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ANISOTROPIC DARK ENERGY COSMOLOGY IN BIANCHI TYPE–III SPACE–TIME WITH COSMIC STRINGS

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Abstract

In this work, we investigate anisotropic dark energy cosmological models in the framework of Bianchi type-III space-time incorporating one-dimensional cosmic strings within Einstein's general relativity. To obtain exact solutions of the field equations, a hybrid expansion law for the average scale factor is assumed, which successfully describes the transition of the universe from an early decelerating phase to the present accelerating phase. We analyze the dynamical behavior of the model through important cosmological parameters such as the dark energy equation of state parameter, skewness parameters, deceleration parameter, and statefinder diagnostics. The evolution of physical quantities indicates that the string tension density and matter energy density dominate at early times but decay as the universe evolves, leading to the dominance of dark energy at late times. The equation of state parameter exhibits a transition from quintessence behavior and asymptotically approaches the cosmological constant value ($\omega_{de} = -1$), consistent with current observational data. Furthermore, the statefinder analysis shows that the model approaches the ΛCDM limit in the late-time universe. The results demonstrate that the proposed anisotropic model with cosmic strings provides a viable description of the observed accelerated expansion of the universe.

Keywords: Bianchi type-*III*, Cosmic string, Dark energy, Skewness parameter, General Relativity, Deceleration parameter.

1 Introduction

Recent cosmological data suggest that the universe's expansion has evolved from a decelerating phase to an accelerating phase in the last few billion years. This is usually associated with the recent supremacy of a portion of energy with negative pressure called Dark energy (DE) (Peebles et al. [1]). Among candidates for dark energy, the simplest one is the cosmological constant Λ proposed by Einstein and a dynamical scalar field such as quintessence (Frieman et al. [2]) and Λ appear in particle physics as vacuum energy with constant energy density ρ constant pressure p and equation of state $\omega = \frac{p}{\rho} = -1$. In particle physics viewpoint Λ is troublesome. Right before the turn of the millennium some cosmological observations have revealed that our universe contains dark energy. This source of energy in the universe satisfies an anomalous equation of state regarding regular cases or radiating and equivalent to the current vacuum energy in our universe. Dark energy (DE) can be accommodated inside general relativity by including an extra term to the Einstein equations which is often called cosmological constant and normally denoted by Λ.

Carroll [3] has investigated the most obvious theoretical candidate for DE is the cosmological constant, which has the equation of state (EoS) $\omega_{de} = -1$. Nevertheless, it suffers from the so-called cosmological constant problem (the problem of fine-tuning) and the universal problem of correlation.

The DE density is akin to pair of such difficulties. Copeland et al. [4] and Wang et al. [5] have been proposed many dynamically varying DE candidates in literature to overcome the problems. Investigation hold to approve a certain agility of cosmic is a late time aspect and has occurred at a Redshift of the order $z_t \sim 1$. It shows that the universe has witnessed a rapid transformation from a decelerated to an accelerated phase in recent times. This cosmic transition aspect asserts a developing deceleration parameter with a flipping sign. The price whereby the conversion arises generally decides the Redshift z_t transfer.

In current times anisotropic cosmological models have generated a lot of research interest. The Friedman-Robertson-Walker (FRW) models endorse the high degree of isotropy of the microwave background radiation. However, certain measurements have indicated dipole anisotropy of the cosmic microwave background radiation (CMB) (Boughn et al. [6]). Though the dipole anisotropic can be clarified easily by the gravitational attraction of the Sun to the background radiation, the quadruple anisotropy is an intrinsic property of the background radiation. The observed quadruple amplitude has a lower value than that expected from the best fit Λ-dominated cold dark matter (ΛCDM) model to the entire power spectrum since the first data of the differential microwave radiometer (DMR) appeared in 1992 (Akarsu et al. [7]; Smoot et al. [8]; Bennet et al. [9]). This anomaly was confirmed with the high resolution data provided by Wilkinson Microwave Anisotropy Probe (WMAP, WMAP3) (Hinshaw et al. [10], [11]). The quadruple amplitude could not be improved even in the most recent data (Hinshaw et al. [11]; Nolta et al. [12]). This low value of quadruple anisotropy seems to be inescapable (Eriksen et al. [13]). Sarala [14] and Ralston and Jain [15] are invited strong evidences in favor of global anisotropy. Since the experimental data favors an anisotropic universe, it is wise to consider spatially homogeneous and anisotropic Bianchi type cosmological models. Bianchi type models are widely studied to investigate different aspects of the universe and Such models play a key role in the elaboration of the earliest stages of development in the universe.

In recent years string cosmologies have been studied extensively. Throughout the early cosmos phase change with randomly broken symmetry, cosmic strings arise as one dimensional topological defect along with other defects like domain walls. Strings can lead to many interesting effects out of these topological faults. The resulting density disruption leads to large-scale structures such as galaxies being created. The strings at later times vibrate more vigorously, lose energy and gradually disappear. String theory also implies that the effects of quantum theory will overcome and substitute the big bang singularity by a bounce. Maybe this can be seen most clearly by observing the physical properties of a string gas over a space-time. Letelier [16] and Stachel [18] has been a good many works on string cosmologies with different space times, either in the frame work of Einstein relativity or modified theories of gravity. Pradhan et al. [19] and Tripathy et al. [20, 21] studied different aspects of cosmic strings in general relativity. As an extensive collection of entities such as galaxies will characterize the universe, string cloud cosmological models can help understand the evolution of the early universe.

