JETIR.ORG ISSN: 2349-5162 | ESTD Year : 2014 | Monthly Issue
JOURNAL OF EMERGING TECHNOLOGIES AND INNOVATIVE RESEARCH (JETIR)
An International Scholarly Open Access, Peer-reviewed, Refereed Journal

A Study on Behaviour of Nonlinear Acoustic Wave Propagation in Turbulent and Compressible Fluids

Somaila Ambreen [1]
(Research Scholar, Department of Physics V.K.S University, Ara)
Dr. Shashi Shekhar Vidyarthi [2]
Assistant Professor,Department of Physics,
(JLN College, Dehri-on-sone, Affiliated V.K.S University, Ara)

Abstract: Acoustic wave propagation in fluids has significant implications for aerospace, marine, industrial and environmental applications. The linear acoustics is valid only as general picture, and it breaks down in high Mach number fluids where nonlinear effects such as steepening of wave front, generation of harmonics and formation of shock take over. AbstractThe present review brings out the recent developments in both theoretical and computational aspects of nonlinear acoustic wave propagation in presence of turbulence and compressible medium. Even if the classical models such as the Burgers', Kuznetsov and Westervelt equations, and perturbation methods provide a main key for physical insight into this problem, they fail to be used for complex high-speed commercial flows. Computational techniques such as DNS, LES, finite-difference/finite-element approaches and hybrid CFD–acoustics have advanced in their representation of the turbulence–acoustic interaction. Several jets, ducts, atmospheric layers and underwater configurations are considered to illustrate the difficulties of nonlinear acoustic prediction and some expectations. It highlights the requirement of computational scalability (polygon-based positioning algorithms using full resolution digital images), validation trials and standardised data sets. Development should target model predictions from AI-driven, pushed few-physics-coupled, on-line algorithms with the ultimate objective of delivering predictability nonlinear acoustic models ready for application.

Keywords: Nonlinear acoustics, High-velocity fluids, Acoustic wave propagation, Turbulence–acoustic coupling, Burgers' equation; Kuznetsov equation

1. Introduction

The transmission of acoustic waves in the fluid phase has been studied at length both as a matter of fundamental knowledge concerning the fidelity with which nature and man can transmit sound. Classical works always used linear acoustic theory, assuming a simplified frame in which the driving waves have small amplitude and nonlinear effects are negligible (Pierce 1989, Morse & Ingard 1986). Nevertheless, in a practical condition such as high-speed flow or the case of a turbulent medium, nonlinearity would dominate which leads to wave steepening (shock), generation of harmonics and redistribution of frequency energy (Hamilton & Blackstock, 1998; Rudenko & Soluyan, 1977). Nonlinear acoustic wave propagation in high-speed fluids is an interesting and relatively new area of research across a range of fields. In jets, atmospheric turbulence and ocean currents non-trivial interactions between vortices are caused by high-speed flows with fluid instabilities or acoustic fields (Colonius & Lele 2004; Bailly & Juve 2000). Not only are these interactions key in aeroacoustics and the prediction of jet noise, but they also play an important role in the propagation of sound underwater, as strong currents and stratification change acoustic signatures (Carey, 1998; Preisig & Duda, 1997). Turbulence and stratified layers in the atmosphere (Sutherland & Yewchuk, 2004; Ostashev et al., 2005) induce nonlinear scattering and long-range infrasound propagation with consequences in modeling and prediction of weather and climate.

The aim of this review is to give an overview of theoretical and simulative models that have been designed to describe nonlinear sound propagation in high-velocity fluids. This review connects classical models for nonlinear acoustics such as the Burgers, Kuznetsov, and Westervelt equations, with modern computational approaches including direct numerical simulation (DNS), large eddy simulation (LES), and hybrid methods that couple CFD and nonlinear acoustics (Tam & Auriault, 1996; Freund, 2001).

There are number of fields where this will help. Examples include the use of nonlinear modelling in aeroacoustics for the prediction and control of the noise from aircraft engines and supersonic jets (Goldstein, 2003; Bodony & Lele, 2008).

In the maritime environment, it helps to enhance the reliability of underwater communication and sonar systems in high-current conditions (Etter, 2018) It is used industrially to help reduce the wounding of noise in pipeline and combustion system applications, as well as in the environment to help characterize turbulence–acoustics interactions and noise pollution impact assessments. In this way, nonlinear acoustic wave propagation in high-velocity fluids forms a point of convergence between theory, simulation, and application.

