JETIR.ORG ISSN: 2349-5162 | ESTD Year : 2014 | Monthly Issue
JOURNAL OF EMERGING TECHNOLOGIES AND INNOVATIVE RESEARCH (JETIR)
An International Scholarly Open Access, Peer-reviewed, Refereed Journal

Characterization of T – Normal graphs

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Abstract

Graph- theoretic properties pertaining to topological separation are important in order to understand how graph elements are structurally related. In this paper, we introduce the concept of T-normal graphs and investigate some of their basic properties, specifically, we derive conditions under which the complete graph k_n is T-normal and consider how the T-normal aspect behaves under the union and Cartesian product of two graphs. Finally, we provide a T-normal graphs using complements of graphs providing amore in depth understanding topological behavior.

Keywords: T-regular graph, complete graph; union; Cartesian product .complement of a graph, clique.**1**

Introduction

All graphs are finite and simple in paper .Let the set of G be denoted by V(G),the set of edges of G denoted by E(G).the maximum degree of G denoted by Δ(G), and the minimum degree by ծ(G).

Given a vertex v in G ,t he degree (Koh et al. 2023) of V in G denoted by $d_G(V)$, is defined as the number of edges incident with V. A pendant vertex (Kottarathil et.al.,2024) is a vertex one in G.A vertex of degree 0 is isolated (Diestel,2025) A graph is an empty graph (KohetAl.,2023) if E(G) is empty ..A complete (Balakrishnan and Ranganathan. 2012).If every pair of depict vertices of G are adjacent in G.A complete graph on n vertices is denoted by K_n. Given two graphs G and H, we say H is an induced sub graph (chartrand et. Al.., 2010) of G if V(H) ⊆ V(G). such that two vertices of H are adjacent a if and only if they are adjacent in G. In this case if V(H)= S, we write H=G[S]or h=(S). An edge cover (Grinberg,2023) of G shall mean a set F of edges of G such that each vertex of G contained in at least one edge e ∈ F. The union (West et. Al,.2001)of two graphs G_1 and G_2 which is represented as G_1 u G_2 is the graph that has edge set $E(G_1)$ U $E(G_2)$ and vertex set $V(G_1)$ U (G_2).if v is a vertex of a graph G, then G – V is the graph obtained from G by Deleting all the vertex V .In general, if S is a set of vertices in G, then G - S is the graph derived from G by deleting all the vertices in S and all the edges incident with any one of the vertices in S(Balakrishnan and Ranganathan,2012).

Given any two graphs G=(V(G), E(G)) and H=(V(G),E(G)) ,their Cartesian product (Walton et al ., 2024)

G □ H is the group with vertex set V(G) x V(H) where the vertex $(u_1,v_1)$is adjacent to the vertex(u_2,v_2) whenever u_1u_2 ∈ E(G) and v_1=,v_2 ,or $u_1=u_2$.and v_1,v_2 e. (H).The an independent edge set (Abughazaleh and Abughneim 2023)(stable set) of a graph G is a subset of the edges of G such that no two edge in the subset share a vertex of G.A property P Is said to be a hereditary property (Dharwadker and pirzada,2011) of a graph G,if a graph G has the property p ,Then every sub graph of G also has property p.Recent research into hereditary graph properties and independent edge sets has illuminated some structural behaviors of graphs (Levitt and Manddrescu,2025; Singh and Sivaraman2014): levit and Mandrescu,2024). These research papers lay in the solution in equation in

the integers groundwork for utilizing properties like T- normality to understand what closure under graph operations and independence condition work.

Lemma 1.1 Let Q be a clique in $G_1 \square G_2$then either the first co-ordinates of all vertices of are same and the sub graph of G induced by the second co-ordinates of all vertices of Q is a clique in G_2, or the second co-ordinates of all vertices of q are same and the sub graph of G induced by the first co-ordinates of all vertices of Q is a clique in G_1.

The remainder of this manuscript is ordered as follows. Section 2 presents the concept of t-normal graphs provides examples remarks, and results regarding their characterization using graph complements, as well as the normality of the union of two graphs 3 provides condition into which the Cartesian product of two is T-normal. We conclude in the solution of equation in the integers with final thoughts and some possibilities for future work.

2. T- normal Graphs

This section defines T- normal graphs and provides some examples. We also give remarks and some basic results on the characterization of T- normal using graph complements and the T-normality of two graphs.

Definition 2.1. A graph G is said to be T- normal if for any two disjoint cliques $Q_1$and Q_2 in G there exists covers D_1 and D_2 of Q_1 and Q_2 respectively in G such that $V(D_1) \cap V(D_2) = \emptyset$

Remark 2.1 Let $Q_1$and Q_2 be two disjoint cliques in G_1 and G_2 respectively *with* $|V(Q_1)|= m$ and $|V(Q_2)|=n$. If both m and n are even then both Q_1 and Q_2 can be covered by its own edges. It is a trivial case. Hence it is enough to consider the cases in which one of m and n are odd in which both m and n are odd.

Example 2.1. Path on three vertices, P_3 is not T-normal.

Consider the path $G=p_3$ with vertices $v_1 – v_2 – v_3$. Consider disjoint cliques $Q_1 = \{v_1\}$ and $Q_2 = \{v_3\}$. Any edge cover of Q_1 must contain v_1v_2,so $V(D_1) \ni \{v_1,v_2\}$. Any edge cover of Q_2 must contain v_2v_3, so (D_2) $\{v_2,v_3\}$. Thus $V(D_1) \cap V(D_2) \subseteq \{v_2\} = \emptyset$ and no disjoint endpoint sets possible .therefore p_3 is not T- normal.