Anisotropic Bianchi-type models with cosmic strings have gained growing significantly in recent years. Due to the prime function, cosmic strings were studied in general relativity in description of the evolution of early phases of the universe. The presence of cosmic strings was explained through the grand unified theories. One dimensional strings are believed to occur as a topological stable defect during the phase transition followed by spontaneous broken symmetries after the big bang explosion as the temperature goes down below some critical point (Kibble [50]). Cosmic strings are assumed to provoke disruptions in density that contribute to galaxies forming (Zel'dovich [51]).

The modern universe's model is centered on huge-scale isotropy and space uniformity. Yet, small anisotropic can be found in the universe. Watanabe et al. [5], Buiny et al. [22], Tripathy [23], Ade et al. [24, 25], Quartin and Notari [26], Armendariz Picon [27], Jaffe et al. [28], Cooray et al. [29], Cooke and Lynden-Bell [30], Colin et al. [31], Salehi and Aftabi [32], Campanelli et al. [33], Campan- elli [34], Gruppuso [35], Koivisto and Mota [36–38], Saadeh et al. [39], Battye and Moss [40], Bunnet al. [41] and Akarsu and Kilinc [42, 43] have been a lot of investigations on the anisotropic features of the universe in recent times. Using the data from WMAP, many authors have claimed a departure from global isotropy, that the non-trivial topology of asymmetric expansion exists in large scale cos- mic geometry (Buiny et al. [22]; Tripathy [23]; Watanabe et al. [44]). The preliminary Planck data show a slight redshift of the power spectrum from exact scale invariance (Ade et al. [24]) even though some other analysis of Planck data go against the global anisotropy (Ade et al. [25]; Quartin and No- tari [26]). Armendariz-Picon [27], Jaffe et al. [28], Cooray et al. [29] and Koivisto and Mota [38] have been associated dark energy with the breakdown of global isotropy and can display anisotropic fea- tures. Employing the Union compilation of type-*I*a Supernovae, Cooke and Lynden-Bell [30] found that a low dark energy anisotropy, typically seen in higher redshift groups with $z > 0.56$, directed roughly towards the cosmic microwave background dipole. The question of the departure from world isotropy will soon be resolved. However, Bianchi type models have been constructed in recent times to handle the issue of anisotropy (Campanelli et al. [33]; Campanelli [34]; Gruppuso [35]; Koivisto and Mota [36, 37]; Battye and Moss [40]; Akarsu and Kilinc [42, 43]). It is certain that Bianchi type models are the generalization of the FRW models where the spatial isotropy is relaxed.

In the present work, we consider a general form of non-interacting dark energy with anisotropic physical force within different direction and the significance of anisotropic outcome act inspected on the dark energy density parameter and the equation of state. In addition to the anisotropic DE fluid, cosmic strings aligned along x-direction are also considered to incorporate some anisotropic effect. Mishra et al. [45], Mishra and Tripathy [46] and Tripathy et al. [47] have investigated the dynamical behaviour of pressure anisotropies either in the framework of general relativity or alternative gravity theory. Cosmological studies involving one dimensional cosmic strings are interesting in the sense that, they may provide some explanation to the Physics of the early universe. Moreover, cosmic strings are believed to arise as topological defects during the spontaneous symmetry breaking. So they also would help build big frameworks, like galaxy clusters. There have been several impres- sive research in upcoming days into various dimensions of cosmological string models. (Tripathy et al. [20, 21, 48, 49]; Saha and Visinescu [52]). Reddy [53], Reddy and Rao [54, 55] have studied string cosmological models in modified theories of gravitation. All of the described models can be investigated as anisotropic models. This is because recent experimental data, as well as theoretical arguments, support the existence of some quantity of anisotropy early in the formation of the universe and Bianchi type models are well established with anisotropic background.

In the literature theoretical models of interacting and non-interacting DE were extensively dis- cussed. Tripathy et al. [56] investigated Bianchi type V DE model with two non-interacting fluids as matter field and found the pressure anisotropy which continues along with the cosmic expansion. We also found that those models were driven by phantom behaviors. In our current work, we have built anisotropic DE model in general relativity on the background of Bianchi *III* and studied the dynamics of the model taking into consideration two different non-interacting fluids that contribute to the matter field. The first is the rising cosmic string fluid, and the second is the DE.

The paper is designed as. In Section 2, we derived specifically field equations with general rela- tivity by the metric Bianchi type *III* in the presence of dark energy and cosmic string fluid. For the Bianchi Type-*III*, we obtained in sections 3, solutions of the field equation and some properties of the model. Finally, the results are addressed in Section 4.

2 The metric and field equations

We consider Bianchi type-*III* space-time in the form

$$ds^2 = dt^2 - A^2dx^2 - B^2e^{2x}dy^2 - C^2dz^2, \quad (1)$$

where A, B, C are functions of the cosmic time t only.

The Einstein's field equations are given by

$$R_{ij} - \frac{1}{2}Rg_{ij} = -\frac{8\pi G}{c^4}T_{ij} \quad (2)$$

where R_{ij}, g_{ij} and R are the Ricci tensor, metric tensor and Ricci scalar respectively. Here we consider $8\pi G = c = 1$, G is the Newtonian gravitational constant and c is the speed of light. The directions are viewed differently and thus define the anisotropic expansions along with the three or- thogonal directions. When the directional factor is equal, the model is reduced to a model Friedmann Robertson-Walker (FRW).

