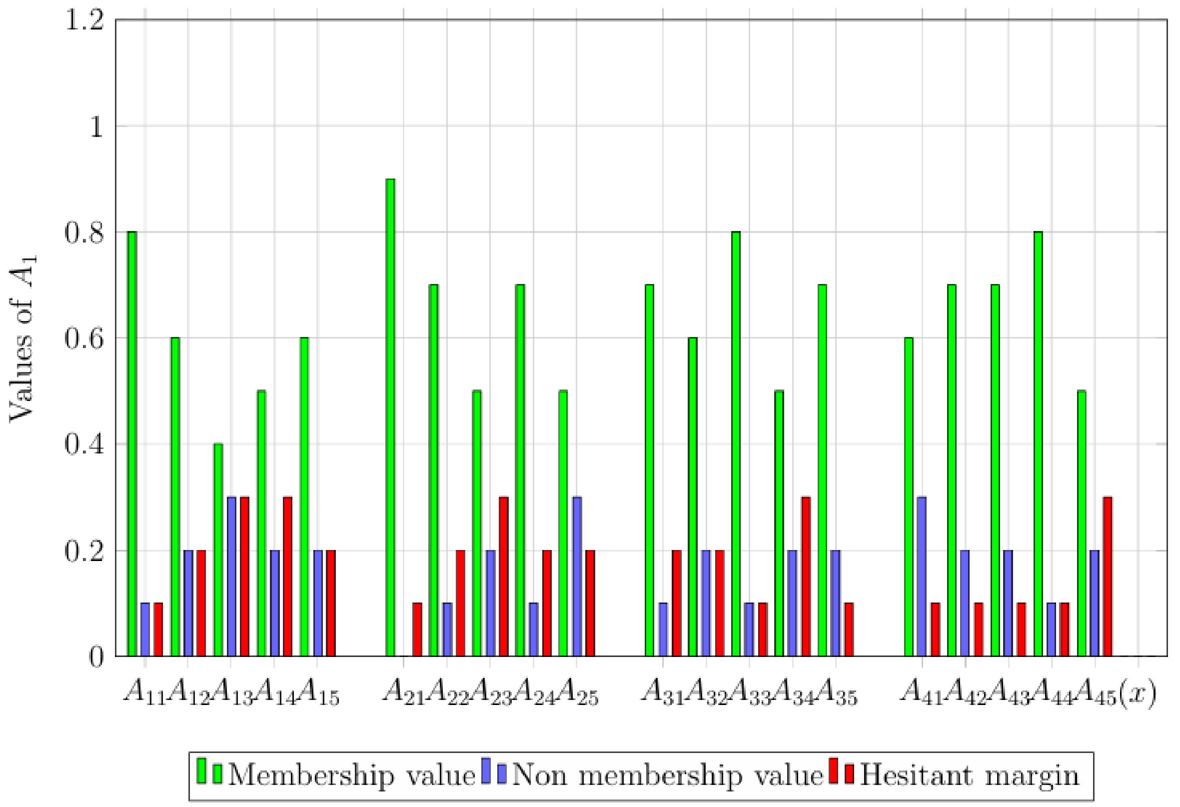
A_4 = {⟨(0.8, 0.9, 1, 0.7, 0.8), (0.2, 0.1, 0, 0.1, 0.1), (0, 0, 0, 0.2, 0.1)⟩,

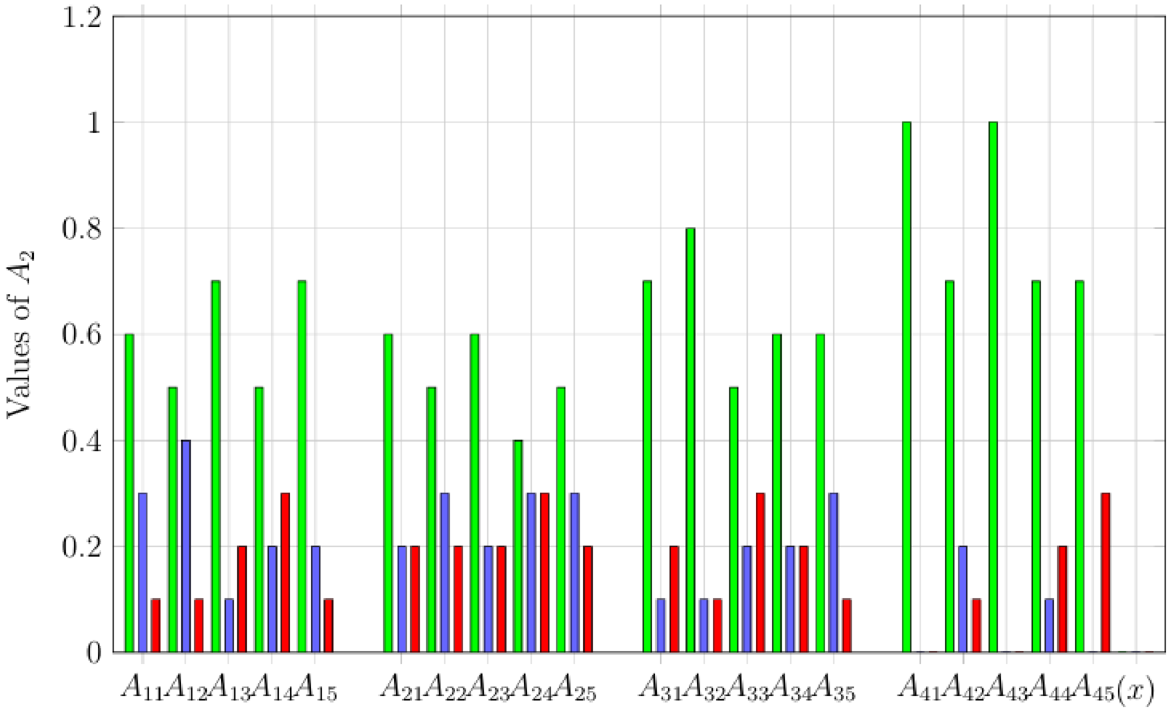
$\langle(0.7, 0.8, 0.9, 0.7, 0.6), (0.1, 0.1, 0, 0.1, 0.2), (0.2, 0.1, 0.1, 0.2, 0.2)\rangle,$
$\langle(0.9, 1, 1, 0.9, 0.7), (0.1, 0, 0, 0, 0.2), (0, 0, 0, 0.1, 0.1)\rangle$
$\langle(0.8, 0.7, 0.8, 0.8, 1), (0.1, 0.2, 0, 0.1, 0), (0.1, 0.1, 0.2, 0.1, 0)\rangle\}$

