ON FUZZY ALMOST GENERALIZED $\tilde{b}$-CONTINUOUS MAPPINGS IN FUZZY TOPOLOGICAL SPACES

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Abstract In this paper, we introduce and characterize the concept of fuzzy almost generalized $\tilde{b}$-continuous mappings. Several interesting properties of these mappings are also given. Examples and counter examples are also given to illustrate the concepts introduced in the paper. We also introduce the concept of fuzzy $fT_{\frac{1}{2}}\tilde{b}$-space, fuzzy $g\tilde{b}$-space, fuzzy regular $g\tilde{b}$-space and fuzzy generalized $\tilde{b}$-compact space. It is seen that a fuzzy almost generalized $\tilde{b}$-continuous mapping from a fuzzy $fT_{\frac{1}{2}}\tilde{b}$-space to another fuzzy topological space becomes fuzzy almost continuous mapping.

Keywords and phrases: $fag\tilde{b}$-continuous, $fg\tilde{b}$-space, $frg\tilde{b}$-space, $fT_{\frac{1}{2}}\tilde{b}$-space.

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1. Introduction

The fuzzy concept has invaded almost all branches of Mathematics since the introduction of the concept of fuzzy sets by Zadeh [29]. Fuzzy sets have applications in many fields such as information [22] and control [23]. The theory of fuzzy topological spaces was introduced and developed by Chang [10] and since then various notions in classical topology have been extended to fuzzy topological spaces. In general topology a new class of generalized open sets in a topological spaces called b-open sets were introduced and studied by Andrijevic [2]. The class of b-open sets is contained in the class of semi-pre open sets and contains all semi-open sets and pre-open sets. In the fuzzy topological space, fuzzy b-open sets and fuzzy b-neighbourhoods were first introduced by Benchalli et.al [8,9] The notion of generalized closed sets in general topology is due to Normal Levine [16]. Later Balasubramanian and Sundaram [6] introduced fuzzy generalized closed sets. The concept of fuzzy almost continuous mapping [1], fuzzy completely continuous mapping [5], fuzzy δ-continuous [13] and fuzzy R mapping were introduced by Azad [1], Bhaumik[5] and Mukerjee [18] and Ganguly and Saha [13,14] respectively.

In this paper, we introduce a new class of mappings viz. fuzzy almost generalized $\tilde{b}$-continuous mapping and establish some of their properties. We also introduce fuzzy $fT_{\frac{1}{2}}b$-space, fuzzy generalized $\tilde{b}$-compact space, fuzzy $g\tilde{b}$ space, fuzzy regular $g\tilde{b}$ space. It is seen that a fuzzy almost generalized $\tilde{b}$-continuous surjection image of a fuzzy generalized $\tilde{b}$-compact space is fuzzy nearly compact.

2. Preliminaries

A family of fuzzy sets of X is called a fuzzy topology [10] on X if 0 and 1 belong to τ and τ is closed with respect to arbitrary union and finite intersection. The members of τ are called fuzzy open sets and their complements are fuzzy closed sets. Throughout this paper, (X,τ), (Y,σ), and (Z,η), (or simply X, Y and Z) mean fuzzy topological space (abbreviated as fts) on which no separation axioms are assumed unless otherwise mentioned. We denote the closure and interior of a fuzzy set λ in X by $Cl(\lambda)$ and $Int(\lambda)$, whereas the constant fuzzy sets taking on the values 0 and 1 on X are denoted by 0_X and 1_X respectively.

This section contains some basic definitions and preliminary results which will be needed in the sequel.

Definition 2.1 1 *[8] Let λ be a fuzzy set of a fts X. Then λ is called*

- a fuzzy b-open (fbo, in short) set of X if $\lambda \leq Cl(Int(\lambda)) \vee Int(Cl(\lambda))$.
- a fuzzy b-closed (fbc, in short) set of X if $Cl(Int(\lambda)) \wedge Int(Cl(\lambda)) \leq \lambda$.

Theorem 2.12 *[8] For a fuzzy set λ in a fts X,*

- λ is fbo $\Leftrightarrow$ $1-\lambda$ is fbc.
- λ is fbc $\Leftrightarrow$ $1-\lambda$ is fbo.

