ON FUZZY UPPER AND LOWER ALMOST CONTRA e^*-CONTINUOUS MULTIFUNCTIONS

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Abstract In this paper, we introduce the concepts of fuzzy upper and fuzzy lower almost contra e^*-continuous multifunctions, fuzzy upper and fuzzy lower weakly contra e^*-continuous multifunction on fuzzy topological spaces in $\hat{S}$ostak sense. Several characterizations and properties of these fuzzy upper (resp. fuzzy lower) almost contra e^*-continuous, fuzzy upper (resp. lower) weakly contra e^*-continuous multifunctions are presented and their mutual relationships are established in L-fuzzy topological spaces. Later, composition and union between these multifunctions have been studied.

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Introduction

Kubiak [17] and $\hat{S}$ostak [24] introduced the notion of (L-)fuzzy topological space as a generalization of L-topological spaces (originally called (L-) fuzzy topological spaces by Chang [5] and Goguen [9]. It is the grade of openness of an L-fuzzy set. A general approach to the study of topological type structures on fuzzy powersets was developed in [[10]-[12], [17], [18], [24]-[26]].

Berge [4] introduced the concept multimapping $F{:}X$¨Y where X and Y are topological spaces. After Chang introduced the concept of fuzzy topology [5], continuity of multifunctions in fuzzy topological spaces have been defined and studied by many authors from different view points (eg. see [2], [3], [20]-[22]). Tsiporkova et al., [30] introduced the continuity of fuzzy multivalued mappings in the Chang's fuzzy topology [5]. Later, Abbas et al., [1] introduced the concepts of fuzzy upper and fuzzy lower semi-continuous multifunctions in L-fuzzy topological spaces. Recently, Sobana et al. [27] introduced the concept of r-fuzzy e-open sets and r-fuzzy e-continuity in $\check{S}$ostak's fuzzy topological spaces. Vadivel et. al., [31] introduced the concept of fuzzy almost e-continuity, fuzzy e-compactness in a fuzzy topological space in the sense of $\hat{S}$ostak [24]. Dhanasekaran et.al [8] introduced the concept of fuzzy upper and fuzzy lower almost contra e-continuous multifunction on fuzzy topological spaces in $\hat{S}$ostak sense.

In this paper, we introduce the concepts of fuzzy upper and fuzzy lower almost contra e^*-continuous multifunctions, fuzzy upper and fuzzy lower weakly contra e^*-continuous multifunction on fuzzy topological spaces in $\hat{S}$ostak sense. Several characterizations and properties of these multifunctions are presented and their mutual relationships are established in L-fuzzy topological spaces. Later, composition and union between these multifunctions have been studied.

Throughout this paper, nonempty sets will be denoted by X, Y etc., $L = [0,1]$ and $L_0 = (0,1]$. The family of all fuzzy sets in X is denoted by L^X. The complement of an L-fuzzy set λ is denoted by λ^c. This symbol ¨ for a multifunction.

For $\alpha \in L$, $\overline{\alpha}(x) = \alpha$ for all $x \in X$. A fuzzy point x_t for $t \in L_0$ is an element of L^X such that $x_t(y) = (t$y=x 0ify x. The family of all fuzzy points in X is denoted by $Pt(X)$. A fuzzy point $x_t \in \lambda$ iff $t \leq \lambda(x)$.

All other notations are standard notations of L-fuzzy set theory.

2.Preliminaries

Definition 2.1 1 *[1] Let $F{:}X$¨Y, then F is called a fuzzy multifunction (FM, for short) if and only if $F(x) \in L^Y$ for each $x \in X$. The degree of membership of y in $F(x)$ is denoted by $F(x)(y) = G_F(x,y)$ for any $(x,y) \in X \times Y$. The domain of F, denoted by $domain(F)$ and the range of F, denoted by $rng(F)$, for any $x \in X$ and $y \in Y$, are defined by :*

$$dom(F)(x) = \bigvee_{y\in Y} G_F(x,y) \text{and} rng(F)(y) = \bigvee_{x\in X} G_F(x,y).$$

Definition 2.2 2 *[1] Let $F{:}X$¨Y be a FM. Then F is called:*

- Normalized iff for each $x \in X$, there exixts $y_0 \in Y$ such that $G_F(x, y_0) = \overline{1}$.
- A crisp iff $G_F(x,y) = \overline{1}$ for each $x \in X$ and $y \in Y$.

Definition 2.3 3 *[1] Let $F{:}X$¨Y be a FM. Then*

- The image of $\lambda \in L^X$ is an L-fuzzy set $F(\lambda) \in L^Y$ defined by

$$F(\lambda)(y) = \bigvee_{x \in X} [G_F(x,y) \wedge \lambda(x)].$$

- The lower inverse of $\mu \in L^Y$ is an L-fuzzy set $F^l(\mu) \in L^X$ defined by
$$F^l(\mu)(x) = \bigvee_{y \in Y} [G_F(x,y) \wedge \mu(y)].$$

- The upper inverse of $\mu \in L^Y$ is an L-fuzzy set $F^u(\mu) \in L^X$ defined by
$$F^u(\mu)(x) = \bigwedge_{y \in Y} [G_F^c(x,y) \vee \mu(y)].$$

Theorem 2.14 *[1] Let $F: X \ddot{} Y$ be a FM. Then*
- $F(\lambda_1) \leq F(\lambda_2)$ if $\lambda_1 \leq \lambda_2$.
- $F^l(\mu_1) \leq F^l(\mu_2)$ and $F^u(\mu_1) \leq F^u(\mu_2)$ if $\mu_1 \leq \mu_2$.
- $F^l(\mu^c) = (F^u(\mu))^c$.
- $F^u(\mu^c) = (F^l(\mu))^c$.
- $F(F^u(\mu)) \leq \mu$ if F is a crisp.
- $F^u(F(\lambda)) \geq \lambda$ if F is a crisp.

Definition 2.4 5 *[1] Let $F: X \ddot{} Y$ and $H: Y \ddot{} Z$ be two FM. Then the composition $H \circ F$ is defined by*
$$((H \circ F)(x))(z) = \bigvee_{y \in Y} [G_F(x,y) \wedge G_H(y,z)].$$

Theorem 2.2 6 *[1] Let $F: X \ddot{} Y$ and $H: Y \ddot{} Z$ be FM. Then we have the following*
- $(H \circ F) = F(H)$.
- $(H \circ F)^u = F^u(H^u)$.
- $(H \circ F)^l = F^l(H^l)$.

Theorem 2.37 *[1] Let $F_i: X \ddot{} Y$ be a FM. Then we have the following*
- $(\bigcup_{i \in \Gamma} F_i)(\lambda) = \bigvee_{i \in \Gamma} F_i(\lambda)$.
- $(\bigcup_{i \in \Gamma} F_i)^l(\mu) = \bigvee_{i \in \Gamma} F_i^l(\mu)$.
- $(\bigcup_{i \in \Gamma} F_i)^u(\mu) = \bigwedge_{i \in \Gamma} F_i^u(\mu)$.