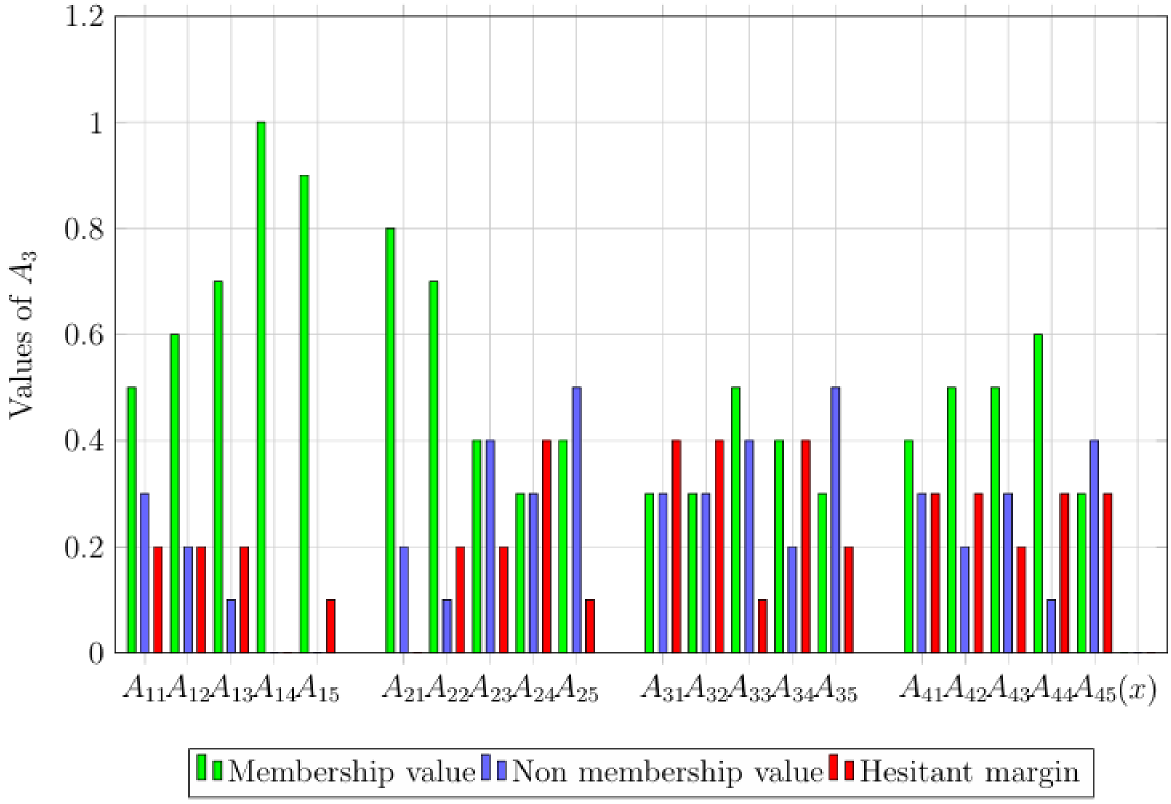
$A_5 = \{\langle(0.5, 0.6, 0.7, 0.6, 0.5), (0.3, 0.1, 0.1, 0.2, 0.4), (0.2, 0.3, 0.2, 0.2, 0.1)\rangle,$
$\langle(0.4, 0.5, 0, 0.7, 0.6), (0.5, 0.3, 0.7, 0.2, 0.1), (0.1, 0.2, 0.3, 0.1, 0.3)\rangle,$
$\langle(0.5, 0.6, 0.5, 0, 0.5), (0.2, 0.3, 0.4, 0.7, 0.2), (0.3, 0.1, 0.1, 0.3, 0.3)\rangle$
$\langle(0.4, 0.6, 0, 0.6, 0.7), (0.3, 0.3, 0.5, 0.1, 0.2), (0.3, 0.1, 0.5, 0.3, 0.1)\rangle\}$