Lemma 2.13 *[8] In a fts X,*

- An arbitrary union of fuzzy b-open sets is a fuzzy b-open set.
- An arbitrary intersection of fuzzy b-closed sets is a fuzzy b-closed set.

Definition 2.24 *[8] Let (X,τ) be a fts. Let λ be a fuzzy set of a fts X.*

- $bInt(\lambda) = \vee \{\mu \in I^X : \mu \leq \lambda, \mu$ is a fbo set $\}$ is called the fuzzy b-interior of λ.
- $bCl(\lambda) = \wedge \{\mu \in I^X : \mu \geq \lambda, \mu$ is a fbc set $\}$ is called the fuzzy b-closure of λ.

Theorem 2.25 *[8]Let (X,τ) be a fts. Let λ be a fuzzy set of a fts X.*

- $bCl(1-\lambda) = 1 - bInt\lambda$.
- $bInt(1-\lambda) = 1 - bCl\lambda$.

Remark 2.16 *[8] Obviously, $bCl(\lambda)$ is the smallest fuzzy b-closed set which contains λ and $bInt(\lambda)$ is the largest fuzzy b-open set which is contained in λ. Also $bCl(\lambda) = \lambda$, for any fuzzy b-closed set λ and $bInt(\lambda) = \lambda$, for any fuzzy b-open set λ.*

Definition 2.37 *A fuzzy subset λ in an fts (X,τ) is called*

- fuzzy semi-open set [1] if $\lambda \leq ClInt(\lambda)$ and a fuzzy semi-closed set if $IntCl(\lambda) \leq \lambda$.
- fuzzy pre-open set [4] if $\lambda \leq IntCl(\lambda)$ and a pre-closed set if $ClInt(\lambda) \leq \lambda$.
- fuzzy regular open (fro, for short) [1] if $\lambda = IntCl(\lambda)$ and a regular closed set if $\lambda = ClInt(\lambda)$.
- fuzzy semi-pre-open [24] if $\lambda \leq ClIntCl(\lambda)$ and a fuzzy semi-pre-closed set if $IntClInt(\lambda) \leq \lambda$.

Theorem 2.38 *[8] In a fts X,*

- Every fpo set is fbo.
- Every fso set is fbo.
- Every fbo (resp. fbc) set is fspo (resp. fspc).

Lemma 2.29 *[8] In a fts, we have the following :*

- Every fuzzy regular open set is fuzzy open.
- Every fuzzy open set is fuzzy α-open.
- Every fuzzy α-open set is both fuzzy semi-open and fuzzy pre-open.
- Every fuzzy semi-open set is fuzzy semi-pre-open.
- Every fuzzy pre-open is fuzzy semi-pre-open.

Definition 2.410 *[28] A fuzzy subset λ in a fts X is called*

- fuzzy generalized $\tilde{b}$-closed (in short, $fg\tilde{b}c$) set iff $fCl(\lambda) \leq \mu$, whenever $\lambda \leq \mu$ and μ is fuzzy b-open set.
- fuzzy generalized $\tilde{b}$-open (in short, $fg\tilde{b}o$) set iff $\mu \leq fInt(\lambda)$ whenever $\mu \leq \lambda$ and μ is fuzzy b-closed set.

The complement of a fuzzy generalized $\tilde{b}$-closed set is called a fuzzy generalized $\tilde{b}$-open set.

Definition 2.5 11 *Let $f:(X,\tau_1) \to (Y,\tau_2)$ be a mapping from a fts (X,τ_1) to another (Y,τ_2). Then f is called*

- fuzzy continuous [10] if $f^{-1}(\lambda)$ is fuzzy open set in X for any fuzzy open set λ in Y.
- fuzzy b-continuous [8] if $f^{-1}(\lambda)$ is a fbo set in X for any fuzzy open set λ in Y.
- fuzzy b-irresolute [9] if $f^{-1}(\lambda)$ is a fbo set in X for any fbo set λ in Y.
- fuzzy b-open [8] if for every fbo set λ in X, $f(\lambda)$ is a fbo set in Y.
- fuzzy almost continuous [1] if $f^{-1}(\lambda)$ is fuzzy open set in X for any fuzzy regular open (fro, for short) set λ in Y.

