LAPLACIAN ENERGY OF BILPOLAR FUZZY GRAPH

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Abstract-In this paper the concept of LaplacianEnergy of Intuitionistic Fuzzy Graph is extended to LaplacianEnergy of Bipolar Fuzzy Graph. Moreover we defined the adjacency matrix of bipolar fuzzy graph and hence introduced the concept of LaplacianEnergy for Bipolar Fuzzy Graph is studied.

Keywords:Energy of a graph, Laplacian Energy, Intuitionistic Fuzzy Graph, Bipolar Fuzzy Graphs.
Subject Classification:05C50, 05C72, 05C35

1.Introduction:

Fuzzy set[1] has developed a potential area of interdisciplinary research. The idea of a fuzzy graph connection was characterized by Zadeh[9]and it has found applications in the analysis of cluster patterns. The spectrum of a graph initially showed up in a paper by Collatz also, Sinogowitz in 1957 [8]. The idea of fuzzy graph was presented by Rosenfeld [2] in 1975 . He also introduce the structure of fuzzy graph by thre relation between the sets in fuzzy,also getting analogs of a few graph theoretical concepts. Followed by him Bhattacharya [3] gave some new ideas in fuzzy graph. Mordeson and Peng introduced some operation on fuzzy graph. Collatz also, Sinogowitz in 1957 [8] given the concept of spectrum of graph.The energy of bipolar fuzzy graph of adjacency matrix G is the sum of absolute value of eigenvalues.

The spectrum of Laplacian matrix L(G)consisting of the numbers $\{\zeta_1,\zeta_2,...,\zeta_n\}$ in the Laplacian spectrum of the graph G.The sum of average degree d(G) of G and the distances of the Laplacian eigenvalues of G is equal to the laplacian energy of bipolar fuzzy G. some essential fuzzy graph theoretic ideas and applications have been demonstrated, numerous creators discovered further outcomes and fuzzy analogs of numerous other graph theoretic ideas. In this paper we are concerned about basic graphs.If G is a (n,m) graph were n vertices and m edges of the graph G.

In this paper we present the idea of Laplacian energy of bipolar fuzzy graph.

Section 2 consists of preliminaries and definition of laplacian energy of fuzzy graph and in section 3, we present the Laplacian energy of a bipolar fuzzy graph and in section 4 , result were discussed.

2. Preliminaries

Definition 2.1: Fuzzy set: [4] Let X be a nonempty set. A fuzzy set A in X is defined as

A = { (x, $\mu_A(x)$)/x $\in$ X} which is characterized by a membership function $\mu_A(x)$:X→[0,1] and a fuzzy set satisfying the following condition, $\mu_A(x)+\gamma_A(x)=1$ where $\gamma_A(x)=1-\mu_A(x)$ is the non membership function .

Definition 2.2: [5] A fuzzy graph with V as the underlying set is a pair of functions G = (σ, μ) where σ : V→[0,1] is a fuzzy subset and μ : V x V→[0,1] is a symmetric fuzzy relation on the fuzzy subset σ for all u,v∈V such that μ(u,v)≤ σ(u) Λ σ(v). The underlying crisp graph $\tilde{G}=(\sigma,\mu)$ of is denoted by G = (V, E) where E ⊆V x V. A fuzzy relation can also be expressed by a matrix called fuzzy relation matrix $M=[a_{ij}]$ where $a_{ij}=\mu(u_i,u_j)$ Throughout this paper, we suppose $\tilde{G}$ is undirected without loops and σ(u)=1 for each u∈V.

Definition 2.3: An edge whose end points are the same is called a loop. A graph without loops and parallel edges is called a simple graph. Two vertices that are connected by an edge is called adjacent. The adjacency matrix $A=[a_{ij}]$ for a graph G = (V, E) is a matrix with n rows and n columns, n = |V| and its entries defined by

$$a_{ij}=\begin{cases}1 \; if \; (v_i,v_j)\in E\\ 0 \; otherwise\end{cases}$$

Definition 2.4: Let A(G) be a adjacency matrix and $D(G)=[d_{ij}]$be a degree matrix of $\tilde{G}=(\sigma,\mu)$ The matrix L(G)=D(G)-A(G) is defined as fuzzy laplacian matrix of graph G.