The energy-momentum tensor is given to two non-interacting fluids in a certain environment is

$$T_{ij} = T_{(cs)ij} + T_{(de)ij}, \quad (3)$$

where $T_{(cs)ij}$ and $T_{(de)ij}$ denote the energy momentum tensor for one dimensional cosmic strings and DE, respectively.

For fluid having one dimensional cosmic string, the energy momentum tensor is given by Letelier [16], Mishra et al. [17] and Stachel [18]

$$T_{(cs)ij} = (\rho + p)u_iu_j - pg_{ij} - \lambda x_ix_j, \quad (4)$$

where $u^iu_i = -x^ix_i = 1$ and $u^ix_i = 0$. In a commoving coordinate system, u^i is the four velocity vector and p is the isotropic pressure of the fluid. x^i signifies the cosmic string path (along x-direction). λ is the tension density of string and the ρ is proper density of energy consisting of massive strings.

The annual improvement to the baryonic energy density arises by particles alone, in the absence of any string phase. In contrast to the usual cosmic fluid isotropic pressure, we want to include a certain amount of anisotropy in the dark energy pressure.

The energy momentum tensor of the dark energy is given by

$$T_{(de)ij} = diag\left[\rho_{de}, -p_{de_x}, -p_{de_y}, -p_{de_z}\right]$$
$$= diag\left[1, -\omega_{de_x}, -\omega_{de_y}, -\omega_{de_z}\right]\rho_{de}$$
$$= diag[1, -(\omega_{de}+\delta), -(\omega_{de}+\gamma), -(\omega_{de}+\eta)]\rho_{de},$$

where $\omega_{de} = \frac{p_{de}}{\rho_{de}}$ is the DE equation of state parameter, p_{de} is pressure of the dark energy and ρ_{de} is the DE density. The skewness parameters are δ, γ and η deviated from ω_{de} along x, y and z axes respectively and when vanish identically, the DE pressure becomes isotropic.

The Einstein's field equations (2) for the metric (1) using equations of (4) and (5) are given by

$$\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}+\frac{\dot{B}\dot{C}}{BC} = \lambda - p - (\omega_{de}+\delta)\rho_{de}, \quad (6)$$
$$\frac{\ddot{A}}{A}+\frac{\ddot{C}}{C}+\frac{\dot{A}\dot{C}}{AC} = -p - (\omega_{de}+\gamma)\rho_{de} \quad (7)$$
$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB} - \frac{1}{A^2} = -p - (\omega_{de}+\eta)\rho_{de} \quad (8)$$
$$\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{A}\dot{C}}{AC}+\frac{\dot{B}\dot{C}}{BC} - \frac{1}{A^2} = \rho + \rho_{de}, \quad (9)$$
$$\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = 0 \quad (10)$$

where an over dot represents the differentiation with respect to cosmic time *t*.
By integrate (10) and ignore the constant of integration, we get

$$A = B. \quad (11)$$

The energy conservation equation is given as $T^{ij}_{;j} = 0$ and it can be defined as

$$\dot{\rho} + 3(\rho+p)H - \lambda H_x + \dot{\rho}_{de} + 3\rho_{de}(\omega_{de}+1)H + \rho_{de}\left(\delta H_x + \gamma H_y + \eta H_z\right) = 0. \quad (12)$$

where H is Hubble's parameter, H_x is directional Hubble's parameter and dot (.) shows derivative with respect to cosmic time *t*.
All sources (cosmic string and DE) will preserve the energy-momentum tensors separately. We get two separate equations from the equation because they are non-interacting fluids (12),

$$\dot{\rho} + 3(\rho+p)H - \lambda H_x = 0, \quad (13)$$

and

$$\dot{\rho}_{de} + 3\rho_{de}(\omega_{de}+1)H + \rho_{de}\left(\delta H_x + \gamma H_y + \eta H_z\right) = 0. \quad (14)$$

We take,

$$\lambda = \xi\rho, \quad (15)$$

$$p = \omega\rho, \quad (16)$$

where ξ and ω are assumed to be non-evolving state parameters.
The spatial volume *III* and the scale factor a for the space-time (1) are defined by

$$V = a^3 = ABC. \quad (17)$$

The physical quantities of observational interest in cosmology such as the average Hubble parameter H, scalar expansion θ, shear scalar σ^2 and the average anisotropic parameter A_m defined as follows

$$H = \frac{\dot{a}}{a} = \left(\frac{1}{3}\right) * \left(\frac{\dot{V}}{V}\right) = \frac{1}{3}\left(H_x + H_y + H_z\right) \quad (18)$$

$$\theta = 3H \quad (19)$$

$$\sigma^2 = \frac{1}{2}\left(\sum_{i=x}^{z} H_i^2 - \frac{\theta^2}{3}\right) = \frac{1}{2}\left(H_x^2 + H_y^2 + H_z^2 - \frac{\theta^2}{3}\right) \quad (20)$$

$$A_m = \left(\frac{1}{3}\right)\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2 \quad (21)$$

where $\Delta H_i = H_i - H$, and $H_i = H_x,\ H_y$ and H_z are directional Hubble parameters in the directions of x, y and z respectively.

Once the cosmic expansion behavior is known, it becomes simpler to study the background cos- mology of Bianchi type *III* universe.