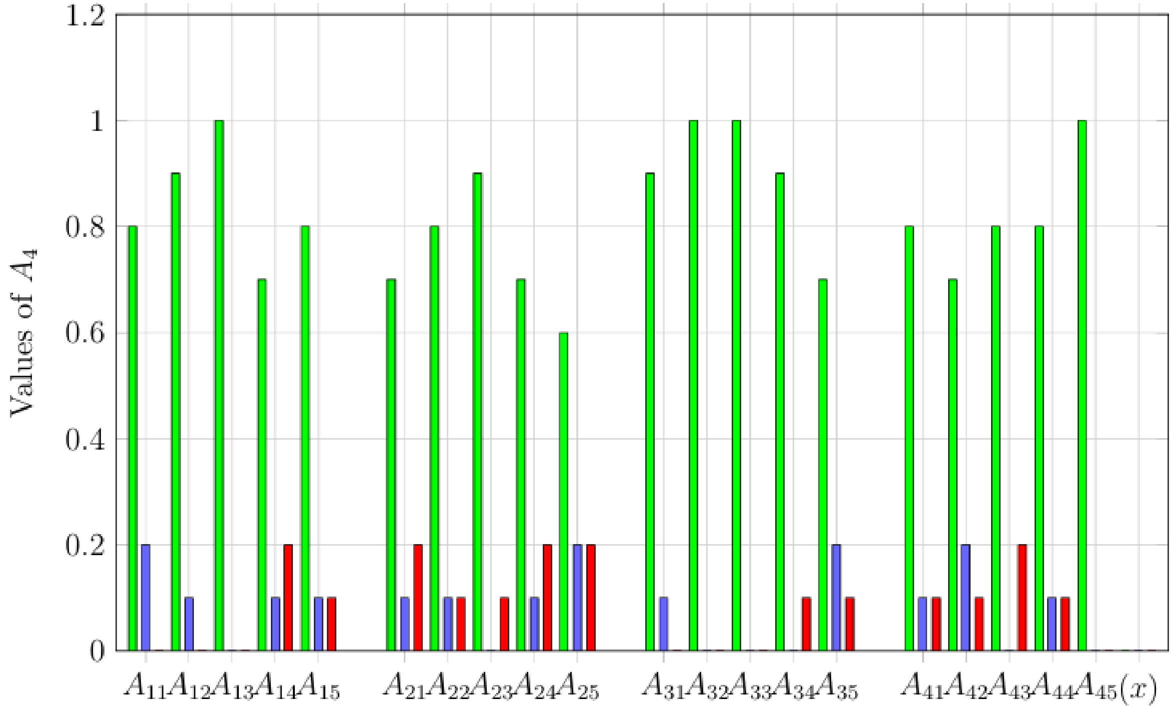
$A_6 = \{\langle(0.3, 0.4, 0.5, 0.6, 0.3), (0.6, 0.5, 0.3, 0.3, 0.5), (0.1, 0.1, 0.2, 0.1, 0.2)\rangle,$
$\langle(0.4, 0.3, 0.7, 0, 0.5), (0.3, 0.4, 0.1, 0.5, 0.3), (0.3, 0.3, 0.2, 0.5, 0.2)\rangle,$
$\langle(0, 0.3, 0.4, 0.5, 0), (0.7, 0.3, 0.5, 0.3, 0.5), (0.3, 0.4, 0.1, 0.2, 0.5)\rangle$
$\langle(0.4, 0.2, 0, 0.6, 0.7), (0.3, 0.4, 0.4, 0.3, 0.2), (0.3, 0.4, 0.6, 0.1, 0.1)\rangle\}$

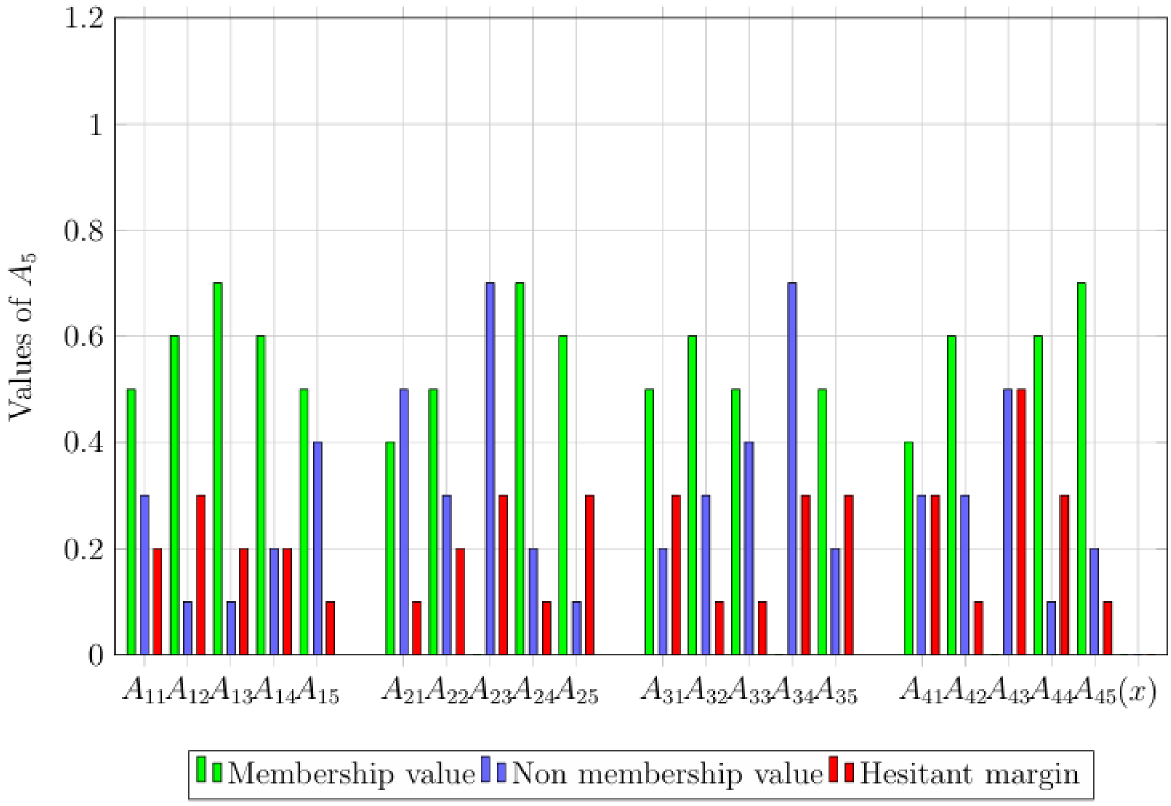
$B = \{\langle(0.5, 0.7, 1, 0.6, 0.7), (0.3, 0.1, 0, 0.2, 0.2), (0.2, 0.2, 0, 0.2, 0.1)\rangle,$
$\langle(0.3, 0.4, 0.5, 0.3, 0.4), (0.6, 0.5, 0.4, 0.4, 0.5), (0.1, 0.1, 0.1, 0.3, 0.1)\rangle,$
$\langle(1, 0.8, 0.9, 0.7, 1), (0, 0.1, 0.1, 0.1, 0), (0, 0.1, 0, 0.2, 0)\rangle$
$\langle(0.2, 0.3, 0.5, 0.3, 0.4), (0.6, 0.7, 0.4, 0.6, 0.3), (0.2, 0, 0.1, 0.1, 0.3)\rangle\}$