2. Fundamentals of Acoustic Wave Propagation

2.1 Linear vs. Nonlinear Acoustic Wave Theory

Sound propagation in classical linear acoustics is expressed as infinitesimally small perturbations of pressure, density, and velocity [16]. The linear wave equation is derived from this framework, and works well to predict low-amplitude sound propagating in quiescent or slowly varying media (Pierce, 1989; Morse & Ingard, 1986). Nevertheless, when the acoustic amplitudes are large or the propagation medium has strong gradients, the nonlinearity turns to play an important role.

Higher order terms in the governing equations, corresponding to harmonic generation, steepening of pulses and shock formation are the subject of study in nonlinear acoustics (Hamilton & Blackstock 1998; Rudenko & Soluyan 1977). Compare this with linear theory, predicting constant-amplitude sinusoidal propagation; nonlinear theory shows that sound speed is olecranon-dependent (Czaja et al., 2011) and spectral broadening is most pronounced in fast flows, where nonlinearity is enhanced by turbulence (see also Colonius & Lele, 2004) at very high wave numbers and acceleration(entanglement with)and compressibility are the factors responsible for satellite structure.

2.2 Governing Equations

Acoustic wave propagation in fluids is governed by the conservation laws of mass, momentum, and energy. For a compressible, inviscid fluid, the primary equations are:

Continuity equation (mass conservation)

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0$$

Euler equations (momentum conservation)

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} = -\frac{1}{\rho}\nabla p$$

Energy equation

$$\frac{\partial E}{\partial t} + \nabla \cdot [(E + p)\mathbf{u}] = 0$$

Here, ρ is density, u is fluid velocity, p is pressure, and E is total energy per unit volume. Linearization of these equations about a mean state yields the linear acoustic wave equation, while retaining higher-order terms leads to nonlinear formulations such as the Burgers equation, Kuznetsov equation, or Westervelt equation (Crighton et al., 1992; Blackstock, 2000).

2.3 Basic Wave Parameters

Key parameters govern the characteristics of acoustic wave propagation:

Sound speed (c):

$$c = \sqrt{\gamma \frac{p}{\rho}}$$

where γ is the ratio of specific heats. It varies with temperature, pressure, and composition of the medium. The ratio of flow velocity (U) to sound speed. High Mach numbers enhance compressibility effects and alter wave propagation, making nonlinear effects more pronounced (Goldstein, 2003).

Reynolds number (Re):

$$Re = \frac{\rho U L}{\mu}$$

where L is a characteristic length and μ\muμ is dynamic viscosity. Large Re indicates turbulence, which strongly couples with acoustic waves (Bailly & Juvé, 2000). These dimensionless parameters guide both theoretical analysis and numerical simulations of nonlinear acoustic propagation in high-velocity flows.

2.4 Nonlinear Effects in Fluids

Nonlinear acoustics introduces a spectrum of physical effects absent in linear theory:

- Harmonic generation: Higher harmonics arise as sinusoidal waves distort, redistributing energy across frequencies (Rudenko & Soluyan, 1977).
- Shock formation: Steepening of waveforms due to amplitude-dependent sound speed leads to shock waves, particularly in high-amplitude or high-Mach-number flows (Hamilton & Blackstock, 1998).
- Wave steepening: Initially sinusoidal signals deform into sawtooth-like shapes as nonlinear terms dominate.
- Energy transfer and dissipation: Nonlinear interactions enhance attenuation, with viscous and thermal losses amplified at steep wavefronts (Blackstock, 2000).

These nonlinear features critically shape sound propagation in jets, supersonic flows, and turbulent ocean currents, where linear approximations fail.