3 Solutions of the field equations and some properties of the model

To solve the system of field equations (6)-(10) completely we need some extra conditions. First, we assume the factor of hybrid scale factor Akarsu et al. [64]. In order to solve this inconsistent systems completely we need more constraints based on the following relations. Furthermore, we believe that the average scale factor is an expanding function of time. Due to the prevailing universe's outcomes, the best preferred for the average scale factors are the de Sitter solution and the power law expansion. The comportment of the power-law and exponential regulation of the average scale factor contribute to a continuous deceleration parameter. Also, the particular selection of scale factors determines whether the directional factor is time-dependent. In the present work, we have considered the specific scale factor, the hybrid average scale factor which at late time results into a constant deceleration parameter. The hybrid scale factor has two factors in the form,

$$a = (t^n e^t)^{\frac{1}{k}}, \quad (22)$$

where $k > 0$ and $n \geq 0$ are constants. Among this merged form, the one form is progressively expand- ing, and the other is an expansion of power law. the power-law (t^n) dominated, the cosmic dynamics in the early phase. Furthermore, the hybrid scale factor aspect element gradually draws away to a later stage of evolution in a more dominant way that could mimic the pace of universe expansion. When $n = 0$, the exponential law recovered. Thus, using a hybrid scale factor, a cosmic transition from early deceleration to late time acceleration can be achieved. This type of ansatz for the scale factor has already been considered by Pradhan et al. [19], Santhi et al. [57], Pradhan and Amirhashchi [58]. Hy- brid expansion law is commonly known as a combination of the exponential and power-law which is
represented in equation(22) and this relation gives a deceleration time-dependent parameter describ- ing the

Universe's transition from early deceleration to the current accelerating phase. Therefore the selection of the average factor is reasonable physically.

Secondly, we take the expansion scalar θ in the model to be proportional to the shear scalar σ this condition leads to (Collins et al. [59])

$$A = C^s, \tag{23}$$

where $s > 0$ and $s \neq 1$ is constant. The model is anisotropic if $s \neq 1$ and isotropic if $s = 1$. Thus, An anisotropic model is of significance to us and any assessment of the s except $s = 1$ is measured.

Using equations (17), (22), (23), we obtained

$$A = B = (t^n e^t)^{\frac{3s}{k(2s+1)}} \tag{24}$$

$$C = (t^n e^t)^{\frac{3}{k(2s+1)}} \tag{25}$$

Therefore the metric (1) with the help of equations (24), and (25) can now be written as

$$ds^2 = dt^2 - (t^n e^t)^{\frac{6s}{(2s+1)k}} dx^2 - (t^n e^t)^{\frac{6s}{(2s+1)k}} e^{2x} dy^2 - (t^n e^t)^{\frac{6}{(2s+1)k}} dz^2. \tag{26}$$

Equation (26) represents Bianchi type-*III* cosmological model.

The directional Hubble's parameters for Bianchi type-*III* model become

$$H_x = H_y = \frac{\dot{A}}{A} = \frac{3s}{k(2s+1)}\left(\frac{n}{t} + 1\right), \quad H_z = \frac{\dot{C}}{C} = \frac{3}{k(2s+1)}\left(\frac{n}{t} + 1\right). \tag{27}$$

The shear scalar σ^2

$$\sigma^2 = \frac{3}{k^2(2s+1)^2}\left(\frac{n}{t} + 1\right)^2 (s-1)^2. \tag{28}$$

The average an anisotropic parameter A_m

$$A_m = 2\left(\frac{s-1}{2s+1}\right)^2. \tag{29}$$

(29)

The ratio of shear scalar (σ) to expansion scalar (θ), which gives

$$\frac{\sigma^2}{\theta^2} = \frac{1}{3}\frac{(s-1)^2}{(2s+1)^2}. \tag{30}$$

It follows from Eq.(28) that for $s \neq 1$, we have $\frac{\sigma^2}{\theta^2} \neq 0$. This implies our model is anisotropic

with time t. As the growth of cosmic time t, all of these quantities decrease identically and vanish for large values of cosmic time t.

The dark energy density (ρ_{de}) obtained from equation (9) as

$$\rho_{de} = \frac{9s}{k^2}\left(\frac{n}{t}+1\right)^2\left(\frac{s+2}{(2s+1)^2}\right)-\left(t^n e^t\right)^{\frac{-6s}{k(2s+1)}}-\rho_0\left(t^n e^t\right)^{\frac{-3}{k}\left(1+\omega-\frac{\xi}{3}\right)}. \tag{31}$$

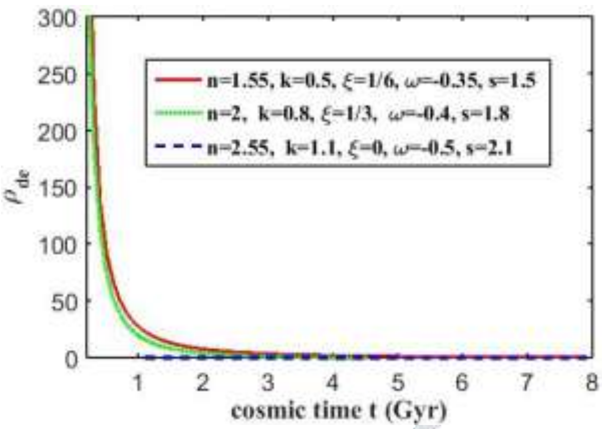

Figure 1: **P**lot of dark energy density (ρ_{de}) versus cosmic time t for $\rho_0 = 0.2$ in Bianchi type-*III* model.

From equation (31), we can see that the dark energy density (ρ_{de}) decreases and converges to a constant as cosmic time t approaches to infinity. in Bianchi type-*III* model dark energy density ρ_{de} versus cosmic time t for $\rho_0 = 0.2$ is ploted in figure 1. It is decided that alike to ρ_{de} of Bianchi type- *III*, decreasing function of time and leads to a small positive values in late time of the universe. We can see that dominance of dark energy density in late time of the cosmos in excess of tension density of the string and proper energy density ρ which are in accept with recent observational data of the universe.