Definition 2.58 *[12,17,19,24] An L-fuzzy topological space (L-fts, in short) is a pair (X, τ), where X is a nonempty set and $\tau: L^X \to L$ is a mapping satisfying the following properties.*
- $\tau(\overline{0}) = \tau(\overline{1}) = 1$,
- $\tau(\mu_1 \wedge \mu_2) \geq \tau(\mu_1) \wedge \tau(\mu_2)$, for any $\mu_1, \mu_2 \in I^X$.
- $\tau(\vee_{i \in \Gamma} \mu_i) \geq \wedge_{i \in \Gamma} \tau(\mu_i)$, for any $\{\mu_i\}_{i \in \Gamma} \subset I^X$,

Then τ is called an L-fuzzy topology on X. For every $\lambda \in L^X$, $\tau(\lambda)$ is called the degree of openness of the L-fuzzy set λ.

A mapping $f: (X, \tau) \to (Y, \eta)$ is said to be continuous with respect to L-fuzzy topologies τ and η iff $\tau(f^{-1}(\mu)) \geq \eta(\mu)$ for each $\mu \in L^Y$.

Theorem 2.4 9 *[6,15,16,19] Let (X, τ) be a an L-fts. Then for each $\lambda \in L^X, r \in L_0$, we define L-fuzzy operators $C_\tau and I_\tau: L^X \times L_0 \to L^X$ as follows:*

$C_\tau(\lambda, r) = \wedge \ \{\mu \in L^X: \lambda \leq \mu, \tau(\overline{1} - \mu) \geq r\}$.

$I_\tau(\lambda, r) = \vee \ \{\mu \in L^X: \lambda \geq \mu, \tau(\mu) \geq r\}$.

For $\lambda, \mu \in L^X$ and $r, s \in L_0$, the operator C_τ satisfies the following conditions:
- $C_\tau(\overline{0}, r) = \overline{0}$,
- $\lambda \leq C_\tau(\lambda, r)$,
- $C_\tau(\lambda, r) \vee C_\tau(\mu, r) = C_\tau(\lambda \vee \mu, r)$,
- $C_\tau(C_\tau(\lambda, r), r) = C_\tau(\lambda, r)$,
- $C_\tau(\lambda, r) = \lambda$ iff $\tau(\lambda^c) \geq r$.
- $C_\tau(\lambda^c, r) = (I_\tau(\lambda, r))^c$ and $I_\tau(\lambda^c, r) = (C_\tau(\lambda, r))^c$.

Definition 2.610 *[1] Let $F: X \ddot{} Y$ be a FM between two L-fts's $(X, \tau), (Y, \eta)$ and $r \in L_0$. Then F is called:*

- Fuzzy upper semi (or Fuzzy upper) (in short, FUS (or FU))-continuous at a L-fuzzy point $x_t \in dom(F)$ iff $x_t \in F^u(\mu)$ for each $\mu \in L^Y$ and $\eta(\mu) \geq r$, there exists $\lambda \in L^X$, $\tau(\lambda) \geq r$ and $x_t \in \lambda$ such that $\lambda \wedge dom(F) \leq F^u(\mu)$. F is FU-continuous iff it is FU-continuous at every $x_t \in dom(F)$.
- Fuzzy lower semi (or Fuzzy lower) (in short, FLS (or FL))-continuous at a L-fuzzy point $x_t \in dom(F)$ iff $x_t \in F^l(\mu)$ for each $\mu \in L^Y$ and $\eta(\mu) \geq r$, there exists $\lambda \in L^X$, $\tau(\lambda) \geq r$ and $x_t \in \lambda$ such that $\lambda \leq F^l(\mu)$. F is FL-continuous iff it is FL-continuous at every $x_t \in dom(F)$.
- Fuzzy continuous if it is FU-continuous and FL-continuous.

Theorem 2.511 *[1] Let $F: X \ddot{} Y$ be a fuzzy multifunction between two L-fts's (X, τ) and (Y, η). Let $\mu \in L^Y$. Then we have the following*
- F is FL-continuous iff $\tau(F^l(\mu)) \geq \eta(\mu)$.
- If F is normalized, then F is FU-continuous iff $\tau(F^u(\mu)) \geq \eta(\mu)$.
- F is FL-continuous iff $\tau(\overline{1} - F^u(\mu)) \geq \eta(\overline{1} - \mu)$.
- If F is normalized, then F is FU-continuous iff $\tau(\overline{1} - F^l(\mu)) \geq \eta(\overline{1} - \mu)$.

Definition 2.7 12 *[13] Let $F: X \multimap Y$ be a FM between two L-fts's $(X,\tau),(Y,\eta)$ and $r \in L_0$. Then F is called:*

• Fuzzy upper almost contra continuous ($FUAC$-continuous, for short) at any L-fuzzy point $x_t \in dom(F)$ iff $x_t \in F^u(\mu)$ for each $\mu \in L^Y$ and μ is r-frc there exist $\lambda \in L^X$, $\tau(\lambda) \geq r$ and $x_t \in \lambda$ such that $\lambda \wedge dom(F) \leq F^u(\mu)$.

• Fuzzy lower almost contra continuous ($FLAC$-continuous, for short) at any L-fuzzy point $x_t \in dom(F)$ iff $x_t \in F^l(\mu)$ for each $\mu \in L^Y$ and μ is r-frc there exist $\lambda \in L^X$, $\tau(\lambda) \geq r$ and $x_t \in \lambda$ such that $\lambda \leq F^l(\mu)$.

• $FUAC$-continuous (resp. $FLAC$-continuous) iff it is $FUAC$-continuous (resp. $FLAC$-continuous) at every $x_t \in dom(F)$.

Definition 2.8 13 *[13] Let $F: X \multimap Y$ be a FM between two L-fts's (X,τ), (Y,η) and $r \in L_0$. Then F is called.*

• Fuzzy upper weakly contra continuous ($FUWC$-continuous, in short) at an L-fuzzy point $x_t \in dom(F)$ iff $x_t \in F^u(\mu)$ for each $\mu \in L^Y$ and μ is r-fuzzy closed, there exist $\lambda \in L^X$, $\tau(\lambda) \geq r$ and $x_t \in \lambda$ such that $\lambda \wedge dom(F) \leq F^u(C_\eta(\mu,r))$.

• Fuzzy lower weakly contra continuous ($FLWC$-continuous, in short) at an L-fuzzy point $x_t \in dom(F)$ iff $x_t \in F^l(\mu)$ for each $\mu \in L^Y$ and μ is r-fuzzy closed, there exist $\lambda \in L^X$, $\tau(\lambda) \geq r$ and $x_t \in \lambda$ such that $\lambda \leq F^l(C_\eta(\mu,r))$.

• $FUWC$-continuous (resp. $FLWC$-continuous) iff it is $FUWC$-continuous (resp. $FLWC$-continuous) at every $x_t \in dom(F)$.

Definition 2.9 14 *[14] Let (X,τ) be a fts. For $\lambda,\mu \in I^X$ and $r \in I_0$, λ is called r-fuzzy regular open (for short, r-fro) (resp. r-fuzzy regular closed (for short, r-frc)) if $\lambda = I_\tau(C_\tau(\lambda,r),r)$ (resp. $\lambda = C_\tau(I_\tau(\lambda,r),r)$).*

Definition 2.10 15 *[14] Let (X,τ) be a fts. Then for each $\mu \in I^X, x_t \in P_t(X)$ and $r \in I_0$,*

• μ is called r-open Q_τ-neighbourhood of x_t if $x_t q \mu$ with $\tau(\mu) \geq r$.