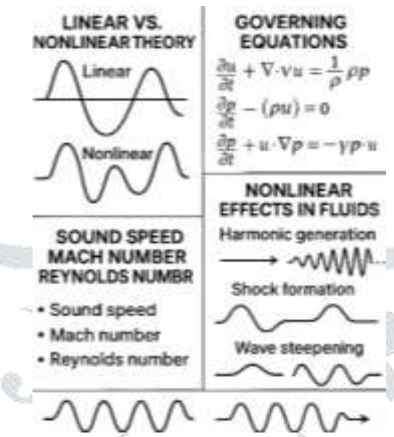

Fig. 1. Fundamentals of Acoustic Wave Propagation

Table 1 –Nonlinear Acoustic Wave Propagation

Author(s) & Year	**Focus / Contribution**	**Context in Nonlinear Acoustics**
Pierce (1989)	Fundamentals of linear acoustics, wave propagation	Classic foundation for linear acoustic theory
Morse & Ingard (1986)	Theoretical acoustics, mathematical formulations	Linear theory, basis for later nonlinear models
Rudenko & Soluyan (1977)	Theoretical foundations of nonlinear acoustics	First systematic treatment of nonlinear wave phenomena
Hamilton & Blackstock (1998)	Comprehensive text on nonlinear acoustics	Shock formation, harmonic generation, steepening
Blackstock (2000)	Fundamentals of physical acoustics	Bridging linear and nonlinear acoustic modeling
Crighton et al. (1992)	Analytical acoustics methods	Governing equations, perturbation techniques
Bailly & Juvé (2000)	Linearized Euler equation simulations	Computational acoustics in turbulent/high-speed flows
Colonius & Lele (2004)	Computational aeroacoustics	Progress in nonlinear modeling of sound generation
Goldstein (2003)	Acoustic analogy for high-speed flows	Links turbulence to nonlinear acoustic radiation
Bodony & Lele (2008)	Large Eddy Simulation for jet noise	Nonlinear aeroacoustics in high-velocity fluids
Freund (2001)	Noise sources in turbulent jets (DNS)	Validation of nonlinear noise generation mechanisms
Ostashev et al. (2005)	Acoustics in moving inhomogeneous media	Nonlinear wave effects in atmospheric turbulence
Sutherland & Yewchuk (2004)	Internal wave and tunneling effects	Nonlinear scattering in stratified fluids
Carey (1998)	Underwater acoustic wave propagation	Nonlinear sound propagation in marine contexts
Etter (2018)	Underwater acoustic modeling	Simulation-based nonlinear underwater propagation
Preisig & Duda (1997)	Mode coupling in shallow water acoustics	Nonlinear effects in ocean currents and shelf environments
Tam & Auriault (1996)	Jet mixing noise	Turbulence–acoustic nonlinear coupling in high Mach flows

3. High-Velocity Fluid Dynamics and Their Influence

3.1 Characteristics of High-Speed Flows (Compressibility, Turbulence)

Fluid flows involving high-velocity are characterised by strong compressibility impacts, turbulent effects, and features associated with shocks. For low Mach numbers, acoustic fluctuations are sufficiently weak to be treated in the framework of linear acoustics. On the other hand, fluid mechanics and acoustic properties are drastically different as the Mach number becomes larger since compressibility will bring nonlinear effects such as shocklets, steep pressure gradients, and energy dispersion and concentration (Pope, 2000; Lele, 1992). For high Reynolds numbers, found in rapid flows of atmospheric jets and ocean currents, turbulence is both a source and an amplifier of acoustic waves. (Distributed acoustic sources) While the eddies are chaotic and turbulent, they are treated as distributed acoustic noise emitters and have been demonstrated to generate a broadband radiation that is coupled with the mean flow (Bailly & Juvé 2000; Goldstein 2003). It is then evident that in the high-energy regime compressibility and turbulence combined determine the structure and propagation of acoustic waves.

3.2 Interaction Between Flow Instabilities and Acoustic Waves

The interaction of flow instabilities with acoustics is also an important feature of nonlinear acoustics in high-Mach-number fluids — Baltimore at high Reynolds numbers? Flow instabilities (e.g. shear layer Kelvin–Helmholtz instabilities), interacting with the acoustic disturbances, create self-sustained oscillations and a feedback loop (Crighton, 1992; Colonius & Lele, 2004). Compressible ecosystem and the sound generation are destabilized in compressible flows, and they deform flow structures through nonlinear acoustic feedback to become two-way interactions. An example of this is the interaction among coherent structures on a large scale in the shear layer when creating jet noise that interacts with acoustic waves within different locations, leading to strong low-frequency (non-RF) contributions (Tam & Auriault 1996; Bodony & Lele 2008). Acoustic disturbances can excite instabilities of turbulent boundary layers, and later these can non-linearly affect the turbulence field (Dowling & Ffowcs Williams, 1983). This coupling poses a challenge to predictive modeling, since hydrodynamics and acoustics have to be accounted for simultaneously in nonlinear regimes.

