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Slightly m^*-open and $g^*\gamma$-open maps

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Abstract

In this paper, we define and study a concepts of slightly $g^*\gamma$ –open maps and slightly m^*-open maps using $g^*\gamma$ –open and m^*-open sets due to Navalagi et.al.[4]. Also we characterize their basic properties.

Keywords: $g^*\gamma$ –open sets, m^*-open sets, slightly m^*-open maps, slightly $g^*\gamma$ –open maps, clopen maps, strongly m^*-open maps, always $g^*\gamma$ –open maps.

1. Introduction

In 1986, D. Andrajevic[1] had defined and studied the concepts of semipreopen and semipreclosed sets in topological spaces. In 1997, A.A.El.Atic [2], has introduced and studied the concept of γ-open and γ-closed sets. Recently in 2019[5], a new class of sets called $g^*\gamma$ – open and m^*-open sets were introduced. Using these sets, we defined $g^*\gamma$ –open maps and m^*-open maps and studied some of their properties. The purpose of this paper is to study of slightly $g^*\gamma$ –open maps and slightly m^*-open maps.

2. Preliminaries

In this paper, X, Y, Z always means a topological spaces on which no separation axioms are assumed. Unless otherwise mentioned.
For a subset A of X, Cl(A) and Int(A) represents the closure of A and interior of A respectively.

The following definitions and results are useful in the sequel:
Definition 2.1: Consider $A\subset X$. Then,the intersection of all semipreclosed sets containing A is called semipreclosure[1] of A and is denoted by spCl(A).

Definition 2.2: Consider $A\subset X$. Then the union of all semipreopen sets contained in A is called semipre-interior[1] and is denoted by spInt(A).

Definition 2.3: Let X be a topological space. A subset A is called semipreopen[1] set if $A \subset Cl(Int(Cl(A)))$.

Definition 2.4: Let X be a topological space. A subset A is called γ-open[2] set if $A \subset Cl(Int(A)) \cup Int(Cl(A))$.
The complement of γ-open set is γ-closed[2] set.

Definition 2.5 : A subset A of a space X is termed as m^* -closed set[4] if $spCl(A) \subset U$ whenever $A \subset U$ and U is γ -open set in X.

Definition 2.6: A subset A of a space X is termed as m^*-open[4] set if $F \subset sp\,Int(A)$ whenever $F \subset A$ and F is γ-closed set in X.

Definition 2.7: A subset A of a space X is termed as generalized star γ-closed[3] (in brief, $g^*\gamma$-closed) set if $Cl(A) \subset U$ whenever $A \subset U$ and U is γ -open set in X.

Definition 2.8: A subset A of a space X is termed as generalized star γ-open[3] (in brief, $g^*\gamma$ -open) set if $F \subset Int(A)$ whenever $F \subset A$ and F is γ-closed set in X.

Definition 2.9: Map $i : X \to Y$ is termed as $g^*\gamma$ -open[3] if the image of open set of X is $g^*\gamma$ -open in Y.

Definition 2.10: Map $i : X \to Y$ is termed as always $g^*\gamma$ -open[3], if the image of each $g^*\gamma$ -open set of X is $g^*\gamma$ -open in Y.

Definition 2.11: Map $i: X \to Y$ is termed as strongly $g^*\gamma$ -open[3], if the image of each $g^*\gamma$ -open set of X is open in Y.

Definition 2.12: Map $i : X \to Y$ is termed as a m^*-open[5] if the image of open set of X is m^*-open set in Y.

Definition 2.13: Map i: $X \rightarrow Y$ is termed as always m^*-open[5], if the image of each m^*-open set of X is m^*-open in Y.

Definition 2.14: Map i: $X \rightarrow Y$ is termed as strongly m^*-open[5], if the image of each m^*-open set of X is open in Y.

Definition 2.15: Map i:$X \rightarrow Y$ is termed as clopen[4], if the image of each open set of X is clopen set in Y.

Definition 2.16: Map i:$X \rightarrow Y$ is termed as slightly open[5], if the image of each clopen set of X is open set in Y.

Definition 2.17: Map i:$X \rightarrow Y$ is termed as always clopen[5], if the image of each clopen set of X is clopen set in Y.

3. On slightly m^*-open and $g^*\gamma$-open maps

Definition 3.1: Map i:$X \rightarrow Y$ is termed as slightly m^*-open, if the image of each clopen set of X is m^*-open set in Y.

Theorem 3.2: The below listed results are identical for a map i:$X \rightarrow Y$:

(i) i is slightly m^*-open.

(ii) $\forall$ clopen set $V \subset X$, i(V) is m^*-open in Y.

(iii) $\forall$ clopen set $V \subset X$, i(V) is m^*-closed in Y.