Definition 2.5: [6] Let X be a nonempty set. A bipolar fuzzy set A in X is an object having the form $A=\{(x,\mu_A^P(x),\mu_A^N(x)/x\in X)\}$

Where $\mu_A^P(x):X\to[0,1]$ and $\mu_A^N(x):X\to[-1,0]$ are mappings.

We use the positive membership degree $\mu_A^P(x)$ to denote the satisfaction degree of an element x to the property corresponding to a bipolar fuzzy set A and the negative membership degree $\mu_A^N(x)$ to denote the satisfaction degree of an element x to some implicit counter property corresponding to a bipolar fuzzy set B . If $\mu_A^P(x) \neq 0$ and $\mu_A^N(x) = 0$, it is the situation that x is regarded as having only positive satisfaction for A. If $\mu_A^P(x) = 0$ and $\mu_A^N(x) \neq 0$, it is the situation that x does not satisfy the property of A but somewhat satisfies the counter property of A. It is possible for an element x to be such that $\mu_A^P(x) \neq 0$ and $\mu_A^N(x) \neq 0$ when the membership function of the property overlaps that of its counter property over some portion of X.

3.LAPLACIAN ENERGY OF BIPOLAR FUZZY GRAPH:

Definition 3.1: [7] A bipolar fuzzy graph G=(V,A,B) is a nonempty set V together with a pair of functions $A = (\mu_A^P, \mu_A^N) : V \to [0,1] \times [-1,0]$ and $B = (\mu_B^P, \mu_B^N) : V \times V \to [0,1] \times [-1,0]$ such that for all $x, y \in V$,

$$\mu_B^P(x,y) \leq \min\left(\mu_A^P(x), \mu_A^P(y)\right)$$

$$\mu_B^N(x,y) \geq \max\left(\mu_A^N(x), \mu_A^N(y)\right)$$

Definition 3.2: The bipolar fuzzy graph is defined as the adjacency matrix. That is for an bipolar fuzzy graph $G = (V, E, \mu^P, \mu^N)$ an bipolar fuzzy adjacency matrix is defined by A(BG)=$[a_{ij}]$ where $a_{ij}(\mu_{ij}^P, \mu_{ij}^N)$.

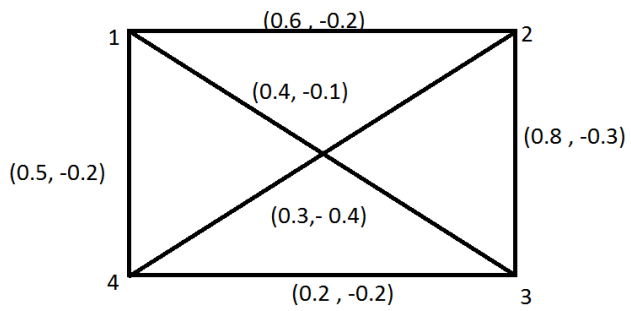

Fig:1

For the Figure 1

$$A(BG) = \begin{bmatrix} 0 & (0.6,-0.2) & (0.4,-0.1) & (0.5,-0.2) \\ (0.6,-0.2) & 0 & (0.8,-0.3) & (0.3,-0.4) \\ (0.4,-0.1) & (0.8,-0.3) & 0 & (0.2,-0.2) \\ (0.5,-0.2) & (0.3,-0.4) & (0.2,-0.2) & 0 \end{bmatrix}$$

Definition 3.3: The Bipolar fuzzy graph of adjacency matrix can be split in to two matrices such as μ_{ij}^P and μ_{ij}^N , where μ_{ij}^P denote membership of positive values , μ_{ij}^N denote membership of negative values.

$$A(BG) = \left[\left(\mu_{ij}^P\right), \left(\mu_{ij}^N\right)\right]$$

where

$$A\left(\mu_{ij}^P\right) = \begin{bmatrix} 0 & 0.6 & 0.4 & 0.5 \\ 0.6 & 0 & 0.8 & 0.3 \\ 0.4 & 0.8 & 0 & 0.2 \\ 0.5 & 0.3 & 0.2 & 0 \end{bmatrix}$$

and

$$A\left(\mu_{ij}^N\right) = \begin{bmatrix} 0 & -0.2 & -0.1 & -0.2 \\ -0.2 & 0 & -0.3 & -0.4 \\ -0.1 & -0.3 & 0 & -0.2 \\ -0.2 & -0.4 & -0.2 & 0 \end{bmatrix}$$

Definition 3.4: The adjacency matrix of bipolar fuzzy graph the eigen value is defined in two ways one is adjacency matrix of positive membership values $A(\mu_{ij}^P)$ and other is adjacency matrix of negative membership values $A(\mu_{ij}^N)$.