Using equation (27) we get

$$\eta = -(\delta+\gamma)s. \tag{32}$$

Using equations (11), (24), (25), (31) and (32), we get the skewness parameters as

$$\delta = \frac{\frac{9\left(\frac{n}{t}+1\right)^2}{k^2(2s+1)^2}(2s^2-s-1)-\frac{3n(s-1)}{k(2s+1)t^2}-\left(t^n e^t\right)^{\frac{-6s}{k(2s+1)}}+(s+1)\xi\rho_0\left(t^n e^t\right)^{\frac{-3}{k}\left(1+\omega-\frac{\xi}{3}\right)}}{(2s+1)\left(\frac{9s}{k^2}\left(\frac{n}{t}+1\right)^2\left(\frac{s+2}{(2s+1)^2}\right)-\left(t^n e^t\right)^{\frac{-6s}{k(2s+1)}}-\rho_0\left(t^n e^t\right)^{\frac{-3}{k}\left(1+\omega-\frac{\xi}{3}\right)}\right)}. \tag{33}$$

$$\gamma = \frac{\frac{9\left(\frac{n}{t}+1\right)^2}{k^2(2s+1)^2}(2s^2-s-1)-\frac{3n(s-1)}{k(2s+1)t^2}-\left(t^n e^t\right)^{\frac{-6s}{k(2s+1)}}-s\xi\rho_0\left(t^n e^t\right)^{\frac{-3}{k}\left(1+\omega-\frac{\xi}{3}\right)}}{(2s+1)\left(\frac{9s}{k^2}\left(\frac{n}{t}+1\right)^2\left(\frac{s+2}{(2s+1)^2}\right)-\left(t^n e^t\right)^{\frac{-6s}{k(2s+1)}}-\rho_0\left(t^n e^t\right)^{\frac{-3}{k}\left(1+\omega-\frac{\xi}{3}\right)}\right)}. \tag{34}$$

$$\eta = \left[\frac{-2\left(\frac{9\left(\frac{n}{t}+1\right)^2}{k^2(2s+1)^2}(2s^2-s-1)-\frac{3n(s-1)}{k(2s+1)t^2}-\left(t^n e^t\right)^{\frac{-6s}{k(2s+1)}}\right)-\xi\rho_0\left(t^n e^t\right)^{\frac{-3}{k}\left(1+\omega-\frac{\xi}{3}\right)}}{(2s+1)\left(\frac{9s}{k^2}\left(\frac{n}{t}+1\right)^2\left(\frac{s+2}{(2s+1)^2}\right)-\left(t^n e^t\right)^{\frac{-6s}{k(2s+1)}}-\rho_0\left(t^n e^t\right)^{\frac{-3}{k}\left(1+\omega-\frac{\xi}{3}\right)}\right)}\right]s. \tag{35}$$

where, $\rho_{de} = \frac{9s}{k^2}\left(\frac{n}{t}+1\right)^2\left(\frac{s+2}{(2s+1)^2}\right)-\left(t^n e^t\right)^{\frac{-6s}{k(2s+1)}}-\rho_0\left(t^n e^t\right)^{\frac{-3}{k}\left(1+\omega-\frac{\xi}{3}\right)}$

From equation (7), we get the equation of state parameter of dark energy (ω_{de}) as

$$\omega_{de} = \frac{\left(\frac{-9}{k^2}\left(\frac{n}{t}+1\right)^2+\frac{6n}{kt^2}\right)\frac{s(s+2)}{(2s+1)}+\left(t^n e^t\right)^{\frac{-6s}{k(2s+1)}}-\left((2s+1)\omega-s\xi\right)\rho_0\left(t^n e^t\right)^{\frac{-3}{k}\left(1+\omega-\frac{\xi}{3}\right)}}{(2s+1)\left(\frac{9s}{k^2}\left(\frac{n}{t}+1\right)^2\left(\frac{s+2}{(2s+1)^2}\right)-\left(t^n e^t\right)^{\frac{-6s}{k(2s+1)}}-\rho_0\left(t^n e^t\right)^{\frac{-3}{k}\left(1+\omega-\frac{\xi}{3}\right)}\right)}. \quad (36)$$

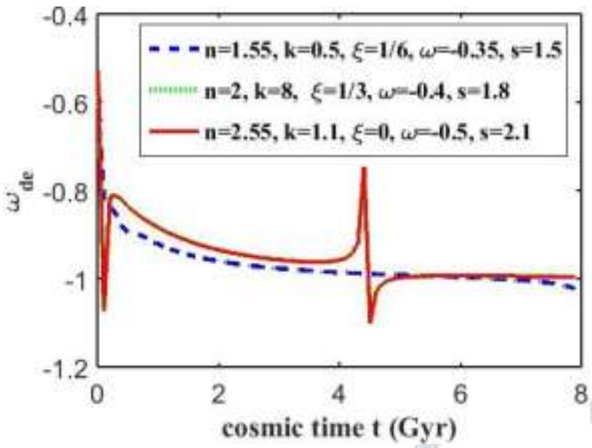

Figure 2: **P**lot of EoS parameter versus cosmic time t of Bianchi type-*III* model for $\rho_0 = 0.2$.