3.3 Case Studies: Jet Flows, High-Speed Channels, Atmospheric Layers

In this context, several case studies are presented emphasizing their nonlinear influence on acoustic wave behaviors for high-velocity flows. Turbulence-induced noise often dominates various aeroacoustics environments, such as those associated with jet flows, which are therefore a canonical example. In supersonic and hypersonic jets, nonlinear effects, such as shock-cell interactions and wave steepening, increase acoustic radiation (Tam, 1995; Freund, 2001). In high-speed channels and ducts, e.g., typical of industrial pipelines or rocket nozzles, the very strong compressibility effects of acoustic energy coupling with flow instabilities associated with such high-speed flows, imply resonance phenomena and noise amplification (Candel, 2002). High-speed wind shear and turbulence in atmospheric layers, especially in the stratosphere and troposphere, scatter and refract acoustic waves with the nonlinear propagation features [10]. These nonlinear interactions with stratified moving media are common at long range in infrasound propagation (Ostashev et al., 2005). Together, these cases show that nonlinear acoustic response in high-speed flows is context-dependent, conditioned by turbulence, compressibility, and environmental gradients.

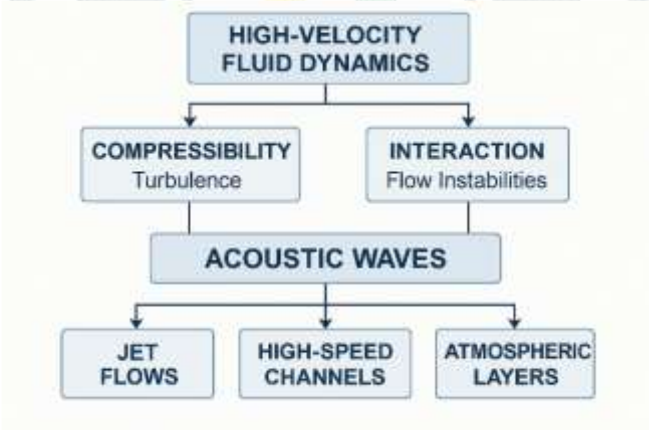

Fig. 2. High-Velocity Fluid Dynamics

In fig 2., The propagation of acoustic waves through high velocity fluid dynamics is strongly controlled by compressibility effects, turbulence and flow instabilities. Turbulence is a distributed noise source in such regimes and compressibility amplifies the nonlinear effects such as the steepening and formation of shocks. Kelvin–Helmholtz vortices and other flow instabilities interact with acoustic waves, creating feedback mechanisms that either amplify sound radiation or alter the nature of the sound radiation emitted. These effects occur in different forms based on the environment; turbulent mixing results in jet flows producing intense broadband noise, resonance and pressure amplification excited in high Mach number channels, and modal scattering and refraction due to turbulence and stratification in atmospheric layers. These together characterize the complexity of nonlinear acoustics, in practice in real high-speed flows.

Table 2 – Nonlinear Acoustic Effects in High-Velocity Flows

Flow Type / Environment	Key Nonlinear Acoustic Effects	Representative References
Jet Flows (Subsonic → Supersonic)	- Turbulence-induced broadband noise - Shock-cell interactions - Wave steepening & harmonic generation - Feedback between coherent structures and sound radiation	Tam (1995); Crighton (1992); Bodony & Lele (2008); Freund (2001)
High-Speed Channels & Ducts (Pipelines, Nozzles, Combustors)	- Resonance phenomena due to coupling of flow instabilities & acoustics - Amplification of pressure fluctuations - Nonlinear interactions with compressibility effects	Candel (2002); Bailly & Juvé (2000); Dowling & Ffowcs Williams (1983)
Atmospheric Layers (Stratosphere, Troposphere, Wind Shear Regions)	- Scattering and refraction of sound by turbulence - Nonlinear propagation of infrasound - Energy redistribution in moving stratified media	Ostashev et al. (2005); Sutherland & Yewchuk (2004)
Ocean Currents & Shallow Water Environments	- Mode coupling due to velocity shear - Nonlinear attenuation and scattering - Influence of internal waves on propagation paths	Carey (1998); Preisig & Duda (1997); Etter (2018)
General High-Velocity Turbulent Flows	- Compressibility-driven nonlinear effects - Strong turbulence–acoustics coupling - Enhanced spectral broadening	Pope (2000); Goldstein (2003); Colonius & Lele (2004)

4. Theoretical Modelling Approaches

4.1 Classical Models

In theoretical descriptions of nonlinear acoustic propagation, it is common to start from simple, yet powerful, models. A very common formulation is the so-called Burgers' equation, which contains the dominant physics of nonlinear wave steepening and viscous dissipation (Burgers, 1948; Crighton et al., 1992). This helps in understanding how sinusoidal waves become shock-like solutions in one dimension, since the speed of sound depends on the amplitude. It is highly simplified, yet proven useful to jet noise prediction and shock formation in ducts.