Definition 3.5: In the bipolar fuzzy graph the Laplacian matrix is defined as the difference between the degree and adjacency matrix, denoted as L(BG)=D(BG)-A(BG)Laplacian matrix of bipolar fuzzy graph can be written as $L(\mu_{ij}^P)$ and $L(\mu_{ij}^N)$, where $L(\mu_{ij}^P)$ denote the positive membership values and $L(\mu_{ij}^N)$ denote the negative membership values.

$$L(BG)=\left[\left(L\left(\mu_{ij}^P\right)\right),\left(L\left(\mu_{ij}^N\right)\right)\right]$$

Example 3.2: The laplacian matrix of bipolar fuzzy graph the membership values are given as

$$L\left(\mu_{ij}^P\right)=\begin{bmatrix} 1.5 & -0.6 & -0.4 & -0.5 \\ -0.6 & 1.7 & -0.8 & -0.3 \\ -0.4 & -0.8 & 1.4 & -0.2 \\ -0.5 & -0.3 & -0.2 & 1 \end{bmatrix} L\left(\mu_{ij}^N\right)=\begin{bmatrix} -0.5 & 0.2 & 0.1 & 0.2 \\ 0.2 & -0.9 & 0.3 & 0.4 \\ 0.1 & 0.3 & -0.6 & 0.2 \\ 0.2 & 0.4 & 0.2 & -0.8 \end{bmatrix}$$

Definition 3.6: Let $BG=(V,E,\mu^P,\mu^N)$ and laplacian matrix of bipolar fuzzy graph is L(BG).In the Bipolar fuzzy graph the laplacian polynomial is the characteristic polynomial of its matrix $\phi(BG,\tilde{\mu})=\det(\tilde{\mu}I_n-L(BG))$ .The roots of $\phi(BG,\tilde{\mu})$ is the bipolar fuzzy laplacianeigen values of bipolar graph.

Theorem 3.1: Let $\tilde{G}=(V,E,\mu^P,\mu^N)$ be the bipolar fuzzy graph with $|V|=n$ vertices and is the laplacianeigen values of me $\tilde{\mu}_1\geq\tilde{\mu}_2\geq\ldots\geq\tilde{\mu}_n$ mbership values of bipolar fuzzy graph then,

$$1.\sum_{i=1}^{n}\tilde{\lambda}_i^2=2\sum_{1\leq i\leq j\leq n}\mu_{ij}^P$$

$$2.\sum_{i=1}^{n}\tilde{\lambda}_{i\ i}^2=2\sum_{1\leq i\leq j\leq n}\left(\mu_{ij}^P\right)^2+\sum_{i=1}^{n}d^2_{\mu_{ij}^P(\tilde{G})}(u_i)$$

Proof:

1,Sine $\tilde{L}\left\lfloor\mu_{ij}(\tilde{G})\right\rfloor$ is symmetric matrix and these laplacian Eigen values are non negative such that

$$tr\left(L\left(\mu_{ij}^P\left(\tilde{G}\right)\right)\right)=\sum_{i=1}^{n}d_{\tilde{G}}(u_{ij})=2\sum_{1\leq i\leq j\leq n}\mu_{ij}^P$$

2, By the definition of laplacian matrix,

$$\begin{bmatrix} d_{\tilde{G}}(u_1) & . & . & . & -\mu(u_1,u_n) \\ . & . & & & . \\ . & & . & & . \\ . & & & . & . \\ -\mu(u_n,u_1) & . & . & . & d_{\tilde{G}}(u_n) \end{bmatrix}$$

Then we obtain

$$tr\left(\tilde{L}_{i,i}^2\right)=\left\lfloor d^2_{\mu_{ij}^P(\tilde{G})}(u_1)+\mu^2(u_1,u_2)+\ldots+\mu^2(u_1,u_n)\right\rfloor+\ldots+\left\lfloor\mu^2(u_1,u_n)+\ldots+d^2_{\mu_{ij}^P(\tilde{G})}(u_n)\right\rfloor$$

$$=2\sum_{1\leq i\leq j\leq n}\left(\mu_{ij}^P\right)^2+\sum_{i=1}^{n}d^2_{\mu_{ij}^P(\tilde{G})}(u_i)$$