From Skewness parameters, It could be said that the behaviors of pressure anisotropies continue nearly unchanged in the late phase of enlargement in the existence of cosmic strings. Although the cosmic strings display their appearance dominantly at an early stage. We also investigated the impact of anisotropy parameter s on particular conditions pressure anisotropies to the behavior of the Skewness parameters of Bianchi type-*III*. Figure 2 displays the EoS parameter plot against the time t of Bianchi type-*III* for different values of parameters. Figure 2 concluded that for $\xi = \frac{1}{6}$, the DE of EoS starts from quintessence region and finally approaches to ΛCDM. But for $\xi = 0$ and $\xi = \frac{1}{3}$ starts in quintessence region goes to phantom regions and vibrating between the two and finally approaches to ΛCDM at late times. Hence the EoS parameter assmble to -1 i.e; $\omega_{de} \simeq -1$ With time t reaching infinity.
Density parameter of the cosmic string fluid Ω_{cs} and the density parameter of DE fluid Ω_{de} is given by

$$\Omega_{de} = \frac{3s(s+2)}{(2s+1)^2} - \frac{k^2}{3\left(\frac{n}{t}+1\right)^2 \left(t^n e^t\right)^{\frac{6s}{(2s+1)k}}} - \frac{k^2\rho_0}{3\left(\frac{n}{t}+1\right)^2}\left(t^n e^t\right)^{\frac{-3}{k}(1+\omega+\xi)}. \quad (37)$$

The total average density parameter is given by

$$\Omega = \frac{3s(s+2)}{(2s+1)^2} - \frac{k^2}{3\left(\frac{n}{t}+1\right)^2 \left(t^n e^t\right)^{\frac{6s}{(2s+1)k}}}. \quad (38)$$

From equation (38), we observed that the total average density parameter Ω is a function of cosmic time t. It means arriving to a constant worth as $t \to \infty$ and free of the parameter ξ. It is consistent with the recent observations by appropriate choices of parameters s for flatness of the universe.

Some other important features of the models

3.1 Statefinders parameters

The accelerated expansion phenomenon of the universe proposed by many dark energy models. To see the reliability of these models, Sahni et al. [60] have proposed the state-finders parameters (r, s). The plane equivalent to these parameters is astral $r-s$ plane and It exhibit the distance between a given dark energy model and the ΛCDM limit. A cosmological plane of these parameters explains known regions of the universe. These take in (r, s)=(1, 0) corresponds to the ΛCDM limit, (r, s)=(1, 1) represents the CDM limit, $s > 0$ and $r < 1$ gives the phantom and quintessence dark energy eras, and $s < 0$ and $r > 1$ show chaplygin gas.

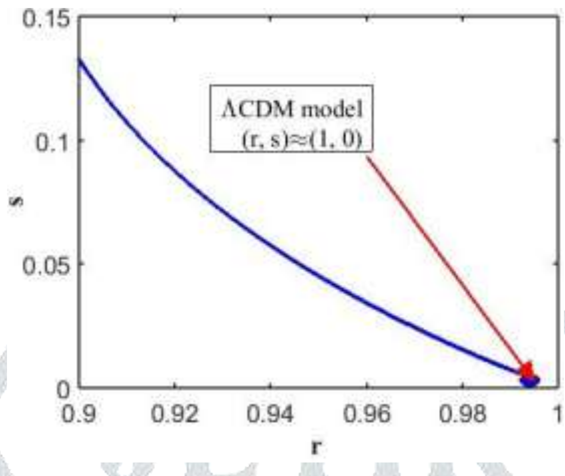

Figure 3: **P**lot of $r-s$ plane for $n = 1.55$ and $k = 0.5$.

The statefinders parameters for our models are

$$\begin{aligned} r &= \frac{\dddot{a}}{aH^3}, \\ &= 1+\frac{2nk^2}{(n+t)^3}-\frac{3nk}{(n+t)^2}. \\ s &= \frac{r-1}{3\left(q-\frac{1}{2}\right)}, \\ &= \frac{2\left(2nk^2-3nk(n+t)\right)}{3(n+t)(2nk-3(n+t)^2)}. \end{aligned} \quad (39)\ \&\ (40)$$

You can obtain the parameters of the state-finders in the plane by plotting r versus s as shown in figure 3. As the $r-s$ plane for the three models is shown to possess the regions of the quintessence and phantom regions. We also note that at late times, our models conform to the average ΛCDM limit.

3.2 Deceleration parameter

Deceleration parameter q and the redshift parameter (z) for all the three models are given by

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = \frac{d}{dt}\left(\frac{1}{H}\right) - 1 = -1 + \frac{nk}{(n+t)^2}, \quad (41)$$

$$z = -1 + \frac{a_0}{a} = -1 + a_0(t^n e^t)^{\frac{-1}{k}}, \quad (42)$$

here a_0 is the present value of the scale factor at $z = 0$.