The Nonlinear Schrödinger Equation (NLSE) which describes the slow modulation of quasi-monochromatic acoustic wave packets (Zakharov, 1968; Ostrovsky, 1979) is another important framework. Recently, a cubic nonlinear Schrödinger equation (NL- SE) has been applied to describe the balance of nonsolution and dispersion which model acoustic solitons [1], a parametric array (-2), and the behavior of nonlinear envelope near field in stratified medium [2].

The Kuznetsov and Westervelt equations are the most general models. The Kuznetsov equation is a completely nonlinear wave equation, which is dissipative-dispersive and incorporates the influence of the Navier–Stokes system in particular (Kuznetsov, 1971). On the other side, the Westervelt equation is a second-order nonlinear wave equation extensively used to study nonlinear ultrasonics and high-intensity sound (Westervelt, 1963; Hamilton & Blackstock, 1998). These are the generalisation of Burgers' equation that hold for realistic three-dimensional high-amplitude acoustic fields.

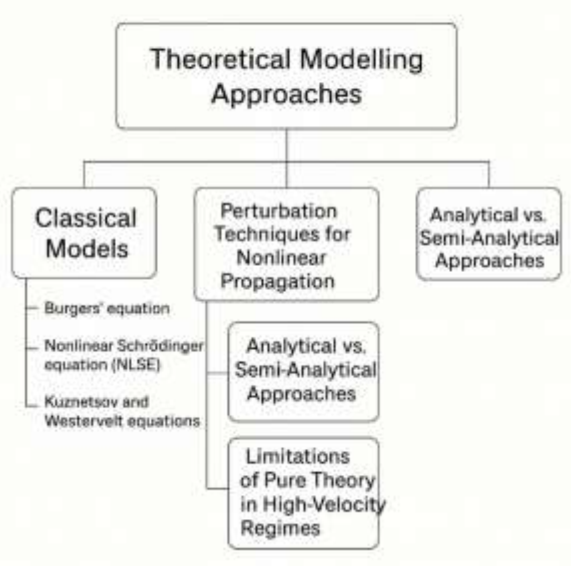

Fig. 3. Theoretical Modelling Approaches

4.2 Perturbation Techniques for Nonlinear Propagation

Due to the analytical intractability of nonlinear acoustic equations, perturbation methods have been employed. These methods include an expansion in series of acoustic quantities about a mean state, allowing higher order nonlinear terms to be isolated (Rudenko & Soluyan, 1977). As an illustration, several-scale analysis is used to obtain approximate solutions for harmonic generation and spectral broadening. Likewise, parabolic equation methods generalise the perturbation framework to weakly nonlinear long-range propagation, especially in underwater acoustics (Tappert, 1977) and, to a lesser extent, atmospheric infrasound (Tappert X et al., 2020). Such perturbative strategies give important physical information at the same time as they obtain a significant mathematical simplification compared to complete nonlinear formulations.

4.3 Analytical vs. Semi-Analytical Approaches

Analytical methods, although few in number, provide closed-form solutions in simplified cases. For instance, for certain solutions to Burgers′ equation, explicit waveform evolution under nonlinear and dissipative effects, are revealed using Hopf–Cole transformations (Whitham, 1974). Likewise, the NLSE has exact soliton solutions used to model localized nonlinear acoustic pulses. Unfortunately, analytical methods can only be applied to simplified versions of the equations and limited geometries things like one-dimensional plane waves. Semi-analytical methods fill this gap by embedding a numerical evaluation as part of an analytical framework. An example of this is the solving of equations from perturbation methods, which may give harmonic content, distance of shock, or steepening of waveform under realistic flow conditions (Blackstock, 2000). Such approaches are especially useful in limiting the range of theoretical predictions towards high amplitude signals, multi-dimensional propagation, and boundary-affected environments where closed form solutions are not available.