Then the laplacian matrix of bipolar fuzzy , the non negative membership function can be similarly proved

$$1.\sum_{i=1}^{n}\delta_i^2=2\sum_{1\leq i\leq j\leq n}\mu_{ij}^N$$

$$2.\sum_{i=1}^{n}\delta_i^2=2\sum_{1\leq i\leq j\leq n}\left(\mu_{ij}^N\right)^2+\sum_{i=1}^{n}d^2_{\mu_{ij}^N(\tilde{G})}(u_i)$$

Where $\delta_1\geq\delta_2\geq\ldots\geq\delta_n$ $\delta_1\geq\delta_2\geq\ldots\ldots\geq\delta_n$ is the laplacianeigen values of membership values of bipolar fuzzy graph.

Definition 3.7: In Bipolar fuzzy graph the Laplacian energy is defined as $\left\lfloor LE\left(\mu_{ij}^P\left(\tilde{G}_1\right)\right),LE\left(\mu_{ij}^N\left(\tilde{G}_1\right)\right)\right\rfloor$

$$LE\left(\mu_{ij}^{P}\left(\tilde{G}\right)\right)=\left|\lambda_i-\frac{2\sum_{1\le i\le j\le n}\mu\left(u_i,u_j\right)}{n}\right|$$

Similarly for $[LE(\mu_{ij}^{N}(\tilde{G}_1))]$

Example:3.3

The laplacian energy of Bipolar fuzzy graph for Fig:1

$$Spec\left(L\left(\mu_{ij}^{p}\left(\tilde{G}_1\right)\right)\right)=\left\{2.4297,1.9395,1.2308,4.8849\times10^{-15}\right\}$$

$$Spec\left(L\left(\mu_{ij}^{N}\left(\tilde{G}_1\right)\right)\right)=\left\{-1.2702,-0.9,-0.6298,1.1102\times10^{-16}\right\}$$

$$LE\left(\mu_{ij}^{P}\left(\tilde{G}_1\right)\right)=3.1384$$

$$LE\left(\mu_{ij}^{N}\left(\tilde{G}_1\right)\right)=1.5403$$

The laplacian energy of bipolar fuzzy graph is [3.1384 , 1.5403].

4. Results

Theorem:4.1 Let $\tilde{G}=(V,E,\mu^P,\mu^N)$ be an bipolar fuzzy graph with $|V|=n$ and $|E|=m$ and $L(BG)=\left[\left(L\left(\mu_{ij}^{P}\right)\right),\left(L\left(\mu_{ij}^{N}\right)\right)\right]$ be bipolar fuzzy laplacian matrix of $\tilde{G}$ then

$$1.\,LE\left(\mu_{ij}^{P}\left(\tilde{G}\right)\right)\le\sqrt{\left(2\sum_{1\le i\le j\le n}\left(\mu_{ij}^{P}\right)^2+\sum_{i=1}^{n}\left(d_{\mu_{ij}^{P}(\tilde{G})}(u_i)-\frac{2\sum_{1\le i\le j\le n}u_{ij}}{n}\right)^2\right)n}$$

$$2.\,LE\left(\mu_{ij}^{N}\left(\tilde{G}\right)\right)\le\sqrt{\left(2\sum_{1\le i\le j\le n}\left(\mu_{ij}^{N}\right)^2+\sum_{i=1}^{n}\left(d_{\mu_{ij}^{N}(\tilde{G})}(u_i)-\frac{2\sum_{1\le i\le j\le n}u_{ij}}{n}\right)^2\right)n}$$

Proof: Apply Cauchy-schwarz inequality to (1,…,1) and $\left(|\zeta_1|,|\zeta_2|,\ldots,|\zeta_n|\right)$, we get

$$\left|\sum_{i=1}^{n}\zeta_i\right|^2\le n\sum_{i=1}^{n}|\zeta_i|^2 \text{ where ,}$$

$$LE\left(\mu_{ij}^{P}\left(\tilde{G}\right)\right)\le\sqrt{n\sum_{i=1}^{n}|\zeta_i|^2}=\sqrt{2Rn}$$