Definition 2.6 12 [1] *A fuzzy subset λ in an fts X is called*

- quasi-coincident (q-coincident, for short) with a fuzzy point x_p (where x is the support and p, $0 < p \leq 1$, is the value of the point) iff $p + \lambda(x) > 1$.
- q-coincident with a fuzzy set μ, denoted by $\lambda q \mu$, iff $\exists\ x \in X$ such that $\lambda(x) + \mu(x) > 1$.
- q- neighbourhood (q- nbd, for short) of fuzzy point x_p iff $\exists$ a fuzzy open set μ such that $x_p q \mu \leq \lambda$.

Definition 2.713 [19] *A family Λ of fuzzy sets in a fuzzy space X is said to be a cover of fuzzy set μ of X if and only if $\mu \leq \vee \{\lambda_i : \lambda_i \in \Lambda\}$. Λ is called fuzzy open cover if each member λ_i is a fuzzy open set. A sub cover of Λ is a subfamily of Λ which is also a cover of μ.*

Definition 2.8 14 *[10] Let (X,τ) be a fts and let $\mu \in I^X$. μ is said to be a fuzzy compact set if for every fuzzy open cover of μ has a finite subcover of μ.*

Definition 2.915 *A fts (X,τ) is said to be*

- a fuzzy nearly compact [14] iff every fuzzy regular open cover of X has a finite subcover.
- a fuzzy Hausdorff [15] if for every pair of fuzzy points x_p, y_q in X with distinct supports, $\exists$ open fuzzy sets U and V such that $x_p \in U$, $y_q \in V$ and $U \wedge V = 0_X$.

3.Fuzzy almost generalized $\tilde{b}$-continuous mappings

Now, we introduce the following definitions.

Definition 3.116 *Let $f:(X,\tau_1) \to (Y,\tau_2)$ be a mapping from a fts (X,τ_1) to another (Y,τ_2). Then f is called a fuzzy generalized $\tilde{b}$ (in short, $fg\tilde{b}$)-continuous mapping iff $f^{-1}(\lambda)$ is $fg\tilde{b}o$ set in X for every fuzzy open set λ in Y.*

Definition 3.2 17 *An fts (X,τ) is said to be $fT_{\frac{1}{2}}\tilde{b}$-space if every $fg\tilde{b}c$ set in X is fuzzy closed.*

Definition 3.318 *A mapping f from an fts (X,τ_1) to another fts (Y,τ_2) is said to be fuzzy almost generalized $\tilde{b}$(in short, $fag\tilde{b}$)-continuous mapping if $f^{-1}(\lambda)$ is $fg\tilde{b}o$ in X for every fuzzy regular open set λ in Y.*

Equivalently, f is said to be fuzzy almost generalized $\tilde{b}$-continuous mapping if $f^{-1}(\lambda)$ is $fg\tilde{b}c$ in X for every fuzzy regular closed set λ in Y.

Example 3.119 *Let λ, μ, γ and δ be fuzzy subsets of $X = Y = \{a,b,c\}$ are defined as follows:*

$\lambda(a) = 0.4$, $\lambda(b) = 0.6$, $\lambda(c) = 0.5$;
$\mu(a) = 0.6$, $\mu(b) = 0.4$, $\mu(c) = 0.4$;
$\gamma(a) = 0.6$, $\gamma(b) = 0.6$, $\gamma(c) = 0.5$;
$\delta(a) = 0.4$, $\delta(b) = 0.4$, $\delta(c) = 0.4$.

Then $\tau_1 = \{0,1,\lambda,\mu,\gamma,\delta\}$ and $\tau_2 = \{0,1,\delta\}$ are fuzzy topologies on X. Consider the mapping $f:(X,\tau_1) \to (X,\tau_2)$ defined by $f(x) = x$, $\forall x \in X$. Then for every fuzzy regular open set δ in X, $f^{-1}(\delta)$ is fuzzy generalized $\tilde{b}$-open (fgbo) set in X. Thus f is a fuzzy almost generalized $\tilde{b}$-continuous mapping from an fts (X,τ_1) to (X,τ_2).