• μ is called r-open R_τ-neighbourhood of x_t if $x_t q \mu$ with $\mu = I_\tau(C_\tau(\mu,r),r)$.

We denoted

$$Q_\tau(x_t,r) = \{\mu \in I^X : x_t q \mu, \tau(\mu) \geq r\},$$

$$R_\tau(x_t,r) = \{\mu \in I^X : x_t q \mu, \mu = I_\tau(C_\tau(\mu,r),r)\}.$$

Definition 2.11 16 *[14] Let (X,τ) be a fts. Then for each $\lambda \in I^X, x_t \in P_t(X)$ and $r \in I_0$,*

• x_t is called r-τ cluster point of λ if for every $\mu \in Q_\tau(x_t,r)$, we have $\mu q \lambda$.

• x_t is called r-δ cluster point of λ if for every $\mu \in R_\tau(x_t,r)$, we have $\mu q \lambda$.

• An δ-closure operator is a mapping $D_\tau: I^X \times I \to I^X$ defined as follows: $\delta C_\tau(\lambda,r)$ or $D_\tau(\lambda,r) = \vee \{x_t \in P_t(X): x_t$ is r-δ-cluster point of $\lambda\}$.

Equivalently, $\delta C_\tau(\lambda,r) = \wedge \{\mu \in I^X : \mu \geq \lambda, \mu \text{isar} - \text{frcset}\}$ and $\delta I_\tau(\lambda,r) = \vee \{\mu \in I^X : \mu \leq \lambda, \mu \text{isar} - \text{froset}\}$.

Definition 2.12 17 *[14] Let (X,τ) be a fuzzy topological space. For $\lambda \in I^X$ and $r \in I_0$, λ is called r-fuzzy δ-closed iff $\lambda = \delta C_\tau(\lambda,r)$ or $D_\tau(\lambda,r)$.*

Definition 2.1318 *[27] Let (X,τ) be a an L-fts. Then for each $\lambda,\mu \in L^X, r \in L_0$. Then λ is called*

• λ is called an r-fuzzy e-open (briefly, r-feo) set if $\lambda \leq C_\tau(\delta_\tau(\lambda,r),r) \vee I_\tau(\delta C_\tau(\lambda,r),r)$.

• λ is called an r-fuzzy e-closed (briefly, r-feo) set if $C_\tau(\delta I_\tau(\lambda,r),r) \wedge I_\tau(\delta C_\tau(\lambda,r),r) \leq \lambda$.

Definition 2.1419 *[27] Let (X,τ) be an L-fts. Then for each $\lambda,\mu \in L^X, r \in L_0$. Then λ is called*

• $eI_\tau(\lambda,r) = \vee \{\mu \in I^X : \mu \leq \lambda, \mu$ is a r-feo set $\}$ is called the r-fuzzy e-interior of λ.

• $eC_\tau(\lambda,r) = \wedge \{\mu \in I^X : \mu \geq \lambda, \mu$ is a r-fec set $\}$ is called the r-fuzzy e-closure of λ.

Definition 2.15 20 *[23,28] Let $F: X \multimap Y$ be a FM between two L-fts's $(X,\tau),(Y,\eta)$ and $r \in L_0$. Then F is called:*

• Fuzzy upper contra e ($FUCe$, in short) (resp. FUe)-continuous any L-fuzzy point $x_t \in dom(F)$ iff $x_t \in F^u(\mu)$ for each $\mu \in L^Y$ and $\eta(\mu^c) \geq r$ (resp. $\eta(\mu) \geq r$) there exist r-fuzzy e-open set (r-feo set, in short), $\lambda \in L^X$ and $x_t \in \lambda$ such that $\lambda \wedge dom(F) \leq F^u(\mu)$.

• Fuzzy lower contra e ($FLCe$, in short) (resp. FLe)-continuous any L-fuzzy point $x_t \in dom(F)$ iff $x_t \in F^l(\mu)$ for each $\mu \in L^Y$ and $\eta(\mu^c) \geq r$ (resp. $\eta(\mu) \geq r$) there exist r-fuzzy e-open set (r-feo set, in short), $\lambda \in L^X$ and $x_t \in \lambda$ such that $\lambda \leq F^l(\mu)$.

• FUCe (resp. FLCe, FUe and FLe)-continuous iff it is FUCe (resp. FLCe, FUe and FLe)-continuous at every $x_t \in dom(F)$.

Definition 2.16 21 *[7] Let $F: X \multimap Y$ be a FM between two L-fts's $(X,\tau),(Y,\eta)$ and $r \in L_0$. Then F is called:*

• Fuzzy upper e^* (in short, FUe^*)-irresolute at any L-fuzzy point $x_t \in dom(F)$ iff $x_t \in F^u(\mu)$ for each $\mu \in L^Y$ and r-fe*o set, there exists r-fe*o set $\lambda \in L^X$ and $x_t \in \lambda$ such that $\lambda \wedge dom(F) \leq F^u(\mu)$.

• Fuzzy lower e^* (in short, FLe^*)-irresolute at any L-fuzzy point $x_t \in dom(F)$ iff $x_t \in F^l(\mu)$ for each $\mu \in L^Y$ and r-fe*o set, there exists r-fe*o set $\lambda \in L^X$ and $x_t \in \lambda$ such that $\lambda \leq F^l(\mu)$.

• FUe^*-irresolute and FLe^*-irresolute iff it is FUe^*-irresolute and FLe^*-irresolute at every $x_t \in dom(F)$.

Definition 2.17 22 *[28] Let (X,τ) be an L-fts. Then for each $\lambda \in L^X$ and $r \in L_0$ we define L-fuzzy operator e-ker$_\tau$: $L^X \times L_0 \to L^X$ as follows:*

e-$ker_\tau(\lambda,r) = \wedge \{\mu \in L^X : \lambda \leq \mu, \mu \text{is } r\text{-feo-set}\}$.

Lemma 2.123 *[28] For λ in an L-fts (X,τ), if λ is r-feo-set then $\lambda = e$-$ker_\tau(\lambda,r)$.*

3.Fuzzy upper and lower almost contra e^*-continuous multifunctions

Definition 3.124 *Let $F: X \multimap Y$ be a FM between two L-fts's $(X,\tau),(Y,\eta)$ and $r \in L_0$. Then F is called:*

• Fuzzy upper almost contra e^*-continuous ($FUACe^*$-continuous, in short) at any L-fuzzy point $x_t \in dom(F)$ iff $x_t \in F^u(\mu)$ for each $\mu \in L^Y$ and μ is r-frc, there exist r-fe*o set $\lambda \in L^X$ and $x_t \in \lambda$ such that $\lambda \wedge dom(F) \leq F^u(\mu)$.

• Fuzzy lower almost contra e^*-continuous ($FLACe^*$-continuous, in short) at any L-fuzzy point $x_t \in dom(F)$ iff $x_t \in$

$F^l(\mu)$ for each $\mu \in L^Y$ and μ is r-frc, there exist r-fe*o set $\lambda \in L^X$ and $x_t \in \lambda$ such that $\lambda \leq F^l(\mu)$.