V_1	V_2	V_3

P_3

Fig. 1. Path p_3

Proposition 2.1 The Cycle on 5 vertices c_5 is T-normal.

Proof –considerthe cycle G =G_5 with vertices v_0,v_1, v_2,v_3, v_4,. As c_5 has no triangle, all cliques are either singletons or edges.

Case1. Two singletons

For Q_1 = {v_1} and Q_2 = {v_2} choose D_1 = {v_0v_4} and D_2= {v_2v_3}. Then V(D_1) ={vo,v_4} and v(D_2) = {v_2,v_3} which are disjoint.

Case2. Singleton and edge

Let Q_1 = {v_o} and Q_2 = {v_2,v_3} Take D_1= {v_0v_4} the endpoint set are again disjoint

Case 3. Two disjoint edges

For Q_1 = {v_o,v_1} and Q_2 = {v_2,v_3} choose D_1= {v_2,v_3}; Here V(D_1) = {V_0,v_1} and V(D_2) = {v_2,v_3} which are disjoint.

Thus every pair of disjoint cliques in c_5 admits covers with disjoint vertex set and hence c_5 is T- normal in the solution in the equation in integers.

In this section we explore the conditions that allow for the T- normal behavior of a Cartesian of two graphs. Theorems and proofs are presented below that characterize these conditions.

Theorem -2.1 Let G_1 and G_2 be two graphs with ƃ(G_1) and ƃ(G_2)≥ 2 ,then G_1 G_2 is normal.

Proof- let Q_1 and Q_2 be two disjoint cliques in G_1 and G_2 respectively. Let [v(Q_1)] = m and [v(Q_2)] = n.By lemma 1.1 either the first co-ordinate of Q1nare same and the span of second co-ordinate of vertices of Q_1 for a clique in G_2, or the second co-ordinate of vertices of Q_1 are same and the span of second co-ordinate of vertices of Q_1forms a clique in G_1 similarly ,the case of also First of all suppose that second co-ordinate of each vertex of Q_1 is x and that of Q_2 is y. Assume that n≥m Let V(Q_1)={(u_1,x),(u_2,x),(u_3,x),………………(u_m,x)} and V(Q_2)= {(v_1,y),(v_2,y),(v_3,y),…………………….(V_n,y).As ƃ(G_1)≥2,m≥2and n≥2. Therefore we can suppose V_1 = u_m .To get covers $D_1$and D_2 of Q_1 and Q_2 respectively in g such that V(D_1)∩ V(D_2) = ∅ ,consider the two cases.

Case1. M is even and n is odd.

As m is even Q_1 can be covered by its own edegs.As ƃ(G_2)=2,there exists a vertex w different from x such that y is adjacent to w . then {(v_1,y)(v_2,y)(v_3,y) ,……………….(v_{n-2},y),(v_{n-1},y),(v_n,y)(v_n,W)} is a cover of Q_2 which is not incident with any edges of Q_1.

2Case2.Both m and n are odd

As \$(G_2)=2, there exists a vertex w of G_2 different from y such that w is adjacent to x .similar, there exists a vertex z of G2 different from x such that z is adjacent to y. then D_1 = {(u_1,x)(u_2,x)(u_3,x),(u4,x),……………………………………………(u_m,x),(u_m,w)

D_2={(v_1,y)(v_2,y)(v_3,y) ,……………….(v_{n-2},y),(v_{n-1},y),(v_n,y)(v_n,W)} are covers of $Q_1$and Q_2 respectively and

V(D_1)∩ V(D_2) = ∅

Now, assume the second co-ordinate of each vertex of Q_1 is and the first co-ordinate of each vertex of Q_2 is u. Also assume n6m. let $V(Q_1)$= $\{(u_1,X),(u_2,x),(u_3,x)\ldots..,(u_n,X)\}$and $V(Q_2)=\{(u,v_1),(u,v_2),(u,V_3)\ldots\ldots\ldots,(u,v_n)$.the vertex u may or not belongs to $\{u_1,u_2,u_3,\ldots..um\}$.if u$\epsilon$ $\{u_1,u_2,u_3,\ldots.um\}$.let u = u_1 Assume y$\in$ $\{v_1,v_2,v_3\ldots\ldots.v_n\}$.To get covers D_1 and D_2 of $Q_1$and Q_2 respectively in G such that $V(D_1)\cap V(D_2)=\emptyset$ consider the solution in the equation in integers in cases.

Hence the theorem.

3. Conclusion

In this paper ,We proposed the concept of T-normal graphs and clarified the notion by using some examples and a few counter example .Specifically ,we proved that cycles G5 is T-normal whilep3 any complete k_n where n≥2 is not T normal because there is forced overlap in any pair of edge covers. We provided commentary and results relating t- normal to properties of complements of graph and also analyzed when the union of two graph produces a T –normal .Finally we explored the condition under which the Cartesian product is t-normal .future work may focus on obtaining a complete characterization of T-normal graphs and identifying minimal forbidden sub graphs for this class. It would also be interesting to investigate the behavior of T-normality under other graph operations such as the lexicographic and strong products and to examine possible algorithmic approaches for recognizing T-normal graph efficiently. The concept of T-normality is certainly applicable and can be analyzed in the context of generalized graphs including. But not inclusive to Fuzzy Graphs (pal et.at.2020), Neutrosophic graphs (AL-Omeri and kaviyarsasu 2024) and Hyper graphs Dai and Gao (2023).

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Acknowledgement

The first author recognizes the financial assistance from the University Grants commission of india, through the Faculty Development programmer and from CSIR.

Competing Interests

Authors have declared that no competing interests exist.

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