Remark 3.120 *It follows from Definition (3.1) and (3.3), that every $fg\tilde{b}$-continuous mapping is an $fag\tilde{b}$-continuous mapping, but the converse may not be true is shown by the following example.*

Example 3.221 *Let λ, μ, γ and δ be fuzzy subsets of $X = \{a,b,c\}$ are defined as follows:*

$\lambda(a) = 0.3$, $\lambda(b) = 0.4$, $\lambda(c) = 0.5$;
$\mu(a) = 0.6$, $\mu(b) = 0.5$, $\mu(c) = 0.5$;
$\omega(a) = 0.4$, $\omega(b) = 0.4$, $\omega(c) = 0.4$;
$\delta(a) = 0.5$, $\delta(b) = 0.4$, $\delta(c) = 0.5$.

Now, $\tau_1 = \{0,1,\lambda,\mu\}$ and $\tau_2 = \{0,1,\omega,\delta\}$ are fuzzy topologies on X.

Consider the mapping $f:(X,\tau_1) \to (X,\tau_2)$ defined by $f(x) = x$, $\forall x \in X$. Then for every fro set δ in Y, $f^{-1}(\delta)$ is fg$\tilde{b}$o set in X. Hence, f is $fag\tilde{b}$-continuous mapping. Since ω is fuzzy open set in Y, but $f^{-1}(\omega)$ is not $fg\tilde{b}$o in X. Hence f is not $fg\tilde{b}$-continuous. Thus $fag\tilde{b}$-continuous mapping ¿ $fg\tilde{b}$-continuous mapping.

Theorem 3.122 *Let $f: X \to Y$ be a fuzzy mapping. Then the following statements are equivalent:*

- f is $fag\tilde{b}$ continuous.
- for each fuzzy point x_p in X and each fro q-nbd μ of $f(x_p)$, there is a $f(x_p)$ q-nbd λ of x_p such that $f(\lambda) \leq \mu$.
- for each fuzzy set λ in X, $f(g\tilde{b}Cl(\lambda)) \leq bClf(\lambda)$.
- for each fuzzy set μ in Y, $g\tilde{b}Cl(f^{-1}(\mu)) \leq f^{-1}(bCl(\mu))$.

Proof. (1) $\Rightarrow$ (2): Let x_p be a fuzzy point of x. Then $f(x_p)$ is a fuzzy point in Y. Now, let $\mu \in Y$ be a fuzzy regular open set such that $f(x_p)q\mu$. For, $\lambda = f^{-1}(\mu)$ as f is $fag\tilde{b}$ continuous, we have λ is $fg\tilde{b}o$ set of X and $x_p \in \lambda$. Therefore, $f(\lambda) = f(f^{-1}(\mu)) \leq \mu$.

(2) $\Rightarrow$ (3): Let $x_p \in g\tilde{b}Cl(\lambda)$. Then $\mu q\lambda$ and $f(\mu)qf(\lambda)$ implies $\alpha qf(\lambda)$, $f(x_p) \in bCl(f(\lambda))$ and $x_p \in f^{-1}(bCl(f(\lambda)))$. Therefore, $g\tilde{b}Cl(\lambda) \leq f^{-1}(bClf(\lambda))$.

(3) $\Rightarrow$ (4) and (4) $\Rightarrow$ (1) are clear.

Theorem 3.223 *A mapping from an $fT_{\frac{1}{2}}\tilde{b}$-space to another fts is fuzzy almost continuous if it is $fag\tilde{b}$-continuous.*

Proof. Let $f:(X,\tau_1) \to (Y,\tau_2)$ be $fag\tilde{b}$-continuous mapping. Let λ be any fuzzy regular closed set in Y. Since f is $fag\tilde{b}$-continuous, $f^{-1}(\lambda)$ is $fg\tilde{b}c$ in X. As X is fuzzy $T_{\frac{1}{2}}\tilde{b}$-space, $f^{-1}(\lambda)$ is fuzzy closed in X Thus f is fuzzy almost continuous.