• Fuzzy upper almost contra e^*-continuous (resp. Fuzzy lower almost contra e^*-continuous) iff it is $FUACe^*$-continuous (resp. $FLACe^*$-continuous) at every $x_t \in dom(F)$.

Proposition 3.1 25 *F is normalized implies F is $FUACe^*$-continuous at an L-fuzzy point $x_t \in dom(F)$ iff $x_t \in F^u(\mu)$ for each $\mu \in L^Y$ and μ is r-frc there exists r-fe*o set $\lambda \in L^X$ and $x_t \in \lambda$ such that $\lambda \leq F^u(\mu)$.*

Theorem 3.1 26 *Let $F: X\ddot{}Y$ be a FM between two L-fts's (X,τ), (Y,η) and $\mu \in L^Y$, then the following are equivalent:*

• F is $FLACe^*$-continuous.
• $F^l(\mu)$ is r-fe*o set, for any μ is r-frc.
• $\overline{1} - F^u(\mu)$ is r-fe*o set, for any μ is r-fro.
• $\overline{1} - F^u(I_\eta(C_\eta(\mu,r),r))$ is r-fe*o-set, for any $\eta(\mu) \geq r$.
• $F^l(C_\eta(I_\eta(\mu,r),r))$ is r-fe*o-set, for any $\eta(\overline{1}-\mu) \geq r$.

Proof. (i) $\Rightarrow$ (ii): Let $x_t \in dom(F), \mu \in L^Y$, μ is r-frc and $x_t \in F^l(\mu)$, then there exist r-fe*o-set $\lambda \in L^X$ and $x_t \in \lambda$ such that $\lambda \leq F^l(\mu)$ and hence $x_t \in e^*I_\tau(F^l(\mu),r)$. Therefore, we obtain $F^l(\mu) \leq e^*I_\tau(F^l(\mu),r)$. Thus $F^l(\mu)$ is r-fe*o set.

(ii) $\Rightarrow$ (i): Let $x_t \in dom(F), \mu \in L^Y$, μ is r-frc and $x_t \in F^l(\mu)$ we have by (ii), $F^l(\mu)$ is r-fe*o-set. Let $F^l(\mu) = \lambda$(say), then there exists r-fe*o-set $\lambda \in L^X$ and $x_t \in \lambda$ such that $\lambda \leq F^l(\mu)$. Thus F is $FLACe^*$-continuous.

(ii) $\Rightarrow$ (iii): Let $\mu \in L^Y$ and μ is r-fro, hence by (ii), $F^l(\overline{1}-\mu) = \overline{1} - F^u(\mu)$ is r-fe*o set.

(iii) $\Rightarrow$ (ii): It is similar to that of (ii) $\Rightarrow$ (iii).

(iii) $\Rightarrow$ (iv): Let $\mu \in L^Y$ and $\eta(\mu) \geq r$. Since $I_\eta(C_\eta(\mu,r),r)$ is r-fro, then $\overline{1} - F^u(I_\eta(C_\eta(\mu,r),r))$ is r-fe*o set.

(iv) $\Rightarrow$ (iii): Obvious.

(iv) $\Rightarrow$ (v): Let $\mu \in L^Y$ and $\eta(\overline{1}-\mu) \geq r$ hence by (iv), $\overline{1} - F^u(I_\eta(C_\eta(\overline{1}-\mu,r),r)) = F^l(C_\eta(I_\eta(\mu,r),r))$ is r-fe*o set.

(v) $\Rightarrow$ (ii): Obvious.

We state the following result without proof in view of the above theorem.

Theorem 3.2 27 *Let $F: X\ddot{}Y$ be a FM and normalized between two L-fts's $(X,\tau),(Y,\eta)$ and $\mu \in L^Y$, then the following are equivalent:*

• F is $FUACe^*$-continuous.
• $F^u(\mu)$ is r-fe*o-set for any μ is r-frc.
• $\overline{1} - F^l(\mu)$ is r-fe*o-set for any μ is r-fro.
• $\overline{1} - F^l(I_\eta(C_\eta(\mu,r),r))$ is r-fe*o set for any $\eta(\mu) \geq r$.
• $F^u(C_\eta(I_\eta(\mu,r),r))$ is r-fe*o set for any $\eta(\overline{1}-\mu) \geq r$.

Proof. This can be proved in a similar way as Theorem (3.1)

Remark 3.128 *The following implications hold.*

• $FUCe$-continuous $\Rightarrow$ $FUACe^*$-continuous $\Leftarrow$ $FUAC$-continuous.
• $FLCe$-continuous $\Rightarrow$ $FLACe^*$-continuous $\Leftarrow$ $FLAC$-continuous.

In general the converses are not true.

Example 3.129 *Let $X = \{x_1,x_2\}$, $Y = \{y_1,y_2,y_3\}$ and $F: X\ddot{}Y$ be a FM defined by $G_F(x_1,y_1) = 0.8$, $G_F(x_1,y_2) = \overline{1}$, $G_F(x_1,y_3) = 0.7$, $G_F(x_2,y_1) = 0.5$, $G_F(x_2,y_2) = \overline{1}$, and $G_F(x_2,y_3) = 0.6$. Let λ_1 and λ_2 be a fuzzy subsets of X be defined as $\lambda_1(x_1) = 0.4$, $\lambda_1(x_2) = 0.3$; $\lambda_2(x_1) = 0.3$, $\lambda_2(x_2) = 0.4$, μ_1 and μ_2 be a fuzzy subsets of Y defined as $\mu_1(y_1) = 0.5$, $\mu_1(y_2) = 0.5$, $\mu_1(y_3) = 0.5$ and $\mu_2(y_1) = 0.5$, $\mu_2(y_2) = 0.6$, $\mu_2(y_3) = 0.7$. We assume that $\overline{1} = 1$ and $\overline{0} = 0$. Define L-fuzzy topologies $\tau: L^X \to L$ and $\eta: L^Y \to L$ as follows:* $\tau(\lambda) = \begin{cases} 1 & \text{if } \lambda = \overline{0}, \text{ or } \overline{1} \\ \frac{1}{2} & \text{if } \lambda = \lambda_1 \\ 0 & \text{otherwise} \end{cases}$ $\eta(\mu) = \begin{cases} 1 & \text{if } \mu = \overline{0}, \text{ or } \overline{1} \\ \frac{1}{2} & \text{if } \mu = \mu_1, \mu_2 \\ 0 & \text{otherwise} \end{cases}$

are fuzzy topologies on X and Y. For $r = \frac{1}{2}$, $\overline{1} - \mu_1$ is $\frac{1}{2}$ frc set in Y and $F^u(\overline{1}-\mu_1) = \mu_1$ is $\frac{1}{2}$-fe*o set in X. Hence F is $FUACe^*$-continuous but not $FUCe$-continuous. As $\overline{1} - \mu_2$ is closed in (Y,η), $F^u(\overline{1}-\mu_2)=\lambda_2$ is not $\frac{1}{2}$-feo set in (X,τ).