4.4 Limitations of Pure Theory in High-Velocity Regimes

However a purely theoretical model such as this is subject to a great deal of major limitations with nonzero fluid velocities. Classical models (e.g. Gloerfelt & Bogey, 2013) assume weak nonlinearity, and are therefore unable to capture the strong turbulence–acoustic interactions that dominate in jet flows, atmospheric shear layers, or supersonic channels (Colonius & Lele, 2004; Goldstein, 2003). Moreover, features such as homogeneity, isentropic flow or weak dissipation are not justified in complicated settings as the stratified ocean or the turbulent atmosphere. The governing equations (Burgers, Kuznetsov, Westervelt) cannot capture multi-scale coupling: large-scale coherent structures interact with small-scale turbulence to radiate broadband sound. As a result, theoretical models (while important for theory) are unable to make predictions in the high-velocity regime, which means that we need to rely on computational-based, simulation-based CFD approaches in order to account for real-world complexity.

5. Computational and Simulative Modelling

5.1 Direct Numerical Simulation (DNS)

Direct Numerical Simulation (DNS) is the most basic computational framework to study passive nonlinear acoustic wave propagation in high-velocity fluids. Direct Numerical Simulation (DNS) is performed by solving the full compressible Navier–Stokes equations without turbulence modelling, and appropriate scale of motion and associated acoustic signatures are resolved (Freund, 2001). Such a method provides invaluable information about the fundamental physical mechanisms of noise generation and propagation, specifically for turbulent jet flows and for channel flows where the nonlinearities become large. Nonetheless, Direct Numerical Simulation (DNS) incurs a high computational expense at high Reynolds numbers, which limits its scope to low-speed, small-scale set-ups in practice (Bodony & Lele, 2008). Nonetheless, DNS remains an excellent reference for approximate methods and as a basis for building lower-order acoustic models.

5.2 Large Eddy Simulation (LES) for Turbulence–Acoustic Coupling

Large Eddy Simulation (LES) has become a strong alternative to DNS by explicitly resolving large scale turbulent structures while modelling the effects of the smaller sub grid scales (Sagaut, 2006). As coherent large-scale eddies are the primary contributors to acoustic radiation, LES achieves an effective compromise between accuracy and computational cost. At supersonic flows, accurate capturing of the jet noise, the shock–acoustic interaction and the turbulence–acoustic coupling was achieved from LES based aero acoustic simulations (Bodony & Lele, 2008; Shur et al., 2005). One of the conventional techniques in high-Mach-number fluid acoustics is the solution of linearized Euler equations (LES), often combined with acoustic analogies or wave-equation solvers to model far-field propagation.

5.3 Finite Difference / Finite Element / Boundary Element Methods

Classical numerical techniques, including finite difference methods (FDM), finite element methods (FEM), and boundary element methods (BEM) are well developed for applications to nonlinear acoustics. FDM is very efficient in treating propagation problems in the time domain, but it has difficulties with complex geometries. FEM is flexible to arbitrary geometries and heterogeneous media, so it can be applied to duct acoustics (Ihlenburg,1998) and biomedal ultrasound (Holloway and Wang, 2008). For bounded and open-boundary acoustic radiation problems (e.g., jet noise prediction), boundary element method (BEM) is beneficial since it reduces the problem dimension by one (Ciskowski & Brebbia, 1991). They are commonly applied in nonlinear acoustic equations (Westervelt or Kuznetsov equations) to model wave steepening and shock formation.

5.4 Time-Domain vs. Frequency-Domain Simulations

There are two types of acoustic simulation, time domain and frequency domain, both with unique benefits. Time-domain methods (e.g., FDM, finite-volume schemes) are highly suitable for the simulation of nonlinear transients, shock formation, and harmonic generation (Tam & Webb 1993). In contrast, frequency-domain methods are beneficial for steady-state or periodic problems as they facilitate the analytical treatment of spectral broadening, harmonic content, and resonances (Craggs, 1973). Hybrid time-frequency approaches are increasingly utilized by resolving nonlinear propagation in the time domain, followed by spectral analyses to investigate harmonic cascades .