Definition 3.4 24 *A mapping $f: X \to Y$ is called $fg\tilde{b}$ (resp. fb)-irresolute mapping if $f^{-1}(\lambda)$ is $fg\tilde{b}o$ (resp. fbo) subset of X for every $fg\tilde{b}o$ (resp. fbo) set λ in Y.*

Example 3.325 *Let* λ, μ, γ, δ, *and* ω *be fuzzy subsets of* $X = \{a, b, c\}$ *and defined as follows:*
$\lambda(a) = 0.3$, $\lambda(b) = 0.4$, $\lambda(c) = 0.5$;
$\mu(a) = 0.6$, $\mu(b) = 0.5$, $\mu(c) = 0.5$;
$\omega(a) = 0.3$, $\omega(b) = 0.4$, $\omega(c) = 0.5$.
Then $\tau_1 = \{0,1,\lambda,\mu\}$ and $\tau_2 = \{0,1,\omega\}$ are fuzzy topologies on X. Consider the mapping $f: (X,\tau_1) \to (X,\tau_2)$ defined by $f(x) = x$, $\forall x \in X$. Then for every fuzzy generalized $\tilde{b}$-open ($fg\tilde{b}o$)(resp. fbo) set ω in X, $f^{-1}(\omega)$ is fuzzy generalized $\tilde{b}$-open ($fg\tilde{b}o$)(resp. fbo) set in X. Thus, f is a fuzzy generalized $\tilde{b}$ ($fg\tilde{b}$)(resp. fb)-irresolute mapping from an fts (X,τ_1) to (X,τ_2).

Theorem 3.326 *Let* $f: X \to Y$ *and* $g: Y \to Z$ *be two fuzzy mappings. Then* $g \circ f: X \to Z$ *is* $fag\tilde{b}$*-continuous if* f *is* $fg\tilde{b}$*-irresolute and* g *is* $fag\tilde{b}$*-continuous.*

Proof. Let λ be any fuzzy regular open set in Z. Since g is $fag\tilde{b}$-continuous, $g^{-1}(\lambda)$ is $fg\tilde{b}o$ in Y. As f is $fg\tilde{b}$-irresolute, $f^{-1}(g^{-1}(\lambda))$ is $fg\tilde{b}o$ in X. This implies $(g \circ f)^{-1}(\lambda)$ is $fg\tilde{b}o$ in X. Thus, $g \circ f$ is $fag\tilde{b}$-continuous.

Theorem 3.427 *If* $f: X \to Y$ *is an fuzzy continuous mapping and* $g: Y \to Z$ *is an* $fag\tilde{b}$*-continuous mapping, where* Y *is fuzzy* $T_{\frac{1}{2}}\tilde{b}$*-space, then* $g \circ f: X \to Z$ *is fuzzy almost continuous mapping.*

Proof. Let λ be any fuzzy regular closed set in Z. Since g is $fag\tilde{b}$-continuous mapping, $g^{-1}(\lambda)$ is $fg\tilde{b}c$ in Y. As Y is fuzzy $T_{\frac{1}{2}}\tilde{b}$-space, $g^{-1}(\lambda)$ is fuzzy closed in Y. This implies $Y - g^{-1}(\lambda)$ is fuzzy open in Y. As f is fuzzy continuous, $f^{-1}(Y - g^{-1}(\lambda))$ is fuzzy open in $X \Rightarrow X - f^{-1}(g^{-1}(\lambda))$ is fuzzy open in $X \Rightarrow f^{-1}(g^{-1}(\lambda))$ is fuzzy closed in $X \Rightarrow (g \circ f)^{-1}(\lambda)$ is fuzzy closed in X. Hence, $g \circ f$ is fuzzy almost continuous.

4. Fuzzy generalized $\tilde{b}$-compact space and fuzzy regular $g\tilde{b}$-space

In this section, we introduce the definition of fuzzy generalized $\tilde{b}$-compact ($fg\tilde{b}$-compact space) and fuzzy regular $g\tilde{b}$-space. Also some interesting theorems involving $fg\tilde{b}$-compact space and fuzzy regular $g\tilde{b}$-space are established here.