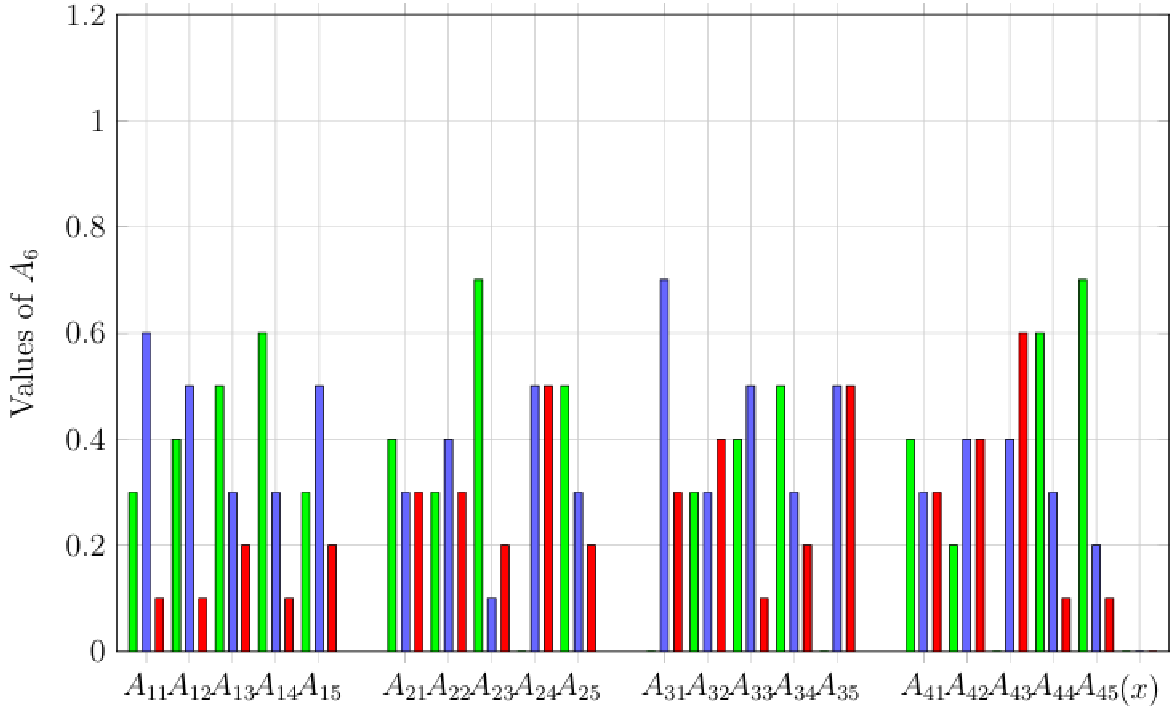


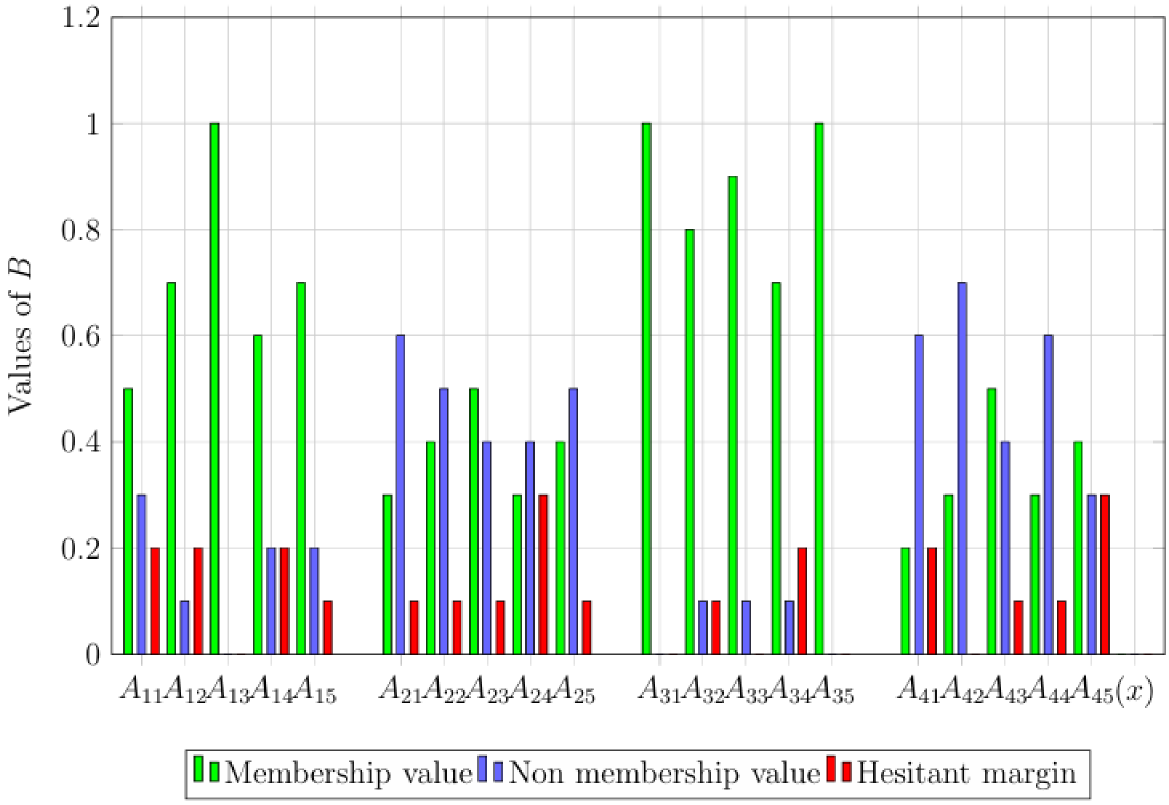

Values of A_2
1.2
1
0.8
0.6
0.4
0.2
0
$A_{11} A_{12} A_{13} A_{14} A_{15}$ $A_{21} A_{22} A_{23} A_{24} A_{25}$ $A_{31} A_{32} A_{33} A_{34} A_{35}$ $A_{41} A_{42} A_{43} A_{44} A_{45} (x)$


Values of A_3
1.2
1
0.8
0.6
0.4
0.2
0
$A_{11} A_{12} A_{13} A_{14} A_{15}$ $A_{21} A_{22} A_{23} A_{24} A_{25}$ $A_{31} A_{32} A_{33} A_{34} A_{35}$ $A_{41} A_{42} A_{43} A_{44} A_{45} (x)$
Membership value
Non membership value
Hesitant margin


Values of A_4
1.2
1
0.8
0.6
0.4
0.2
0
$A_{11} A_{12} A_{13} A_{14} A_{15}$ $A_{21} A_{22} A_{23} A_{24} A_{25}$ $A_{31} A_{32} A_{33} A_{34} A_{35}$ $A_{41} A_{42} A_{43} A_{44} A_{45} (x)$


Values of A_5
1.2
1
0.8
0.6
0.4
0.2
0
$A_{11} A_{12} A_{13} A_{14} A_{15}$ $A_{21} A_{22} A_{23} A_{24} A_{25}$ $A_{31} A_{32} A_{33} A_{34} A_{35}$ $A_{41} A_{42} A_{43} A_{44} A_{45} (x)$
