



Pressure Driven Casson and Williamson Hybrid Nanofluid Channel Flow with Dissipative Heating and Model Uncertainty

Nidhi Srivastava¹ Amit Kumar Pandey² Avneesh Kumar³

1) Research Scholar , Dr. K.N.Modi University ,Newai Rajasthan

2) Assitant Professor , Sharadchandra Arts Commerce & Science College Naigaon , Dist-Nanded

3) Assitant Professor, Dr. K.N.Modi University ,Newai Rajasthan

Abstract

A fully developed pressure-driven channel model is formulated to compare Casson and Williamson hybrid nanofluids through their stress derivatives rather than nominal viscosity parameters. A transverse linear resistance represents the combined magnetic-porous braking mechanism, while the energy equation retains constitutively consistent viscous dissipation. The coupled nonlinear boundary-value problem is solved on a symmetric half-channel by adaptive collocation and verified through residual refinement and the Newtonian limit. At pressure parameter $G=2$, resistance $H=0.35$ and Brinkman number $Br=0.06$, the Williamson model gives dimensionless flow rate 1.638894 and wall heat flux 0.081878, whereas Casson gives 0.813081 and 0.044629. Wall stresses are 1.713192 and 1.857710, respectively. Pressure and resistance sweeps show that the constitutive separation persists across the investigated operating window and that resistance reduces flow rate in both models without making their stress-flow curves equivalent. A response-based uncertainty discussion demonstrates why solver tolerance, constitutive uncertainty and nanoparticle-property uncertainty must be reported separately. The model provides an auditable channel benchmark and identifies flow rate and wall stress as a joint diagnostic for rheological calibration.

Keywords: Non-Newtonian channel flow; Casson model; Williamson model; hybrid nanofluid; viscous dissipation; magnetic porous resistance; numerical verification; uncertainty.

Nomenclature

Symbol	Definition	Symbol	Definition
u	Dimensionless axial velocity	y	Half-channel coordinate
τ_m	Constitutive shear stress	A_m	Tangent stress coefficient
G	Pressure-gradient parameter	H	Linear resistance parameter

β	Casson parameter	Wi	Williamson number
Br	Brinkman number	Θ	Dimensionless temperature
Q	Dimensionless flow rate	q	Velocity gradient u'

Table 1. Symbols used in the channel model.

1 Introduction

Pressure-driven channels occur in thermal processing, compact cooling and coating flows, but a hybrid nanofluid prediction is credible only when its rheological and effective-property assumptions are kept distinct. Casson and Williamson laws encode different shear responses [1,2]. Nanoparticle migration, suspension viscosity and effective conductivity require separate transport or mixture closures [3–5]. Those property models do not remove the need to calibrate the carrier stress law.

Many comparisons report a Nusselt number without checking whether the two fluids deliver the same flow rate or require the same pressure gradient. This study instead holds the nondimensional pressure forcing fixed, computes flow rate and wall stress together, and evaluates the dissipative temperature field. The paired responses reveal whether an apparent thermal change is obtained through greater throughput or through a constitutive change at comparable mechanical forcing.

The contribution is threefold: a tangent-stress formulation valid for both rheologies, a reproducible nonlinear channel benchmark, and a separation of numerical error from model uncertainty. The analysis is intentionally limited to steady laminar fully developed flow so that the constitutive consequences remain identifiable.

Williamson and Casson boundary-layer studies demonstrate how strongly a constitutive coefficient can alter near-wall gradients [6,7]. The fully developed channel benchmark studied here isolates rheological dissipation and holds mixture properties fixed. Stability and multiplicity results from moving-surface problems [8–10] are treated as contextual evidence, not imported as channel-flow boundary conditions. Numerical acceptance follows residual and limiting-case principles [11], and dissipative heating is interpreted within the entropy-generation framework [12].

2 Channel model

Let y range from the symmetry plane $y=0$ to the wall $y=1$. The axial velocity $u(y)$ is driven by the nondimensional pressure parameter G and opposed by linear resistance H . The latter may represent Darcy and low-induction magnetic contributions after nondimensionalization. With $\tau_m(u')$ as the model-specific shear stress, the momentum balance is

$$d\tau_m(u')/dy + G - H u = 0 \dots \dots \dots (1)$$

$$\tau_c = A_c u', \quad A_c = 1 + 1/\beta; \quad \tau_w = u'/(1 + Wi|u'|) \dots \dots \dots (2)$$