Definition 4.1 28 *A family* Λ *of fuzzy sets in a fuzzy topological space* X *is said to be* $fg\tilde{b}o$*-cover if each member* λ_i *is a fuzzy* $fg\tilde{b}o$ *set.*

Definition 4.229 *An fts* (X,τ) *is said to be* $fg\tilde{b}$*-compact space if every* $fg\tilde{b}o$ *cover of* X *has a finite subcover.*

Theorem 4.130 *An* $fag\tilde{b}$ *continuous surjection image of a* $fg\tilde{b}$*-compact space is fuzzy nearly compact.*

Proof. Let $f: X \to Y$ be an $fag\tilde{b}$ continuous surjection mapping and $\{\lambda_i\}_{i\in\Lambda}$ be any fro cover of Y. Then $\{f^{-1}(\lambda_i)\}_{i\in\Lambda}$ is a family of $fg\tilde{b}o$ cover of X. X is $fg\tilde{b}$-compact. So, there exists a finite subset Λ_0 of Λ such that $\{f^{-1}(\lambda_i)\}_{i\in\Lambda_0}$ covers X. Since f is 1-1, onto, $\{\lambda_i\}_{i\in\Lambda_0}$ will be finite fro subcover of the fro cover $\{\lambda_i\}_{i\in\Lambda}$ of Y. So, Y is fuzzy nearly compact.

Definition 4.331 *An fts* (X,τ) *is said to be fuzzy regular* $g\tilde{b}$*(in short,* $rg\tilde{b}$*)-space if every* $fg\tilde{b}o$ *set in* X *is fro.*

Remark 4.132 *There are spaces which are not fuzzy* $rg\tilde{b}$*-spaces.*

Example 4.133 *Let* λ, μ, γ *and* δ *be fuzzy subsets of* $X = \{a, b, c\}$ *defined as follows:*
$\lambda(a) = 0.3$, $\lambda(b) = 0.4$, $\lambda(c) = 0.5$;
$\mu(a) = 0.7$, $\mu(b) = 0.4$, $\mu(c) = 0.5$;
$\gamma(a) = 0.8$, $\gamma(b) = 0.5$, $\gamma(c) = 0.5$;
$\delta(a) = 0.8$, $\delta(b) = 0.4$, $\delta(c) = 0.5$.
Then $\tau = \{0,1,\lambda,\mu\}$ is a fuzzy topology on X. Since γ and δ are $fg\tilde{b}o$ sets in X, but not fuzzy regular open in (X,τ). Hence the topological space (X,τ) is not fuzzy $rg\tilde{b}$-space.

Definition 4.434 *An fts* (X,τ) *is said to be fuzzy* $g\tilde{b}$*-space if every* $fg\tilde{b}o$ *set in* X *is fuzzy open.*

Remark 4.235 *There are spaces which are not fuzzy* $g\tilde{b}$*-spaces is shown by the example below.*

Example 4.236 *Let* λ, μ, γ *and* δ *be fuzzy subsets of* $X = \{a, b, c\}$ *defined as follows:*
$\lambda(a) = 0.3$, $\lambda(b) = 0.5$, $\lambda(c) = 0.2$;
$\mu(a) = 0.4$, $\mu(b) = 0.5$, $\mu(c) = 0.5$;
$\gamma(a) = 0.5$, $\gamma(b) = 0.7$, $\gamma(c) = 0.5$;
$\delta(a) = 0.4$, $\delta(b) = 0.6$, $\delta(c) = 0.5$.
Then $\tau = \{0,1,\lambda,\mu\}$ is a fuzzy topology on X. Since γ and δ are $fg\tilde{b}o$ sets in X but not fuzzy open sets in X. This implies the fuzzy topological space (X,τ) is not fuzzy $g\tilde{b}$-space.

Remark 4.337 *It is obvious from Definition (4.3) and (4.4) that every fuzzy* $rg\tilde{b}$ *space is fuzzy* $g\tilde{b}$ *space but not the converse is shown by the following example.*

Example 4.338 *Let* λ *and* μ *be fuzzy subsets of* $X = \{a, b, c\}$ *defined as follows:*
$\lambda(a) = 0.2,\ \lambda(b) = 0.2,\ \lambda(c) = 0.5;$
$\mu(a) = 0.3,\ \mu(b) = 0.2,\ \mu(c) = 0.5.$
Then $\tau = \{0, 1, \lambda, \mu\}$ is a fuzzy topology on X. Now (X, τ) is fuzzy $g\tilde{b}$-space but not fuzzy $rg\tilde{b}$-space since λ is fuzzy open and $fg\tilde{b}o$ but not fro set in (X, τ).