Example 3.230 *Let $X = \{x_1,x_2\}$, $Y = \{y_1,y_2,y_3\}$ and $F: X\ddot{}Y$ be a FM defined by $G_F(x_1,y_1) = 0.8$, $G_F(x_1,y_2) = 1$, $G_F(x_1,y_3) = 0.3$, $G_F(x_2,y_1) = 0.5$, $G_F(x_2,y_2) = \overline{1}$, and $G_F(x_2,y_3) = 0.6$. Let λ_1 and λ_2 be a fuzzy subset of X be defined as $\lambda_1(x_1) = 0.4$, $\lambda_1(x_2) = 0.1$; $\lambda_2(x_1) = 0.3$, $\lambda_2(x_2) = 0.3$, μ_1 and μ_2 be a fuzzy subsets of Y defined as $\mu_1(y_1) = 0.5$, $\mu_1(y_2) = 0.5$, $\mu_1(y_3) = 0.5$ and $\mu_2(y_1) = 0.7$, $\mu_2(y_2) = 0.7$, $\mu_2(y_3) = 0.7$. We assume that $\overline{1} = 1$ and $\overline{0} = 0$. Define L-fuzzy topologies $\tau: L^X \to L$ and $\eta: L^Y \to L$ as follows:* $\tau(\lambda) = \begin{cases} 1 & \text{if } \lambda = \overline{0}, \text{ or } \overline{1} \\ \frac{1}{2} & \text{if } \lambda = \lambda_1 \\ 0 & \text{otherwise} \end{cases}$ $\eta(\mu) = \begin{cases} 1 & \text{if } \mu = \overline{0}, \text{ or } \overline{1} \\ \frac{1}{2} & \text{if } \mu = \mu_1, \mu_2 \\ 0 & \text{otherwise} \end{cases}$

are fuzzy topologies on X and Y. For $r = \frac{1}{2}$, as $\overline{1} - \mu_1$ is $\frac{1}{2}$-frc set in Y and $F^l(\overline{1}-\mu_1) = \mu_1$ is $\frac{1}{2}$-fe*o set in X. Hence F is $FLACe^*$-continuous but not $FLCe$-continuous because $\overline{1} - \mu_2$ is closed in Y, $F^l(\overline{1}-\mu_2) = \lambda_2$ is not $\frac{1}{2}$-feo set in X.

Example 3.331 *Let* $X=\{x_1,x_2\}$, $Y=\{y_1,y_2,y_3\}$ *and* $F: X\ddot{}Y$ *be a FM defined by* $G_F(x_1,y_1)=0.4$, $G_F(x_1,y_2)=0.6$, $G_F(x_1,y_3)=0.2$, $G_F(x_2,y_1)=0.2$, $G_F(x_2,y_2)=0.1$, *and* $G_F(x_2,y_3)=0.3$. *Let* λ *be a fuzzy subset of* X *defined as* $\lambda(x_1)=0.2$, $\lambda(x_2)=0.1$ *and* μ *be a fuzzy subset of* Y *defined as* $\mu(y_1)=0.3$, $\mu(y_2)=0.4$, $\mu(y_3)=0.5$. *We assume that* $\overline{1}=1$ *and* $\overline{0}=0$. *Define* L *-fuzzy topologies* $\tau: L^X \to L$ *and* $\eta: L^Y \to L$ *as follows:*

$$\tau(\lambda)=\begin{cases}1 & if\ \lambda=\bar{0},\ or\ \bar{1}\\ \frac{1}{2} & if\ \lambda=\lambda\\ 0 & otherwise\end{cases}\qquad \eta(\mu)=\begin{cases}1 & if\ \mu=\bar{0},\ or\ \bar{1}\\ \frac{1}{2} & if\ \mu=\mu\\ 0 & otherwise\end{cases}$$

are fuzzy topologies on X and Y. For $r=\frac{1}{2}$, then F is $FUACe^*$-continuous but not $FUAC$-continuous because $\overline{1}-\mu$ is $\frac{1}{2}$-frc in Y and $F^u(\overline{1}-\mu)=(0.6_{x_1},0.7_{x_2})$ is not $\frac{1}{2}$-fuzzy open set in X.

Example 3.432 *Let* $X=\{x_1,x_2\}$, $Y=\{y_1,y_2,y_3\}$ *and* $F: X\ddot{}Y$ *be a FM defined by* $G_F(x_1,y_1)=0.2$, $G_F(x_1,y_2)=1$, $G_F(x_1,y_3)=0$, $G_F(x_2,y_1)=0.5$, $G_F(x_2,y_2)=0$, *and* $G_F(x_2,y_3)=0.3$. *Let* λ_1 *and* λ_2 *be a fuzzy subsets of* X *be defined as* $\lambda_1(x_1)=0.3$, $\lambda_1(x_2)=0.5$; $\lambda_2(x_1)=0.2$, $\lambda_2(x_2)=0.5$ *and* μ *be a fuzzy subset of* Y *defined as* $\mu(y_1)=0.4$, $\mu(y_2)=0.1$, $\mu(y_3)=0.1$. *We assume that* $\overline{1}=1$ *and* $\overline{0}=0$. *Define* L *-fuzzy topologies* $\tau: L^X \to L$ *and* $\eta: L^Y \to L$ *as follows:*

$$\tau(\lambda)=\begin{cases}1 & if\ \lambda=\bar{0},\ or\ \bar{1}\\ \frac{1}{2} & if\ \lambda=\lambda_1\\ 0 & otherwise\end{cases}\qquad \eta(\mu)=\begin{cases}1 & if\ \mu=\bar{0},\ or\ \bar{1}\\ \frac{1}{2} & if\ \mu=\mu\\ 0 & otherwise\end{cases}$$

are fuzzy topologies on X and Y. For $r=\frac{1}{2}$, then F is $FLACe^*$-continuous but not $FLAC$-continuous because $\overline{1}-\mu$ is $\frac{1}{2}$-frc in Y, $F^l(\overline{1}-\mu)=\lambda_2$ is not $\frac{1}{2}$-fuzzy open set in X.

Theorem 3.3 33 *Let* $F: X\ddot{}Y$ *be a FM between two* L*-fts's* (X,τ) *and* (Y,η). *If* $e^*C_\tau(F^u(\mu),r)\le F^u(e^*\text{-}ker_\eta(\mu,r))$ *for any* $\mu\in L^Y$, *then* F *is* $FLACe^*$*-continuous.*

Proof. Suppose that $e^*C_\tau(F^u(\mu),r)\le F^u(e^*\text{-}ker_\eta(\mu,r))$ for any $\mu\in L^Y$. Let $\nu\in L^Y$ and ν is r-fe*o by Lemma (3.1), we have,

$e^*C_\tau(F^u(\nu),r)\le F^u(e^*\text{-}ker_\eta(\nu,r))=F^u(\nu)$.

This implies that $e^*C_\tau(F^u(\nu),r)=F^u(\nu)$ and hence $\overline{1}-F^u(\nu)$ is r-fe^*o-set. Thus by Theorem (3.1) (iii) F is $FLACe^*$-continuous.