Membership value
Non membership value
Hesitant margin





The Normalized Euclidean distance.

$$d_{n-E}(A,B) = \sqrt{\frac{1}{nk}\sum_{j=1}^{n}\sum_{i=1}^{k}\left(\left|t_{iA}(x_j) - t_{iB}(x_j)\right|^2 + \left|f_{iA}(x_j) - f_{iB}(x_j)\right|^2 + \left|\pi_{iA}(x_j) - \pi_{iB}(x_j)\right|^2\right)}$$

1) $d_{n-E}(A1,B)$ = square root of $\{\frac{1}{20}[(0.3^2 + 0.1^2 + 0.6^2 + 0.1^2 + 0.1^2) +$

$$(0.2^2 + 0.1^2 + 0.1^2 + 0^2 + 0^2) + (0.1^2 + 0^2 + 0.3^2 + 0.1^2 + 0.1^2) + (0.6^2 + 0.3^2 + 0^2 + 0.4^2 + 0.1^2) + (0.6^2 + 0.4^2 + 0.2^2 + 0.3^2 + 0.2^2) + (0^2 + 0.1^2 + 0.2^2 + 0.1^2 + 0.1^2) + (0.3^2 + 0.2^2 + 0.1^2 + 0.2^2 + 0.3^2) + (0.1^2 + 0.1^2 + 0^2 + 0.1^2 + 0.2^2) + (0.2^2 + 0.1^2 + 0^2 + 0.1^2 + 0.2^2) + (0.4^2 + 0.4^2 + 0.2^2 + 0.5^2 + 0.1^2) + (0.3^2 + 0.5^2 + 0.2^2 + 0.5^2 + 0.1^2) + (0.1^2 + 0.1^2 + 0^2 + 0^2 + 0^2)]\}$$