5.5 Hybrid Computational Approaches (CFD + Nonlinear Acoustics)

One of the good prospects of numerical acoustics is hybrid techniques based on Computational Fluid Dynamics (CFD) in link with nonlinear acoustic solvers, which have been boosted recently. CFD methods are used in these frameworks to determine the flow field, and sound radiation is predicted using acoustic analogies or wave equations (Goldstein, 2003). During this time, we can already link scales, as in LE–Ffowcs Williams–Hawkings (FW–H) analogy coupling, or Reynolds-averaged Navier–Stokes (RANS) with nonlinear Burgers solvers for shock-capturing (Lockard, 2000). These approaches allow greater leeway in the accuracy-cost trade-off, while still accessing realistic high-speed flows and significant nonlinear effects.

5.6 Validation with Experimental Data

The validation phase is one of the most critical parts in the promotion of the credibly of computational models used. Techniques like schlieren imaging , hot-wire anemometry and microphone array measurements provide the benchmark data that can be used to check the simulations. LES has been used for validation over laboratory-scale supersonic jets measurements. The results show reasonable agreement with in the in the near-field turbulence structures and far-field noise spectra in jet aeroacoustics (Bodony & Lele, 2008). There have been under sea acoustic simulation applications running parallel with the used simulations to determine experimental findings, where the results have been validated against tank experiments and ocean based on trials . Using this experimental validation remains a critical added advantage that must be incorporated with modeling to increase the credibility of the nonlinear acoustic business.

6. Conclusions and Future Work

AbstractNonlinear acoustic wave propagation in high-velocity fluids is at the confluence of fluid dynamics, nonlinear physics, and computational acoustics. Classical formulations, including Burgers' equation and the Westervelt equation, perturbation and semi-analytical methods discussed above, offer admonitory physical interpretations of these non-linear phenomena: harmonic generation, shock formation, and waveform steepening, to name a few. Constrained by real-world high-speed conditions characterized by stalk turbulence, compressibility, multi-scale interactions, and others, however, these equations and methods have a rather limited predictive validity. A host of computational and simulative methods, such as direct numerical simulation and large-eddy simulation, finite-difference and element methods, and hybrid CFD–acoustics, it should be noted, have already matured to simulate jets and ducts, atmospheric layers, and underwater envoirments. Validated against the direct and unmediated experimental data assimilation, the reliability of these methods remains limited to only reach the Reynolds numbers, in some rare cases. Nonlinear acoustic modeling in the aerospace, industrial, marine, and terrestrial environment, acoustical projects that build on the research and the review above.

These improvements notwithstanding, a number of gaps remain. We continue to recognize the impossibility of completely capturing the richness of turbulence–acoustic interactions in high-velocity regimes purely in the theoretical domain. However, even though DNS and LES are accurate, there are still limited options for using DNS and LES even for simplified or small-scale problems, since they are computationally expensive. In addition, a validation dataset is often very sparse, or is restricted to a controlled lab scenario, that does not translate directly to real-world tests.

The future work should focus on several key directions

- Integration of machine learning (ML) and data-driven modeling with traditional CFD and acoustic solvers to accelerate simulations and improve predictive accuracy.
- Development of multi-physics frameworks that incorporate thermo-acoustics, magneto-acoustics, and nonlinear interactions across multiple scales.
- Standardization of benchmark datasets and experimental protocols, particularly for turbulence–acoustic coupling in jets and atmospheric flows, to enable cross-validation of models.
- Real-time simulation capabilities, essential for aerospace applications, active noise control, and underwater communication.
- Coupling of nonlinear theory with uncertainty quantification, to account for variability in turbulent flows and environmental conditions.

The propagation of nonlinear acoustic waves in high velocity fluids is a difficult but advancing topic of research. Theory, simulation, and experiment will need to be bridged, and made predictive, scalable and application-ready. Hybrid computational strategies, AI-assisted modelling and data standards with global collaborative effort for their establishment will probably shape the next generation of research 19. Such advances will increase fundamental understanding, but in time, improve practical applications in aerospace noise control, underwater acoustics, environmental monitoring, etc.