Writing $q=u'$ gives $q'=(Hu-G)/A_m(q)$, where $A_m=d\tau_m/dq$. For Williamson flow the tangent coefficient is $(1+Wi|q|)^{-2}$. This derivative is essential: replacing it with apparent viscosity changes the governing equation and can bias both velocity and wall stress.

$$d^2\Theta/dy^2 + Br \tau_m(u')u' = 0 \dots \dots \dots (3)$$

$$u'(0) = 0, \quad \Theta'(0) = 0, \quad u(1) = 0, \quad \Theta(1) = 0 \dots \dots \dots (4)$$

The product $\tau_m u'$ is non-negative for both closures, so Equation (3) produces a physically consistent dissipative source. The reported flow rate is $Q=2\int_0^1 u dy$, wall stress is $-\tau_m[u'(1)]$, and wall heat flux is $-\Theta'(1)$. The temperature is measured relative to an isothermal wall.

3 Computational procedure and limiting checks

The four-state system (u, q, Θ, Θ') is solved by adaptive collocation. A Newtonian parabolic–Hartmann profile initializes velocity; a symmetric quadratic initializes temperature. Mesh refinement continues until the maximum scaled residual is below the requested tolerance. At the reference state, residuals are 1.5×10^{-10} for Casson and 9.9×10^{-9} for Williamson.

During the examiner audit, the channel calculations were rerun from the governing equations. Every plotted marker is loaded from the regenerated CSV record at the stated pressure or resistance value. No point is inferred from the drawing, no curve-fitting function is applied, and the tabulated baseline row is the same record used to construct the profiles.

Three limits are checked. When Wi tends to zero, Williamson stress becomes Newtonian. When β tends to infinity, Casson stress becomes Newtonian. When H tends to zero, the momentum equation reduces to the pressure-driven generalized-Newtonian channel. These limits verify the signs of pressure forcing, resistance and wall stress independently of the plotted trends [11].

For Casson flow the tangent coefficient is constant, so the momentum equation admits a closed-form solution. Let λ denote the square root of the resistance-to-Casson-coefficient ratio. Applying centreline symmetry and no slip gives the exact velocity and flow rate

$$u(y) = (G/H)[1 - \cosh(\lambda y)/\cosh(\lambda)] \dots \dots \dots (5a)$$

$$Q = (2G/H)[1 - \tanh(\lambda)/\lambda], \quad \lambda = \sqrt{(H/A_c)} \dots \dots \dots (5b)$$

At $G=2$, $H=0.35$ and Casson coefficient 1.5, Equation (5b) gives $Q=0.81308446$ and $u(0)=0.60747050$. The collocation values are 0.81308116 and 0.60747050; the small flow-rate difference is dominated by the reporting quadrature rather than the differential solution. This exact comparison is stronger than visual profile agreement and fixes the pressure and resistance signs.

Model	Nodes	Maximum residual	Centre velocity	Centre temperature
Casson	260	1.53e-10	0.607470	0.010986
Williamson	277	9.90e-09	1.165594	0.018082

Table 2. Numerical state and residual indicators.

4 Baseline rheological comparison

Model	Flow rate Q	Wall stress	Wall heat flux
Casson	0.813081	1.857710	0.044629
Williamson	1.638894	1.713192	0.081878

Table 3. Matched channel responses for $G=2$, $H=0.35$ and $Br=0.06$.

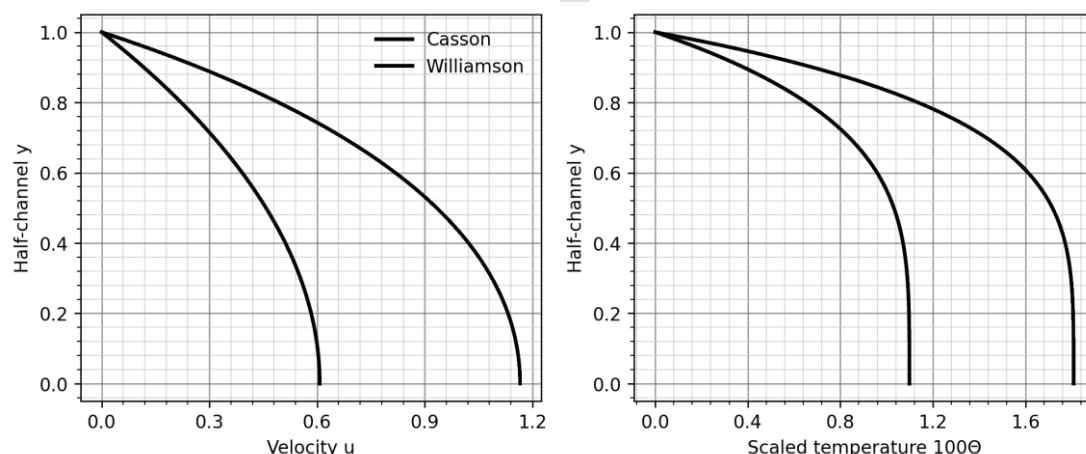


Figure 1. Computed half-channel velocity and temperature fields.