Theorem 3.434 *Let* $F: X\ddot{}Y$ *be a FM and normalized between two* L*-fts's* (X,τ) *and* (Y,η). *If* $e^*C_\tau(F^l(\mu),r)\le F^l(e^*\text{-}ker_\eta(\mu,r))$ *for any* $\mu\in L^Y$ *then* F *is* $FUACe^*$*-continuous.*

Proof. Suppose that $e^*C_\tau(F^l(\mu),r)\le F^l(e^*\text{-}ker_\eta(\mu,r))$ for any $\mu\in L^Y$. Let $\nu\in L^Y$ and ν is r-fe*o by Lemma (2.1), we have

$e^*C_\tau(F^l(\nu),r)\le F^l(e^*\text{-}ker_\eta(\nu,r))=F^l(\nu)$.

This implies that $e^*C_\tau(F^l(\nu),r)=F^l(\nu)$ and hence $\overline{1}-F^l(\nu)$ is r-fe^*o-set. Thus by Theorem (3.2)(iii), F is $FUACe^*$-continuous.

Theorem 3.535 *Let* $\{F_i\}_{i\in\Gamma}$ *be a family of* $FLACe^*$*-continuous between two* L*-fts's* (X,τ) *and* (Y,η). *Then* $\cup_{i\in\Gamma}F_i$ *is* $FLACe^*$*-continuous.*

Proof. Let $\mu\in L^Y$ and μ is r-frc, then $(\cup_{i\in\Gamma}F_i)^l(\mu)=\vee_{i\in\Gamma}(F_i^l(\mu))$ by Theorem (2.3)(ii). Since $\{F_i\}_{i\in\Gamma}$ is a family of $FLACe^*$-continuous between two L-fts's (X,τ) and (Y,η), then $F_i^l(\mu)$ is r-fe*o-set for each $i\in\Gamma$. Then for each $\mu\in L^Y$ and μ is r-frc, we have, $(\cup_{i\in\Gamma}F_i)^l(\mu)=\vee_{i\in\Gamma}(F_i^l(\mu))$ is r-fe*o set. Hence $\cup_{i\in\Gamma}F_i$ is $FLACe^*$-continuous.

Theorem 3.636 *Let* F_1 *and* F_2 *be two normalized* $FUACe^*$*-continuous between two* L*-fts's* (X,τ) *and* (Y,η). *Then* $F_1\cup F_2$ *is* $FUACe^*$*-continuous*

Proof. Let $\mu\in L^Y$ and μ is r-frc, then $(F_1\cup F_2)^u(\mu)=F_1^u(\mu)\wedge F_2^u(\mu)$ by Theorem (2.3)(iii). Since F_1 and F_2 be two normalized $FUACe^*$-continuous between two L-fts's (X,τ) and (Y,η), then $F_i^u(\mu)$ is r-fe*o-set for each $i\in\{1,2\}$. Then for each $\mu\in L^Y$ and μ is r-frc, we have $(F_1\cup F_2)^u(\mu)=F_1^u(\mu)\wedge F_2^u(\mu)$ is r-fe*o-set. Hence $F_1\cup F_2$ is $FUACe^*$-continuous.

Theorem 3.737 *Let* $F: X\ddot{}Y$ *and* $H: Y\ddot{}Z$ *be two FM's and let* (X,τ), (Y,η) *and* (Z,δ) *be three* L*-fts's. If* F *is* FLe^**-irresolute and* H *is* $FLACe^*$*-continuous, then* $H\circ F$ *is* $FLACe^*$*-continuous.*

Proof. Let $\nu\in L^Z$, ν is $r\text{-}frc$. Since H is $FLACe^*$-continuous, then from Theorem (3.1), $H^l(\nu)$ is r-fe*o set in Y. Also, F is FLe^*-irresolute implies $F^l(H^l(\nu))$ is r-fe*o set in X. Hence, we have $(H\circ F)^l(\nu)=F^l(H^l(\nu))$ is $r\text{-}fe^*o$. Thus $H\circ F$ is $FLACe^*$-continuous.

Theorem 3.8 38 *Let* $F: X\ddot{}Y$ *and* $H: Y\ddot{}Z$ *be two FM's and let* (X,τ), (Y,η) *and* (Z,δ) *be three* L*-fts's. If* F *and* H *are normalized,* F *is* FUe^**-irresolute and* H *is* $FUACe^*$*-continuous, then* $H\circ F$ *is* $FUACe^*$*-continuous.*

Proof. Proof is similar to the above Theorem (3.7)

Theorem 3.939 *Let* $F: X\ddot{}Y$ *and* $H: Y\ddot{}Z$ *be two FM's and let* (X,τ), (Y,η) *and* (Z,δ) *be three* L*-fts's. If* H *is normalized and* H *is* $FUACe^*$*-continuous and* F *is* FLe^**-irresolute, then* $H\circ F$ *is* $FLACe^*$*-continuous.*

Proof. Let $\nu \in L^Z$, ν is r-frc. Since H is $FUACe^*$-continuous, then from Theorem (3.2), $H^u(\nu)$ is r-fe*o set in Y. Also, F is FLe^*-irresolute implies $F^l(H^u(\nu))$ is r-fe*o set in X. Hence, we have $(H \circ F)^l(\nu) = F^l(H^u(\nu))$ is r-fe^*o. Thus $H \circ F$ is $FLACe^*$-continuous.

We state the following result without proof in view of the above Theorem.

Theorem 3.1040 *Let $F: X\ddot{} Y$ and $H: Y\ddot{} Z$ be two FM's and let (X,τ), (Y,η) and (Z,δ) be three L-fts's. If F is normalized, F is FUe^*-irresolute and H is $FLACe^*$-continuous, then $H \circ F$ is $FUACe^*$-continuous.*

4.Fuzzy upper and lower weakly contra e^*-continuous multifunctions

Definition 4.141 *Let $F: X\ddot{} Y$ be a FM between two L-fts's (X,τ), (Y,η) and $r \in L_0$. Then F is called.*

- Fuzzy upper weakly contra e^*-continuous ($FUWCe^*$-continuous, in short) at an L-fuzzy point $x_t \in dom(F)$ iff $x_t \in F^u(\mu)$ for each $\mu \in L^Y$ and μ is r-fuzzy closed, there exists r-fe^*o-set $\lambda \in L^X$ and $x_t \in \lambda$ such that $\lambda \wedge dom(F) \leq F^u(C_\eta(\mu,r))$.
- Fuzzy lower weakly contra e^*-continuous ($FLWCe^*$-continuous, in short) at an L-fuzzy point $x_t \in dom(F)$ iff $x_t \in F^l(\mu)$ for each $\mu \in L^Y$ and μ is r-fuzzy closed, there exists r-fe*o-set $\lambda \in L^X$ and $x_t \in \lambda$ such that $\lambda \leq F^l(C_\eta(\mu,r))$.
- $FUWCe^*$-continuous (resp. $FLWCe^*$-continuous) iff it is $FUWCe^*$-continuous (resp. $FLWCe^*$-continuous) at every $x_t \in dom(F)$.