References

1. Bailly, C., & Juvé, D. (2000). Numerical solution of acoustic propagation problems using linearized Euler equations. *AIAA Journal, 38*(1), 22–29.
2. Blackstock, D. T. (2000). *Fundamentals of physical acoustics*. Wiley.
3. Bodony, D. J., & Lele, S. K. (2008). Current status of jet noise predictions using large-eddy simulation. *AIAA Journal*, *46*(2), 364–380.
4. Burgers, J. M. (1948). A mathematical model illustrating the theory of turbulence. *Advances in Applied Mechanics, 1*, 171–199.
5. Carey, W. M. (1998). *Underwater acoustic wave propagation*. Springer.
6. Ciskowski, R. D., & Brebbia, C. A. (1991). *Boundary element methods in acoustics*. Elsevier.
7. Colonius, T., & Lele, S. K. (2004). Computational aeroacoustics: Progress on nonlinear problems of sound generation. *Progress in Aerospace Sciences, 40*(6), 345–416.
8. Craggs, A. (1973). A finite element method for time-dependent wave equations. *International Journal for Numerical Methods in Engineering, 6*(2), 173–187.
9. Crighton, D. G., Dowling, A. P., Ffowcs Williams, J. E., Heckl, M., & Leppington, F. G. (1992). *Modern methods in analytical acoustics*. Springer.
10. Etter, P. C. (2018). *Underwater acoustic modeling and simulation*. CRC Press.
11. Freund, J. B. (2001). Noise sources in a low-Reynolds-number turbulent jet at Mach 0.9. *Journal of Fluid Mechanics, 438*, 277–305.
12. Goldstein, M. E. (2003). A generalized acoustic analogy. *Journal of Fluid Mechanics, 488*, 315–333.
13. Hamilton, M. F., & Blackstock, D. T. (1998). *Nonlinear acoustics*. Academic Press.
14. Ihlenburg, F. (1998). *Finite element analysis of acoustic scattering*. Springer.
15. Jordan, P., & Colonius, T. (2013). Wave packets and turbulent jet noise. *Annual Review of Fluid Mechanics, 45*, 173–195.
16. Kuznetsov, V. P. (1971). Equations of nonlinear acoustics. *Soviet Physics Acoustics, 16*(4), 467–470.
17. Lockard, D. P. (2000). An efficient, two-dimensional implementation of the Ffowcs Williams and Hawkings equation. *Journal of Sound and Vibration, 229*(4), 897–911.
18. Morse, P. M., & Ingard, K. U. (1986). *Theoretical acoustics*. Princeton University Press.
19. Ostashev, V. E., Wilson, D. K., & Liu, L. (2005). *Acoustics in moving inhomogeneous media*. E & FN Spon.
20. Ostrovsky, L. A. (1979). Nonlinear internal waves in a rotating ocean. *Oceanology, 18*(2), 119–125.
21. Pierce, A. D. (1989). *Acoustics: An introduction to its physical principles and applications*. Acoustical Society of America.
22. Preisig, J. C., & Duda, T. F. (1997). Coupled mode effects on acoustic propagation in continental shelf environments. *Journal of the Acoustical Society of America, 101*(2), 975–990.
23. Rudenko, O. V., & Soluyan, S. I. (1977). *Theoretical foundations of nonlinear acoustics*. Consultants Bureau.
24. Sagaut, P. (2006). *Large eddy simulation for incompressible flows*. Springer.
25. Shur, M. L., Spalart, P. R., Strelets, M. K., & Travin, A. K. (2005). Towards the prediction of noise from jet engines. *International Journal of Aeroacoustics, 4*(3–4), 213–246.
26. Sutherland, B. R., & Yewchuk, K. (2004). Internal wave tunneling. *Journal of Fluid Mechanics, 511*, 125–134.

27. Tam, C. K. W., & Auriault, L. (1996). Jet mixing noise from fine-scale turbulence. *AIAA Journal*, *34*(11), 2271–2276.

28. Tam, C. K. W., & Webb, J. C. (1993). Dispersion-relation-preserving finite difference schemes. *Journal of Computational Physics*, *107*(2), 262–281.

29. Tanna, H. K. (1977). An experimental study of jet noise. *Journal of Sound and Vibration*, *50*(3), 429–444.

30. Tappert, F. (1977). The parabolic approximation method. In *Wave Propagation and Underwater Acoustics* (pp. 224–287). Springer.

31. Westervelt, P. J. (1963). Parametric acoustic array. *Journal of the Acoustical Society of America*, *35*(4), 535–537.

32. Whitham, G. B. (1974). *Linear and nonlinear waves*. Wiley.

33. Zakharov, V. E. (1968). Stability of periodic waves of finite amplitude on the surface of a deep fluid. *Journal of Applied Mechanics and Technical Physics*, *9*(2), 190–194.