=square root of $\{\frac{1}{20}[(0.09 + 0.01 + 0.36 + 0.01 + 0.01) + (0.04 + 0.01 + 0.09) + (0.01 + 0.09 + 0.01 + 0.01) +$(0.36+0.09+0.16+0.01) + (0.36+0.16+0.04+0.09+0.04) +(0.01+0.04+0.01+0.01) + (0.09+0.04+0.01+0.04+0.09) + (0.01+0.01+0.01+0.04) + (0.04+0.01+0.01+0.01+0.01) + (0.16+0.16+0.04+0.25+0.01) + (0.09+0.25+0.04+0.25+0.01) + (0.01+0.01)]}

=square root of $\{\frac{1}{20}[0.48 + 0.14 + 0.12 + 0.62 + 0.69 + 0.07 + 0.27 + 0.07 + 0.08 +$ 0.62+0.64+0.02]}

$$= \sqrt{\frac{1}{20}(3.82)} = \sqrt{0.191} = 0.4370$$

$\therefore\ d_{n-E}(A_1, B) = 0.4370$

2) $d_{n-E}(A2, B) =$ square root of $\{\frac{1}{20}[(0.1^2 + 0.2^2 + 0.3^2 + 0.1^2 + 0^2) + (0^2 + 0.3^2 + 0.1^2 + 0^2 + 0^2) + (0.1^2 + 0.1^2 + 0.2^2 + 0.1^2 + 0^2) + (0.3^2 + 0.1^2 + 0.1^2 + 0.1^2 + 0.1^2) + (0.4^2 + 0.2^2 + 0.2^2 + 0.1^2 + 0.2^2) + (0.1^2 + 0.1^2 + 0.1^2 + 0^2 + 0.1^2) + (0.3^2 + 0^2 + 0.4^2 + 0.1^2 + 0.4^2) + (0.1^2 + 0^2 + 0.1^2 + 0.1^2 + 0.3^2) + (0.2^2 + 0^2 + 0.3^2 + 0^2 + 0.1^2) + (0.8^2 + 0.4^2 + 0.5^2 + 0.4^2 + 0.3^2) + (0.6^2 + 0.5^2 + 0.4^2 + 0.5^2 + 0.3^2) + (0.2^2 + 0.1^2 + 0.1^2 + 0.1^2 + 0^2)]\}$

=square root of $\{\frac{1}{20}[(0.01 + 0.04 + 0.09 + 0.01) + (0.09 + 0.01) +$ (0.01+0.01+0.04+0.01)+(0.09+0.01+0.01+0.01+0.01)+(0.16+0.04+0.04+0.01+0.04) +(0.01+0.01+0.01+0.01)+(0.09+0.16+0.01+0.16)+(0.01+0.01+0.01+0.09)+ (0.04+0.09+0.01)+(0.64+0.16+0.25+0.16+0.09)+(0.36+0.25+0.16+0.25+0.09) +(0.04+0.01+0.01+0.01)]}

=square root of $\{\frac{1}{20}[0.15 + 0.10 + 0.07 + 0.13 + 0.29 + 0.04 + 0.42 + 0.12 + 0.14 + 1.3 + 1.11 + 0.07]\}$

$$= \sqrt{\frac{1}{20}(3.94)} = \sqrt{0.197} = 0.4438$$

$\therefore\ d_{n-E}(A_2, B) = 0.4438$

3) $d_{n-E}(A3, B) =$ square root of $\{\frac{1}{20}[(0^2 + 0.1^2 + 0.3^2 + 0.4^2 + 0.2^2) + (0^2 + 0.1^2 + 0.1^2 + 0.2^2 + 0.2^2) + (0^2 + 0^2 + 0.2^2 + 0.2^2 + 0^2) + (0.5^2 + 0.3^2 + 0.1^2 + 0^2 + 0^2) + (0.4^2 + 0.4^2 + 0^2 + 0.1^2 + 0^2) + (0.1^2 + 0.1^2 + 0.1^2 + 0.1^2 + 0^2) + (0.5^2 + 0.5^2 + 0.4^2 + 0.3^2 + 0.7^2) + (0.3^2 + 0.2^2 + 0.3^2 + 0.1^2 + 0.5^2) + (0.2^2 + 0.3^2 + 0.1^2 + 0.2^2 + 0.2^2) + (0.2^2 + 0.2^2 + 0^2 + 0.3^2 + 0.1^2) + (0.3^2 + 0.5^2 + 0.1^2 + 0.5^2 + 0.1^2) + (0.1^2 + 0.3^2 + 0.1^2 + 0.2^2 + 0^2)]\}$

=square root of $\{\frac{1}{20}[(0.01 + 0.09 + 0.16 + 0.04) + (0.01 + 0.01 + 0.04 + 0.04) +$ (0.04+0.04)+(0.25+0.09+0.01)+(0.16+0.16+0.01)+(0.01+0.01+0.01+0.01)+ (0.25+0.25+0.16+0.09+0.49)+(0.09+0.04+0.09+0.01+0.25)+

(0.04+0.09+0.01+0.04+0.04)+(0.04+0.04+0.09+0.01)+
(0.09+0.25+0.01+0.25+0.01) + (0.01+0.09+0.01+0.04)]}