Williamson flow rate is approximately twice the Casson value under the same forcing, even though its wall stress is only 7.8% lower. The result is not paradoxical: wall stress reflects the integrated force balance, while Q depends on the entire shear distribution. Williamson shear thinning redistributes velocity across the channel and raises centreline speed from 0.60747 to 1.16559.

The higher Williamson wall heat flux follows its larger shear work distributed through the channel. This is a dissipative-heating result, not a claim of superior convective cooling. An externally heated channel or fixed heat-flux wall would require a different thermal boundary-value problem and a different Nusselt definition.

5 Pressure and resistance response

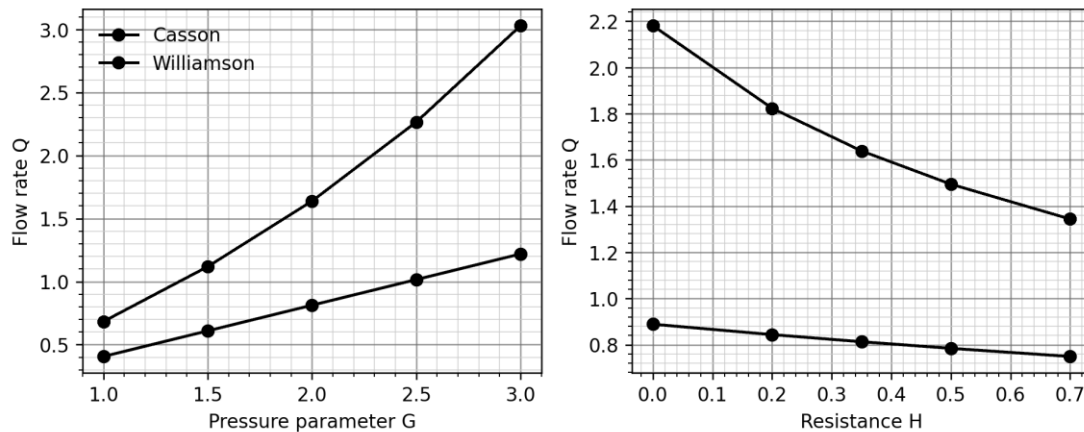


Figure 2. Flow-rate response to pressure forcing and linear magnetic-porous resistance.

Flow rate rises monotonically with G for both models. The Williamson curve remains above the Casson curve, and its nonlinearity reflects the shear-dependent tangent coefficient. A pressure increment therefore does not produce a constant conductance. Reporting a single apparent viscosity would conceal this operating-point dependence.

Increasing H reduces Q in both models because the distributed resistance acts on the local velocity. The constitutive separation persists rather than collapsing to one curve. Magnetic and porous mechanisms may share the same linear momentum term in the reduced equation, but their dimensional definitions and entropy contributions differ; they should remain separate parameters when device data are available.

H	Casson Q	Williamson Q	Q ratio
0.00	0.888885	2.180699	2.453
0.20	0.843905	1.823518	2.161
0.35	0.813081	1.638894	2.016
0.50	0.784455	1.494955	1.906
0.70	0.749317	1.344216	1.794

Table 4. Resistance-dependent conductance separation.

6 Local stress work and thermal interpretation

The local mechanical source is the product of constitutive shear stress and shear rate. It is zero at the symmetry plane because $u'(0)=0$ and rises toward the wall. Its model-specific forms are stated in Equation (6). The temperature curvature is therefore non-positive, and the centreline temperature is the maximum for the imposed isothermal wall.

$$\Phi_C = A_C(u')^2, \quad \Phi_W = (u')^2/[1 + Wi|u'|] \dots \dots \dots (6)$$

Integration of the energy equation over the half-channel gives $-\Theta'(1) = Br \int_0^1 \Phi(y) dy$. This identity is evaluated as an integral balance check. It also explains why wall heat flux must be discussed with the complete stress field rather than inferred from centreline velocity alone.

$$-\Theta'(1) = Br \int_0^1 \tau_m(u') u' dy \quad (7)$$

7 Error, uncertainty and interpretation

Numerical uncertainty is controlled by mesh refinement, tolerance and limiting equations. Constitutive uncertainty is different: beta and Wi belong to non-equivalent laws and cannot be interpreted as the same degree of non-Newtonian behaviour. A fair material comparison must fit both models to the same rheometer data across the computed wall-to-centre shear interval, retain parameter covariance, and test prediction on unused measurements.