Since

$$R=\sum_{1\le i\le j\le n}\left(\mu_{ij}^{P}\right)^2+\sum_{i=1}^{n}\left(d_{\mu_{ij}^{P}(\tilde{G})}(u_i)-\frac{2\sum_{1\le i\le j\le n}u_{ij}}{n}\right)^2 \text{ then}$$

$$LE\left(\mu_{ij}^{P}\left(\tilde{G}\right)\right)\le\sqrt{\left(2\sum_{1\le i\le j\le n}\left(\mu_{ij}^{P}\right)^2+\sum_{i=1}^{n}\left(d_{\mu_{ij}^{P}(\tilde{G})}(u_i)-\frac{2\sum_{1\le i\le j\le n}u_{ij}}{n}\right)^2\right)n}$$

Similarly, we can prove

$$LE\left(\mu_{ij}^{N}\left(\tilde{G}\right)\right)\le\sqrt{\left(2\sum_{1\le i\le j\le n}\left(\mu_{ij}^{N}\right)^2+\sum_{i=1}^{n}\left(d_{\mu_{ij}^{N}(\tilde{G})}(u_i)-\frac{2\sum_{1\le i\le j\le n}u_{ij}}{n}\right)^2\right)n}$$

Theorem:4.2 Let $\tilde{G}=(V,E,\mu^P,\mu^N)$ be an bipolar fuzzy graph and $L(BG)=\left[\left(L(\mu_{ij}^P)\right),\left(L(\mu_{ij}^N)\right)\right]$ be bipolar fuzzy laplacian matrix of $\tilde{G}$ then

$$1.\ LE\left(\mu_{ij}^P(\tilde{G})\right)\geq 2\sqrt{\left(\sum_{1\leq i\leq j\leq n}(\mu_{ij}^P)^2+\sum_{i=1}^{n}\left(d_{\mu_{ij}^P(\tilde{G})}(u_i)-\frac{2\sum_{1\leq i\leq j\leq n}u_{ij}}{n}\right)^2\right)}$$

$$2.\ LE\left(\mu_{ij}^N(\tilde{G})\right)\geq 2\sqrt{\left(\sum_{1\leq i\leq j\leq n}(\mu_{ij}^N)^2+\sum_{i=1}^{n}\left(d_{\mu_{ij}^N(\tilde{G})}(u_i)-\frac{2\sum_{1\leq i\leq j\leq n}u_{ij}}{n}\right)^2\right)}$$

Proof: By the definition 3.7,

$$\left(LE\left(\mu_{ij}^P(\tilde{G})\right)\right)^2=\left|\sum_{i=1}^{n}\varsigma_i\right|^2+2\sum_{1\leq i\leq j\leq n}|\varsigma_i||\varsigma_j|\geq 4R$$

, then

$$LE\left(\mu_{ij}^P(\tilde{G})\right)\geq 2\sqrt{\left(\sum_{1\leq i\leq j\leq n}(\mu_{ij}^P)^2+\sum_{i=1}^{n}\left(d_{\mu_{ij}^P(\tilde{G})}(u_i)-\frac{2\sum_{1\leq i\leq j\leq n}u_{ij}}{n}\right)^2\right)}$$

Similarly, we can prove

$$LE\left(\mu_{ij}^N(\tilde{G})\right)\geq 2\sqrt{\left(\sum_{1\leq i\leq j\leq n}(\mu_{ij}^N)^2+\sum_{i=1}^{n}\left(d_{\mu_{ij}^N(\tilde{G})}(u_i)-\frac{2\sum_{1\leq i\leq j\leq n}u_{ij}}{n}\right)^2\right)}$$

Example:4.1 The bipolar fuzzy graph in Fig:1

$LE(\mu_{ij}^P(\tilde{G}_1))=3.1384$ and its lower bound = 2.5846 and upper bound =3.6551 . $LE(\mu_{ij}^N(\tilde{G}_1))=1.5403$ and its lower bound = 1.3115 and upper bound = 1.8547 .

5.Conclusion : In this paper ,the energy of bipolar fuzzy graph and Laplacian matrix are defined. Laplacian energy bounds is defined for bipolar fuzzy graph. A few outcomes on Laplacian spectra of Bipolar fuzzy graph may uncover more comparable to result of these kinds and will be talked about in the forward coming papers.

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