=square root of $\{\frac{1}{20}[0.30 + 0.10 + 0.08 + 0.35 + 0.33 + 0.04 + 1.24 + 0.48 +$ $0.22 +$
0.18+0.61+0.15]}

$$= \sqrt{\frac{1}{20}(4.08)} = \sqrt{0.204} = 0.4516$$

$\therefore\ d_{n-E}(A_3, B) = 0.4516$

4) $\mathrm{d_{n-E}}(\mathrm{A4, B})$ = square root of $\{\frac{1}{20}[(0.3^2 + 0.2^2 + 0^2 + 0.1^2 + 0.1^2) +$
$(0.1^2 + 0^2 + 0^2 + 0.1^2 + 0.1^2) + (0.2^2 + 0.2^2 + 0^2 + 0^2 + 0^2) +$
$(0.4^2 + 0.4^2 + 0.4^2 + 0.4^2 + 0.2^2) + (0.5^2 + 0.4^2 + 0.4^2 + 0.3^2 + 0.3^2) +$
$(0.1^2 + 0^2 + 0^2 + 0.1^2 + 0.1^2) + (0.1^2 + 0.2^2 + 0.1^2 + 0.2^2 + 0.3^2) +$
$(0.1^2 + 0.1^2 + 0.1^2 + 0.1^2 + 0.2^2) + (0^2 + 0.1^2 + 0^2 + 0.1^2 + 0.1^2) +$
$(0.6^2 + 0.4^2 + 0.3^2 + 0.5^2 + 0.6^2) + (0.5^2 + 0.5^2 + 0.4^2 + 0.5^2 + 0.3^2)$
$+ (0.1^2 + 0.1^2 + 0.1^2 + 0^2 + 0.3^2)]\}$

=square root of $\{\frac{1}{20}[(0.09 + 0.04 + 0.01 + 0.01) + (0.01 + 0.01 + 0.01) +$
(0.04+0.04)+(0.16+0.16+0.16+0.16+0.04)+ (0.25+0.16+0.16+0.09+0.09)+
(0.01+0.01+0.01)+(0.01+0.04+0.01+0.04+0.09)+(0.01+0.01+0.01+0.01+0.04)+
(0+0.01+0+0.01+0.01)+(0.36+0.16+0.09+0.25+0.36)+
(0.25+0.25+0.16+0.25+0.09)+(0.01+0.01+0.01+0.09)]}
=square root of $\{\frac{1}{20}[0.15 + 0.03 + 0.08 + 0.68 + 0.75 + 0.03 + 0.19 + 0.08 +$
$0.03 + 1.22 + 1 + 0.12]\}$

$$= \sqrt{\frac{1}{20}(4.36)} = \sqrt{0.218} = 0.4669$$

$\therefore\ d_{n-E}(A_4, B) = 0.4669$

5) $\mathrm{d_{n-E}}(\mathrm{A5, B})$ = square root of $\{\frac{1}{20}[(0^2 + 0.1^2 + 0.3^2 + 0^2 + 0.2^2) +$
$(0^2 + 0^2 + 0.1^2 + 0^2 + 0.2^2) + (0^2 + 0.1^2 + 0.2^2 + 0^2 + 0^2) +$
$(0.1^2 + 0.1^2 + 0.5^2 + 0.4^2 + 0.2^2) + (0.1^2 + 0.2^2 + 0.3^2 + 0.2^2 + 0.4^2) +$
$(0^2 + 0.1^2 + 0.2^2 + 0.2^2 + 0.2^2) + (0.5^2 + 0.2^2 + 0.4^2 + 0.7^2 + 0.5^2) +$
$(0.2^2 + 0.2^2 + 0.3^2 + 0.6^2 + 0.2^2) + (0.3^2 + 0^2 + 0.1^2 + 0.1^2 + 0.3^2) +$
$(0.2^2 + 0.3^2 + 0.5^2 + 0.3^2 + 0.3^2) + (0.3^2 + 0.4^2 + 0.1^2 + 0.5^2 + 0.1^2)$
$+ (0.1^2 + 0.1^2 + 0.4^2 + 0.2^2 + 0.2^2)]\}$

=square root of $\{\frac{1}{20}[(0.01 + 0.09 + 0.04) + (0.01 + 0.04) +$
(0.01+0.01+0.25+0.16+0.04)+(0.36+0.16+0.04+0.09+0.04)+(0.01+0.04+0.09+0.04+0.16)+
(0.01+0.04+0.04+0.04)+(0.25+0.04+0.16+0.49+0.25)+(0.04+0.04+0.09+0.36+0.04)+
(0.09+0.01+0.01+0.09)+(0.04+0.09+0.25+0.09+0.09)+(0.09+0.16+0.01+0.25+0.01)+
(0.01+0.01+0.16+0.04+0.04)]}

=square root of $\{\frac{1}{20}[0.14 + 0.05 + 0.05 + 0.47 + 0.34 + 0.13 + 1.19 + 0.57 +$
0.20 +0.56+0.52+0.26]}

$$= \sqrt{\frac{1}{20}(4.48)} = \sqrt{0.224} = 0.4733$$

$\therefore\ d_{n-E}(A_5, B) = 0.4733$

6) $d_{n-E}(A6, B) = \text{square root of} \{\frac{1}{20}[(0.2^2 + 0.3^2 + 0.5^2 + 0^2 + 0.4^2) + (0.3^2 + 0.4^2 + 0.3^2 + 0.1^2 + 0.3^2) + (0.1^2 + 0.1^2 + 0.2^2 + 0.1^2 + 0.1^2) + (0.1^2 + 0.1^2 + 0.2^2 + 0.3^2 + 0.1^2) + (0.3^2 + 0.1^2 + 0.3^2 + 0.1^2 + 0.2^2) + (0.2^2 + 0.2^2 + 0.1^2 + 0.2^2 + 0.1^2) + (1^2 + 0.5^2 + 0.5^2 + 0.2^2 + 0.1^2) + (0.7^2 + 0.2^2 + 0.4^2 + 0.2^2 + 0.5^2) + (0.3^2 + 0.3^2 + 0.1^2 + 0^2 + 0.5^2) + (0.2^2 + 0.1^2 + 0.5^2 + 0.3^2 + 0.3^2) + (0.3^2 + 0.3^2 + 0^2 + 0.3^2 + 0.1^2) + (0.1^2 + 0.4^2 + 0.5^2 + 0^2 + 0.2^2)]\}$