Proposition 4.1 42 *F is normalized, then F is $FUWCe^*$-continuous at a fuzzy point $x_t \in dom(F)$ iff $x_t \in F^u(\mu)$ for each $\mu \in L^Y$ and μ is r-fuzzy closed, there exists r-fe*o-set $\lambda \in L^X$ and $x_t \in \lambda$ such that $\lambda \leq F^u(C_\eta(\mu,r))$.*

Theorem 4.143 *Let $F: X\ddot{} Y$ be a FM between two L-fts's (X,τ), (Y,η) and $\mu \in L^Y$. Then F is $FLWCe^*$-continuous if and only if $F^l(\mu) \leq e^*I_\tau(F^l(C_\eta(\mu,r)),r)$ for any $\mu \in L^Y$ and μ is r-fuzzy closed.*

Proof. Let F be $FLWCe^*$-continuous, $\mu \in L^Y$ and μ is r-fuzzy closed. If $x_t \in F^l(\mu)$, there exixts r-fe*o set $\lambda \in L^X$ and $x_t \in \lambda$ such that $\lambda \leq F^l(C_\eta(\mu,r)),r)$ and hence $\lambda \leq e^*I_\tau(F^l(C_\eta(\mu,r)),r)$. Thus $F^l(\mu) \leq e^*I_\tau(F^l(C_\eta(\mu,r)),r)$.

Conversely, let $x_t \in dom(F)$, $\mu \in L^Y$, μ is r-fuzzy closed and $x_t \in F^l(\mu)$. Then

$$x_t \in F^l(\mu) \leq e^*I_\eta(F^l(C_\eta(\mu,r)),r) = \lambda(say).$$

Thus, $x_t \in \lambda$ and λ is r-fe*o set such that

$$\lambda = e^*I_\tau(F^l(C_\eta(\mu,r)),r) \leq F^l(C_\eta(\mu,r)).$$

Hence, F is $FLWCe^*$-continuous.

Theorem 4.244 *Let $F: X\ddot{} Y$ be a FM and normalized between two L-fts's (X,τ), (Y,η) and $\mu \in L^Y$. Then F is $FUWCe^*$-continuous if and only if $F^u(\mu) \leq e^*I_\tau(F^u(C_\eta(\mu,r)),r)$ for any $\mu \in L^Y$ and μ is r-fuzzy closed.*

Proof. This can be proved in a similar way as the above Theorem (4.1)

Remark 4.145 *The following implications hold.*

- $FUWC$-continuous $\Rightarrow$ $FUWCe$-continuous $\Rightarrow$ $FUACe^*$-continuous.
- $FLWC$-continuous $\Rightarrow$ $FLWCe$-continuous $\Rightarrow$ $FLACe^*$-continuous.

The Converse of the above Remark (4.1) need not be true as shown by the following examples.

Example 4.146 *Let $X = \{x_1,x_2\}$, $Y = \{y_1,y_2,y_3\}$ and $F: X\ddot{} Y$ be a FM defined by $G_F(x_1,y_1) = 0.1$, $G_F(x_1,y_2) = \overline{1}$, $G_F(x_1,y_3) = \overline{0}$, $G_F(x_2,y_1) = 0.6$, $G_F(x_2,y_2) = \overline{1}$, and $G_F(x_2,y_3) = 0.3$. Let λ_1 and λ_2 be a fuzzy subsets of X defined as $\lambda_1(x_1) = 0.2$, $\lambda_1(x_2) = 0.3$; $\lambda_2(x_1) = 0.9$, $\lambda_2(x_2) = 0.6$ and μ be a fuzzy subset of Y defined as $\mu(y_1) = 0.4$, $\mu(y_2) = 0.1$, $\mu(y_3) = 0.2$. We assume that $\overline{1} = 1$ and $\overline{0} = 0$. Define L-fuzzy topologies $\tau: L^X \to L$ and $\eta: L^Y \to L$ as follows:*

$$\tau(\lambda) = \begin{cases} 1 & \text{if } \lambda = \overline{0}, \text{ or } \overline{1} \\ \frac{1}{2} & \text{if } \lambda = \lambda_1, \lambda_2 \\ 0 & \text{otherwise} \end{cases} \qquad \eta(\mu) = \begin{cases} 1 & \text{if } \mu = \overline{0}, \text{ or } \overline{1} \\ \frac{1}{2} & \text{if } \mu = \mu \\ 0 & \text{otherwise} \end{cases}$$

are fuzzy topologies on X and Y. For $r=\frac{1}{2}$, then F is $FUWCe$-continuous but not $FUWC$-continuous because $\overline{1} - \mu$ is $\frac{1}{2}$-fuzzy closed in Y and $F^u(\overline{1} - \mu) = \lambda_2$ is not $\frac{1}{2}$-fuzzy open set in X.

Example 4.247 *Let $X = \{x_1,x_2\}$, $Y = \{y_1,y_2,y_3\}$ and $F: X\ddot{} Y$ be a FM defined by $G_F(x_1,y_1) = 0.1$, $G_F(x_1,y_2) = \overline{1}$, $G_F(x_1,y_3) = \overline{0}$, $G_F(x_2,y_1) = 0.6$, $G_F(x_2,y_2) = \overline{1}$, and $G_F(x_2,y_3) = 0.3$. Let λ_1 and λ_2 be a fuzzy subsets of X be defined as $\lambda_1(x_1) = 0.2$, $\lambda_1(x_2) = 0.3$; $\lambda_2(x_1) = 0.9$, $\lambda_2(x_2) = 0.9$ and μ be a fuzzy subset of Y defined as $\mu(y_1) = 0.4$, $\mu(y_2) = 0.1$, $\mu(y_3) = 0.2$. We assume that $\overline{1} = 1$ and $\overline{0} = 0$. Define L-fuzzy topologies $\tau: L^X \to L$ and $\eta: L^Y \to L$ as follows:*

$$\tau(\lambda) = \begin{cases} 1 & \text{if } \lambda = \overline{0}, \text{ or } \overline{1} \\ \frac{1}{2} & \text{if } \lambda = \lambda_1, \lambda_2 \\ 0 & \text{otherwise} \end{cases} \qquad \eta(\mu) = \begin{cases} 1 & \text{if } \mu = \overline{0}, \text{ or } \overline{1} \\ \frac{1}{2} & \text{if } \mu = \mu \\ 0 & \text{otherwise} \end{cases}$$

are fuzzy topologies on X and Y. For $r = \frac{1}{2}$, then F is $FLWCe$-continuous but not $FLWC$-continuous because $\overline{1} - \mu$ is $\frac{1}{2}$-fuzzy closed in Y, $F^l(\overline{1} - \mu) = \lambda_2$ is not $\frac{1}{2}$-fuzzy open set in X.