Theorem 4.239 *Let* $f: X \to Y$ *be* $fg\tilde{b}$*-continuous surjection mapping. Then* Y *is fuzzy connected if* X *is fuzzy connected space and fuzzy* $rg\tilde{b}$*-space.*

Proof. Let X be a fuzzy connected space and Y be not fuzzy connected. Then there exists a fuzzy subset λ of Y such that $\lambda \neq 0_Y, 1_Y$ and λ is both fuzzy open and fuzzy closed in $Y \Rightarrow f^{-1}(\lambda)$ is both fgbo and fgbc in $X \Rightarrow f^{-1}(\lambda)$ is both fro and frc in $X \Rightarrow f^{-1}(\lambda)$ is both fuzzy open and fuzzy closed in X. Also, $f^{-1}(\lambda) \neq 0_X, 1_X$. This implies that X is not a fuzzy connected space, a contradiction. Therefore, Y is fuzzy connected.

Theorem 4.340 *Let* $f: X \to Y$ *be an* $fag\tilde{b}$*-continuous surjective mapping and* X *be fuzzy regular* $g\tilde{b}$*-space as well as fuzzy nearly compact space. Then* Y *is fuzzy nearly compact.*

Proof. Let $\{\lambda_i\}_{i \in \Lambda}$ be any fro cover of Y. Since f is $fag\tilde{b}$-continuous, $f^{-1}(\lambda_i)_{i \in \Lambda}$ is a family of $fg\tilde{b}o$ sets in X. X is fuzzy $rg\tilde{b}$-space. So,$f^{-1}(\lambda_i)_{i \in \Lambda}$ is a family of fro sets in X. X is fuzzy nearly compact. So, there exists a finite subset Λ_0 of Λ such that $X \leq [f^{-1}(\lambda_i)_{i \in \Lambda_0}]$. Then $Y = f(X) \leq (\lambda_i)_{i \in \Lambda_0}$. So, $\{\lambda_i\}_{i \in \Lambda_0}$ is a finite subcover of fro sets of Y. So, Y is fuzzy nearly compact.

Theorem 4.441 *If* $f: X \to Y$ *is an* $fg\tilde{b}$*-continuous injective mapping and* Y *is a fuzzy Hausorff space, then* X *is fuzzy Hausdorff if it is fuzzy regular* $g\tilde{b}$*-space.*

Proof. Let x_p, y_q be two distinct fuzzy points in X. Then $f(x_p) \neq f(y_q)$ in Y. Since Y is fuzzy Hausdorff, there exists fuzzy open nbds U and V of $f(x_p)$ and $f(y_q)$ respectively such that $U \wedge V = 0_X$. Since f is an $fg\tilde{b}$-continuous, $f^{-1}(U)$ and $f^{-1}(V)$ are $fg\tilde{b}o$ in $X \Rightarrow f^{-1}(U)$ and $f^{-1}(V)$ are fro in X. [Since X is fuzzy regular $g\tilde{b}$-space] $\Rightarrow f^{-1}(U)$ and $f^{-1}(V)$ are fuzzy open in X and contains respectively the fuzzy points x_p and y_q. Now, $x_p \in f^{-1}(U) = \lambda$, say $y_q \in f^{-1}(V) = \mu$, say. So, $\lambda \wedge \mu = f^{-1}(U) \wedge f^{-1}(V) = f^{-1}(U \wedge V) = f^{-1}(0_X) = 0_X$. Thus, X is fuzzy Hausdorff.

Conclusion:

It is interesting to work under a new class of mappings viz. fuzzy almost generalized $\tilde{b}$-continuous mapping, fuzzy $fT_{\frac{1}{2}}\tilde{b}$-space, fuzzy generalized $\tilde{b}$-compact space, fuzzy $g\tilde{b}$-space, fuzzy regular $g\tilde{b}$-space. It is seen that a fuzzy almost generalized $\tilde{b}$-continuous surjection image of a fuzzy generalized $\tilde{b}$-compact space is fuzzy nearly compact.

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