(iv) $\forall$ clopen set $V \subset X$, i(V) is m^*-clopen in Y.

Proof: (i)$\Rightarrow$(ii): Let V be a clopen subset of X. Then by (i), i(V) is m^*-open set in Y.

(ii) $\Rightarrow$ (iii): Let V be a clopen subset of X. Then (X-V) is clopen. By (ii), i(X-V)=i(X)-i(V)=Y- i(V) is m^*-open in Y. Then i(V) is m^*-closed.

(iii)$\Rightarrow$(iv): It can be proved easily.

(iv)$\Rightarrow$ (i): Let V be a clopen subset of X. Then by (iv), i(V) is m^*-clopen set in Y. But every m^*-clopen set is m^*-open set, then i(V) is m^*-open set in Y. Hence, i is slightly m^*-open.

Theorem 3.3: Let i:$X \rightarrow Y$ and j:$Y \rightarrow Z$ be maps. The below listed results are true:

(i) Whenever i being slightly m^*-open and j being always m^*-open, $j \circ i$: $X \rightarrow Z$ is slightly m^*-open map.

(ii) Whenever i being slightly open and j being m^*-open, $j \circ i$: $X \rightarrow Z$ is slightly m^*-open map.

(iii) Whenever i being clopen and j being slightly m^*-open, $j \circ i$: $X \rightarrow Z$ is m^*-open map.

(iv) Whenever i being slightly m^*-open and j being strongly m^*-open, $j \circ i$: $X \rightarrow Z$ is slightly open map.

(v) If i is always clopen and j being slightly m^*-open, $j \circ i$: $X \rightarrow Z$ is slightly m^*-open map.

Proof: (i) Let H be a clopen subset of X. Since, i is slightly m^*-open map, i(H) is m^*-open set in Y. Again, j is always m^*-open and i(H) is m^*-open set in Y, then $j(i(H))=(j \circ i)(H)$ is m^*-open set in Z. This shows that $j \circ i$ is slightly m^*-open map.

(ii), (iii), (iv) and (v) are similar to prove.

Definition 3.4: Map $i:X \rightarrow Y$ is termed as slightly $g^*\gamma$-open, if the image of each clopen set of X is $g^*\gamma$-open set in Y.

Definition 3.5: Let $A \subset X$ is termed as $g^*\gamma$-clopen, if it is $g^*\gamma$-open as well as $g^*\gamma$-closed.

Theorem 3.6: The below listed results are identical for a map $i:X \rightarrow Y$:

(i) i is slightly $g^*\gamma$ -open.

(ii) $\forall$ clopen set $V \subset X$, i(V) is $g^*\gamma$ -open in Y.

(iii) $\forall$ clopen set $V \subset X$, i(V) is $g^*\gamma$ -closed in Y.

(iv) $\forall$ clopen set $V \subset X$, i(V) is $g^*\gamma$ -clopen in Y.

Proof: (i)$\Rightarrow$(ii): Let V be a clopen subset of X. Then by (i), i(V) is $g^*\gamma$ -open set in Y.

(ii) $\Rightarrow$ (iii): Let V be a clopen subset of X. Then (X-V) is clopen. By (ii), i(X-V)=i(X)-i(V)=Y-i(V) is $g^*\gamma$ -open in Y. Then i(V) is $g^*\gamma$ -closed.

(iii)$\Rightarrow$(iv): It can be proved easily.

(iv)$\Rightarrow$ (i): Let V be a clopen subset of X. Then by (iv), i(V) is $g^*\gamma$ -clopen set in Y. But every $g^*\gamma$ -clopen set is $g^*\gamma$ -open set, then i(V) is $g^*\gamma$ -open set in Y. Hence, i is slightly $g^*\gamma$ -open.

Theorem 3.7: Let $i:X \rightarrow Y$ and $j:Y \rightarrow Z$ be maps. The below listed results are true:

(i) Whenever i being slightly $g^*\gamma$ -open and j being always $g^*\gamma$ -open, $j \circ i: X \rightarrow Z$ is slightly $g^*\gamma$ -open map.

(ii) Whenever i being slightly open and j being $g^*\gamma$ -open, $j \circ i: X \rightarrow Z$ is slightly $g^*\gamma$ - open map.

(iii) Whenever i being clopen and j being slightly $g^*\gamma$ -open, $j \circ i: X \rightarrow Z$ is $g^*\gamma$ -open map.

(iv) Whenever i being slightly $g^*\gamma$ -open and j being strongly $g^*\gamma$ -open, $j \circ i: X \rightarrow Z$ is slightly open map.

(v) Whenever i being always clopen and j being slightly $g^*\gamma$ –open, $j \circ i: X \rightarrow Z$ is slightly $g^*\gamma$ -open map.

Proof: It holds from the definitions.

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