=square root of $\{\frac{1}{20}[(0.04 + 0.09 + 0.25 + 0.16) + (0.09 + 0.16 + 0.09 + 0.01 + 0.09) +$
(0.01+0.01+0.04+0.01+0.01)+(0.01+0.01+0.04+0.09+0.01)+(0.09+0.01+0.09+0.01+0.04)+
(0.04+0.04+0.01+0.04+0.01)+(1+0.25+0.25+0.04+1)+(0.49+0.04+0.16+0.25)+
(0.09+0.09+0.01+0.25)+(0.04+0.01+0.25+0.09+0.09)+
(0.09+0.09+0.09+0.01)+(0.01+0.16+0.25+0.04)]}

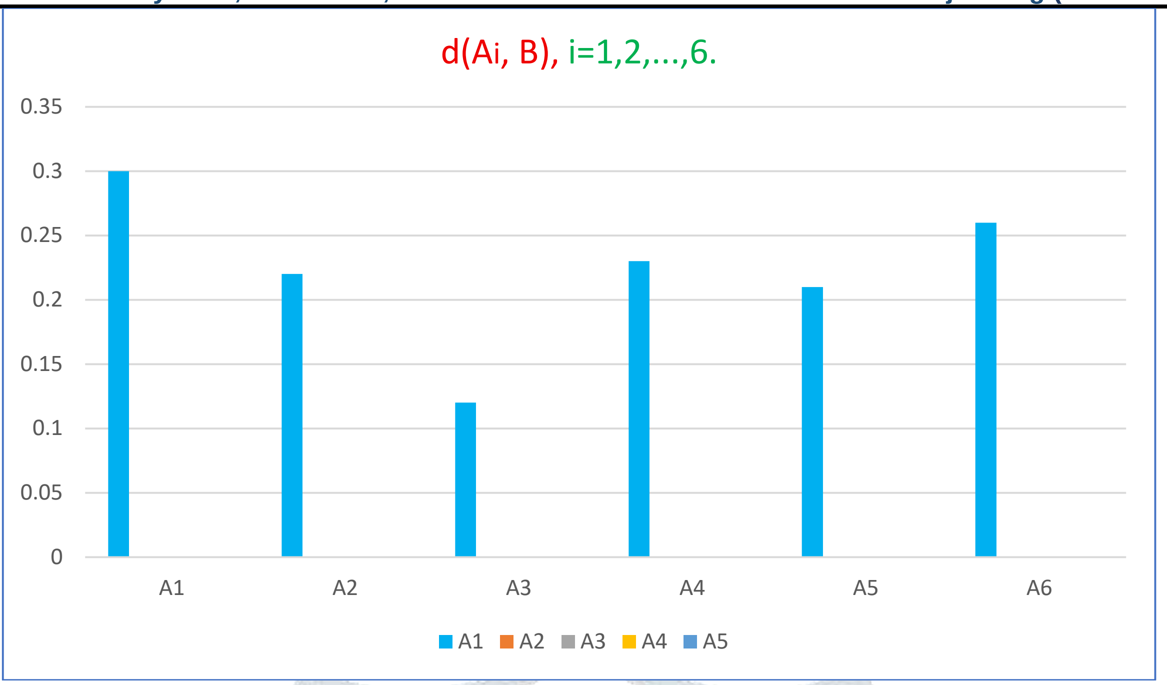
=square root of $\{\frac{1}{20}[0.54 + 0.44 + 0.08 + 0.16 + 0.24 + 0.14 + 2.54 + 0.98 + 0.44 + 0.48 + 0.28 + 0.46]\}$

$$= \sqrt{\frac{1}{20}(6.78\,)} = \sqrt{0.339} = 0.5822$$

$$\therefore\ d_{n-E}(A_6, B) = 0.5822$$

similarly calculating remaining values, we have

1	$d_{n-E}(A_1, B)$	0.4370
2	$d_{n-E}(A_2, B)$	0.4438
3	$d_{n-E}(A_3, B)$	0.4516
4	$d_{n-E}(A_4, B)$	0.4669
5	$d_{n-E}(A_5, B)$	0.4733
6	$d_{n-E}(A_6, B)$	0.5822



From the table and graph, the smallest distance is 0.4370 for A_1, meaning B most likely belongs to pattern A_1, the largest distance is 0.5822 for A_6, indicating B is least similar to A_6.

6 Conclusion

This is a typical pattern recognition step where classification is based on the minimum distance. This confirms the effectiveness of the proposed distance based Multi vague pattern recognition model, which can be applied in real life domains like material classification, medical diagnosis, and decision-making systems where uncertainty is inherent. This Multi vague sets applied to pattern recognition is more effective method than that of fuzzy set model and vague set model.
Overall, the model demonstrates improved classification accuracy by capturing Multi-dimensional uncertainty, offering a strong alternative to traditional pattern recognition approaches.

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