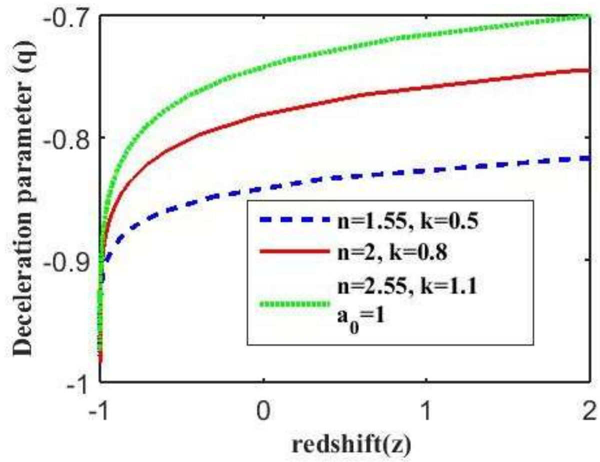

Figure 4: **P**lot of deceleration parameter (q) versus redshift (z) for the three models

A estimate of the rate at which the universe expansion occurs by the DP. The $-ve$ value of q indicate inflation (Riess et al. [61]; Bennett et al. [62]) and it implies that the universe has accelerated expansion, while positive value of q indicates the standard decelerating model. From Eq. (41), we noticed that $q > 0$ for $t < \sqrt{nk-n}$ and $q < 0$ for $t > \sqrt{nk-n}$. The value of deceleration parameter lies in $-1 \leq q < 0$ and Recent observations of SNe Ia, expose that the present Universe is accelerating. Figure 4 depicts the behavior of the deceleration parameter against Redshift (z) for different chosen values of k and n. From figure 4 it is observed that the values of the deceleration parameter $q < 0$ for small values of z indicating that the universe appears to be expanding in accelerating rate at present epoch and late times. the current value of DP reaches the value -1 at late times in our models and it matches with the observed value $q_0 \approx -0.73$ (Cunha et al. [63]). Consequently, our derived models are in good agreement with recent modern cosmology observations.

4 Conclusion

In this paper, we have studied anisotropic dark energy cosmological models in Bianchi type-III space- time in the presence of one-dimensional cosmic strings within the framework of general relativity. By assuming a hybrid scale factor, we obtained exact solutions to the Einstein field equations and ana- lyzed the physical and geometrical behavior of the model. The analysis shows that both the proper energy density and string tension density are dominant in the early universe and decrease with cos- mic time, eventually vanishing at late times. This indicates that cosmic strings play a significant role during the early stages of cosmic evolution but become negligible in the late universe. In contrast, dark energy dominates the late-time dynamics, driving the accelerated expansion. The anisotropy parameter depends on the model parameter and vanishes for a specific choice, leading to an isotropic universe. The skewness parameters reveal that pressure anisotropies persist but remain nearly con- stant during the late-time evolution. The equation of state parameter evolves dynamically, showing quintessence and phantom behaviors in the early phase and converging to the cosmological constant value ($\omega_{de} \approx -1$) at late times, indicating consistency with observational data. The statefinder di- agnostic (r, s) confirms that the model approaches the ΛCDM scenario in the late-time limit. The deceleration parameter exhibits a transition from deceleration to acceleration, supporting the current understanding of cosmic expansion. Additionally, the total density parameter tends to unity at late times, implying a flat universe in agreement with recent observations. Overall, the present model successfully describes the transition of the universe from an anisotropic early phase dominated by cosmic strings to a dark energy dominated accelerated phase, and it provides a consistent framework compatible with modern cosmological observations.

References

[1] Peebles, P.J.E.,Ratra, B.: Rev. Mod. Phys. **75**, 559 (2003). astro-ph/0207347.

[2] Frieman, J. A., Hill, C. T., Stebbins, A., Waga, I.: Phys. Rev. Lett. 75, 2077 (1995). astro-ph/9505060.

[3] Carroll, S.M.: Living Rev. Relativ. **4**, 1 (2001)

[4] Copeland, E.J., Sami, M., Tsujikawa, S.: Int. J. Mod. Phys. D **15**, 1753 (2006)

[5] Wang, B., Abdalla, E., Atrio-Barandela, F., Pavon, D.: Rep. Prog. Phys.**79**(9), 096901 (2016)

[6] Boughn, P., Cheng, E.S., Wilkinson, D.T.: Astrophys. J. **243**, L113 (1981)

[7] Akarsu, O., Kilinc, C.B.: Gen. Relativ. Gravit. **42**, 119 (2010)

[8] Smoot, G.F., et al.: Astrophys. J.**396**, L1 (1992)

[9] Bennet, C.L., et al.: Astrophys. J. **464**, L1-L4 (1996). arXiv:astro-ph/9601067

[10] Hinshaw, G., et al.: Astrophys. J. Suppl. Ser. **148**, 135 (2003).arXiv:astro-ph/0302217v1

[11] Hinshaw, G., et al.: Astrophys. J. Suppl. Ser. **180**, 225 (2009). arXiv:0803.0732 [astro-ph]

[12] Nolta, M.R., et al.: Astrophys. J. Suppl. Ser. **180**, 296 (2009). arXiv:0803.0593 [astro-ph]

[13] Eriksen, H.K., et al.: Astrophys. J. Suppl. Ser. **180**, 225 (2009). arXiv:0803.0732 [astro-ph]

[14] Jain, P., Sarala, S.: J. Astrophys. Astr. **27**, 443 (2006)

[15] Ralston, J.P., Jain, P.: Int. J. Mod. Phys. D**8**, 537 (1999)

[16] Letelier, P.S.: Phys. Rev. D **28**, 2414 (1983)

[17] Mishra, B., Tripathy, S.K., Ray, P.P.: Arxiv preprint (2017). arXiv:1701.08632

[18] Stachel, J.: Phys. Rev. D **21**, 2171 (1980)

[19] Pradhan, A., Saha, B., Rikhvitsky, V.: Indian J. Phys. **89**, 503 (2015)

[20] Tripathy, S.K., Nayak, S.K., Sahu, S.K., Routray, T.R.: Int. J. Theor. Phys. **48**, 213 (2009a)

[21] Tripathy, S.K., Nayak, S.K., Sahu, S.K., Routray, T.R.: Astrophys. Space Sci. **323**, 91 (2009b)