Example 4.348 *Let* $X = \{x_1, x_2\}$, $Y = \{y_1, y_2, y_3\}$ *and* $F: X\ddot{}Y$ *be a FM defined by* $G_F(x_1, y_1) = 0.1$, $G_F(x_1, y_2) = \overline{1}$, $G_F(x_1, y_3) = \overline{0}$, $G_F(x_2, y_1) = 0.5$, $G_F(x_2, y_2) = \overline{0}$, *and* $G_F(x_2, y_3) = \overline{1}$. *Let* λ_1 *and* λ_2 *be a fuzzy subsets of* X *be defined as* $\lambda_1(x_1) = 0.3$, $\lambda_1(x_2) = 0.5$; $\lambda_2(x_1) = 0.4$, $\lambda_2(x_2) = 0.4$ *and* μ_1 *and* μ_2 *be a fuzzy subsets of* Y *defined as* $\mu_1(y_1) = 0.5$, $\mu_1(y_2) = 0.5$, $\mu_1(y_3) = 0.5$ *and* $\mu_2(y_1) = 0.4$, $\mu_2(y_2) = 0.4$, $\mu_2(y_3) = 0.4$ *We assume that* $\overline{1} = 1$ *and* $\overline{0} = 0$. *Define L-fuzzy topologies* $\tau: L^X \to L$ *and* $\eta: L^Y \to L$ *as follows:*

$$\tau(\lambda) = \begin{cases} 1 & if\ \lambda = \overline{0},\ or\ \overline{1} \\ \frac{1}{2} & if\ \lambda = \lambda_1 \\ 0 & otherwise \end{cases} \qquad \eta(\mu) = \begin{cases} 1 & if\ \mu = \overline{0},\ or\ \overline{1} \\ \frac{1}{2} & if\ \mu = \mu_1, \mu_2 \\ 0 & otherwise \end{cases}$$

are fuzzy topologies on X and Y. For $r = \frac{1}{2}$, then F is

(i) $FUACe^*$-continuous but not $FUWCe$-continuous because μ_2 is $\frac{1}{2}$-fuzzy closed in Y and $F^u(\mu_2) = \lambda_2$ is not $\frac{1}{2}$-feo set in X.

(ii) $FLACe^*$-continuous but not $FLWCe$-continuous because μ_2 is $\frac{1}{2}$-fuzzy closed in Y and $F^u(\mu_2) = \lambda_2$ is not $\frac{1}{2}$-feo set in X.

References

[1] S. E. Abbas, M. A . Hebeshi and I. M. Taha, *On fuzzy upper and lower semi-continuous multifunctions*, The Journal of Fuzzy Mathematics, **22** (4) (2014), 951--962.

[2] K. M. A. Al-hamadi and S. B. Nimse, *On fuzzy α-continuous multifunctions*, Miskolc Mathematical Notes, **11** (2) (2010), 105-112.

[3] M. Alimohammady, E.Ekici, S.Jafari and M. Roohi, *On fuzzy upper and lower contra continuous multifunctions*, Iranian Journal of Fuzzy Systems , **8** (3) (2011), 149-158.

[4] C. Berge, *Topological spaces including a treatment of multi-valued functions*, Vector Spaces and Convexity, Oliver, Boyd London, (1963).

[5] C. L. Chang, *Fuzzy topological spaces*, J. Math. Anal. Appl., **24** (1968), 182--189.

[6] K. C. Chattopadhyay and S. K. Samanta, *Fuzzy topology : fuzzy closure operator, fuzzy compactness and fuzzy connectedness, Fuzzy sets and systems*, **54** (2) (1993), 207--212.

[7] P. Dhanasekaran, M. Angayarkanni, B. Vijayalakshmi and A. Vadivel, *On fuzzy upper and lower e^*(δs and δp)-irresolute multifunctions*, (submitted).

[8] P. Dhanasekaran, M. Angayarkanni, B. Vijayalakshmi and A. Vadivel, *On fuzzy upper and lower almost contra e-continuous multifunctions*, (submitted).

[9] J. A. Goguen, *The fuzzy Tychonoff Theorem*, J. Math. Anal. Appl., **43**(3) (1973), 734--742.

[10] U. Höhle, *Upper semicontinuous fuzzy sets and applications*, J. Math. Anal. Appl., **78** (1980), 659--673.

[11] U. Höhle and A. P. Šostak, *A general theory of fuzzy topological spaces*, Fuzzy Sets and Systems, **73** (1995), 131--149.

[12] U. Höhle and A. P. Šostak, *Axiomatic Foundations of Fixed-Basis fuzzy topology*, The Handbooks of Fuzzy sets series, Volume 3, Kluwer Academic Publishers, (1999), 123--272.

[13] M. A . Hebeshi and I. M. Taha, *Weaker forms of contra-continuous fuzzy multifunctions*, The Journal of Fuzzy Mathematics, **23** (2) (2015), 341--354.

[14] Y. C. Kim and J. W. Park, *r-fuzzy δ-closure and r-fuzzy θ-closure sets*, J. Korea Fuzzy Logic and Intelligent systems, **10**(6) (2000), 557-563.

[15] Y. C. Kim, A. A. Ramadan and S. E. Abbas, *Weaker forms of continuity in Šostak's fuzzy topology*, Indian J. Pure and Appl. Math., **34** (2) (2003), 311--333.

[16] Y. C. Kim, *Initial L-fuzzy closure spaces*, Fuzzy Sets and Systems., **133** (2003), 277-297.

[17] T. Kubiak, *On fuzzy topologies*, Ph.D. Thesis, A. Mickiewicz, Poznan, (1985).

[18] T. Kubiak and A.P. Šostak, *Lower set valued fuzzy topologies*, Questions Math., **20** (3) (1997), 423-429.

[19] Y. Liu and M. Luo, *Fuzzy topology*, World Scientific Publishing Singapore., (1997), 229-236.

[20] R. A. Mahmoud, *An application of continuous fuzzy multifunctions*, Chaos, Solitons and Fractals, **17** (2003), 833-841.

[21] M. N. Mukherjee and S. Malakar, *On almost continuous and weakly continuous fuzzy multifunctions*, Fuzzy Sets and Systems, **41** (1991), 113--125.

[22] N. S. Papageorgiou, *Fuzzy topolgy and fuzzy multifunctions*, J. Math. Anal. Appl., **109** (1985), 397-425.

[23] A. Prabhu, A. Vadivel and B. Vijayalakshmi, *On fuzzy upper and lower e-continuous multifunctions*, (submitted).

[24] A. P. Šostak, *On a fuzzy topological structure*, Suppl. Rend. Circ. Matem. Palermo Ser II **11** (1985), 89--103.

[25] A. P. Šostak, *Two decades of fuzzy topology : Basic ideas, Notion and results*, Russian Math. Surveys, **44** (6) (1989), 125--186.

[26] A. P. Šostak, *Basic structures of fuzzy topology*, J. Math. Sciences **78** (6) (1996), 662--701.

[27] D. Sobana, V. Chandrasekar and A. Vadivel, *Fuzzy e-continuity in Šostak's fuzzy topological spaces*, (Submitted).

[28] M. Sujatha, M. Angayarkanni, B. Vijayalakshmi and A. Vadivel, *On fuzzy upper and lower contra e-continuous multifunctions*, (submitted).

[29] E. Tsiporkova, B. De Baets and E. Kerre, *A fuzzy inclusion based approach to upper inverse images under fuzzy multivalued mappings*, Fuzzy sets and systems, **85** (1997), 93--108.

[30] E. Tsiporkova, B. De Baets and E. Kerre, *Continuity of fuzzy multivalued mappings*, Fuzzy sets and systems, **94** (1998), 335--348.

[31] A. Vadivel and B. Vijayalakshmi, *Fuzzy Almost e-continuous mappings and fuzzy e-connectedness in smooth topological spaces*, accepted in The Journal of Fuzzy Mathematics.

[32] C. K. Wong, *Fuzzy topology: product and quotient theorems*, J. Math. Anal. Appl, **45** (1974), 512-521.