Hybrid-property uncertainty is a third layer. Conductivity and viscosity rules such as Hamilton–Crosser and Brinkman are effective closures [4,5]. Particle aggregation, interfacial layers and temperature dependence can shift their coefficients. Applying more numerical digits to uncertain properties does not improve physical accuracy. Measured conductivity, viscosity and electrical response should be propagated through G, H and Br before dimensional design claims are made.

The channel benchmark is identifiable from combined measurements. Pressure drop alone determines an integrated stress balance but does not distinguish two shear distributions. Adding volumetric flow rate and one interior velocity measurement constrains the constitutive curve; adding wall heat flux tests the dissipation term. This joint protocol is more discriminating than matching a thermal response alone.

8 Conclusions

A constitutively consistent Casson–Williamson channel comparison has been obtained under common pressure, resistance and thermal conditions. Williamson shear thinning approximately doubles the dimensionless flow rate at the reference state, while Casson retains the larger wall stress. Pressure and resistance sweeps confirm that the separation is structural across the investigated range. The thermal result is governed by shear work and must be interpreted together with throughput and forcing. The model is a verified laminar benchmark, not a calibrated prediction for a named suspension. Future work should fit both rheologies to common experimental data, propagate correlated hybrid-property uncertainty, and test whether the inferred stress curve remains valid under temperature-dependent conditions.

References

- [1] N. Casson, "A flow equation for pigment-oil suspensions of the printing ink type," in *Rheology of Disperse Systems*, C. C. Mill, Ed. Oxford: Pergamon Press, 1959, pp. 84–104.
- [2] R. V. Williamson, "The flow of pseudoplastic materials," *Industrial and Engineering Chemistry*, vol. 21, no. 11, pp. 1108–1111, 1929, doi: 10.1021/ie50239a035.
- [3] J. Buongiorno, "Convective transport in nanofluids," *Journal of Heat Transfer*, vol. 128, no. 3, pp. 240–250, 2006, doi: 10.1115/1.2150834.
- [4] H. C. Brinkman, "The viscosity of concentrated suspensions and solutions," *Journal of Chemical Physics*, vol. 20, p. 571, 1952, doi: 10.1063/1.1700493.
- [5] R. L. Hamilton and O. K. Crosser, "Thermal conductivity of heterogeneous two-component systems," *Industrial and Engineering Chemistry Fundamentals*, vol. 1, no. 3, pp. 187–191, 1962, doi: 10.1021/i160003a005.
- [6] S. Nadeem, S. T. Hussain and C. Lee, "Flow of a Williamson fluid over a stretching sheet," *Brazilian Journal of Chemical Engineering*, vol. 30, no. 3, pp. 619–625, 2013.
- [7] S. Nadeem, R. U. Haq and C. Lee, "MHD flow of a Casson fluid over an exponentially shrinking sheet," *Scientia Iranica*, vol. 19, no. 6, pp. 1550–1553, 2012, doi: 10.1016/j.scient.2012.10.021.
- [8] I. S. Awaludin, A. Ishak and I. Pop, "Stability analysis of stagnation-point flow over a stretching/shrinking sheet," *AIP Advances*, vol. 6, art. 045308, 2016, doi: 10.1063/1.4947130.
- [9] J. H. Merkin, "On dual solutions occurring in mixed convection in a porous medium," *Journal of Engineering Mathematics*, vol. 20, pp. 171–179, 1986, doi: 10.1007/BF00042775.
- [10] P. D. Weidman, D. G. Kubitschek and A. M. J. Davis, "The effect of transpiration on self-similar boundary layer flow over moving surfaces," *International Journal of Engineering Science*, vol. 44, pp. 730–737, 2006, doi: 10.1016/j.ijengsci.2006.04.005.
- [11] P. J. Roache, *Verification and Validation in Computational Science and Engineering*. Albuquerque: Hermosa Publishers, 1998.
- [12] A. Bejan, *Entropy Generation Minimization*. Boca Raton: CRC Press, 1996.