[22] Buiny, R., Berera, A., Kephart, T.W.: Phys. Rev. D **73**, 063529(2006)

[23] Tripathy, S.K.: Astrophys. Space Sci. **350**, 367 (2014)

[24] Ade, P.A.R., et al. (Planck Collaboration): Astron. Astrophys. **571**, A16 (2014)

[25] Ade, P.A.R., et al. (Planck Collaboration): Astron. Astrophys. **594**, A16 (2016)

[26] Quartin, M., Notari, A.: J. Cosmol. Astropart. Phys. 2015(01), 008 (2015).arXiv:1408.5792

[27] Armendariz-Picon, C.: J. Cosmol. Astropart. Phys. 0407, 007 (2004)

[28] Jaffe, T.R., Banday, A.J., Eriksen, H.K., Gorski, K.M., Hansen, F.K.: Astrophys. J. **629**, L1 (2005)

[29] Cooray, A., Holz, D.E., Caldwell, R.: J. Cosmol. Astropart. Phys. **1011**, 015 (2010)

[30] Cooke, R., Lynden-Bell, D.: Mon. Not. R. Astron. Soc. **401**, 1409 (2010)

[31] Colin, J., Mohayaee, R., Sarkar, S., Shafieloo, A.: Mon. Not. R. Astron. Soc. **414**, 264 (2011)

[32] Salehi, A., Aftabi, S.: J. High Energy Phys. **2016**(09), 140 (2016)

[33] Campanelli, L., Cea, P., Tedesco, L.: Phys. Rev. Lett. **97**, 131302 (2006)

[34] Campanelli, L.: Phys. Rev. D **80**, 063006 (2009)

[35] Gruppuso, A.: Phys. Rev. D **76**, 083010 (2007)

[36] Koivisto, T., Mota, D.F.: Astrophys. J. **679**, 1 (2008b)

[37] Koivisto, T., Mota, D.F.: Phys. Rev. D **73**, 083502 (2006)

[38] Koivisto, T., Mota, D.F.: J. Cosmol. Astropart. Phys. **0806**, 018 (2008a)

[39] Saadeh, D., Feeny, S.M., Pontzen, A., Peiris, H.V., McEwen, J.D.: Phys. Rev. Lett. **117**, 131302 (2016)

[40] Battye, R., Moss, A.: Phys. Rev. D 80, 023531 (2009)

[41] Bunn, E.F., Ferreira, P.G., Silk, J.: Phys. Rev. Lett. **77**, 2883 (2016)

[42] Akarsu, O., Kilinc, C.B.: Gen. Relativ. Gravit. **42**, 119 (2010a)

[43] Akarsu, O., Kilinc, C.B.: Gen. Relativ. Gravit. **42**, 763 (2010b)

[44] Watanabe, M., Kanno, S., Soda, J.: Phys. Rev. Lett. **102**, 191302(2009)

[45] Mishra, B., Sahoo, P.K., Tripathy, S.K.: Astrophys. Space Sci. **356**, 163(2015)

[46] Mishra, B., Tripathy, S.K.: Mod. Phys. Lett. A **30**, 1550175 (2015)

[47] Tripathy, S.K., Mishra, B., Sahoo, P.K.: Eur. Phys. J. Plus **132**, 388 (2017).arXiv:1503.05109v1

[48] Tripathy, S.K., Sahu, S.K., Routray, T.R.: Astrophys. Space Sci. **315**, 105 (2008)

[49] Tripathy, S.K., Nayak, S.K., Sahu, S.K., Routray, T.R.: Astrophys.Space Sci. **323**, 281 (2009c)

[50] Kibble, T.W.B.: J. Phys. A, Math. Gen. 9, 1378 (1976)

[51] Zel'dovich, Ya.B.: Mon. Not. R. Astron. Soc. 192, 663 (1980)

[52] Saha, B., Visinescu, M.: Astrophys. Space Sci. **315**, 99 (2008)

[53] Reddy, D.R.K.: Astrophys. Space Sci.286, 365(2003)

[54] Reddy, D.R.K., Rao, M.V.S.: Astrophys. Space Sci.302, 157 (2006a)

[55] Reddy, D.R.K., Rao, M.V.S.: Astrophys. Space Sci. 305, 183 (2006b)

[56] Tripathy, S. K., Mishra, B., Sahoo, P.K. arXiv:1503.05109v1 [General Relativity-qc], 2015.

[57] Santhi, M. Vijaya, Aditya, Y., Rao, V.U.M.: Astrophys Space Sci. **142**, 361(2016)

[58] Pradhan, A., Amirhashchi, H.: Mod. Phys. Lett. A**26**, 2261 (2011)

[59] Collins, C.B., Glass, E.N., Wilkinson, D.A.: Gen. Relativ. Gravit. **12**, 805 (1980)

[60] Sahni, V., et al.: J. Exp. Theor. Phys. Lett. **77**, 201 (2003)

[61] Riess, A.G., et al.: Astron. J. **116**, 1009 (1998)

[62] Bennett, C.L., et al.: Astrophys. J. Suppl. Ser. **148**, 1 (2003)

[63] Cunha, C.E., Lima, M., Ogaizu, H., Frieman, J., Lin, H.: Mon. Not. R. Astron. Soc. **396**, 2379 (2009)

[64] Akarsu, O., Kumar, S., Myrzakulov, R., Sami, M., Xu, L.: J. Cosmol.Astropart. Phys. **01**, 022 (2014)