UPPER AND LOWER E-CONTINUOUS INTUITIONISTIC FUZZY MULTIFUNCTIONS

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Abstract In this paper, we introduce upper and lower e-continuous intuitionistic fuzzy multifunction from a topological space to an intuitionistic fuzzy topological space.

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Introduction

After the introduction of fuzzy sets by Zadeh [15] in 1965 and fuzzy topology by Chang [4] in 1967, several researches were conducted on the generalizations of the notions of fuzzy sets and fuzzy topology. The concept of intuitionistic fuzzy sets (IFS, for short) was introduced by Atanassov [1,2,3] as a generalization of fuzzy sets. In the last 27 years various concepts of fuzzy mathematics have been extended for intuitionistic fuzzy sets. In 1997 Coker [5] introduced the concept of intuitionistic fuzzy topological spaces (IFTS, for short) as a generalization of fuzzy topological spaces. In 1999, Ozbakir and Coker [8] introduced the concept intuitionistic fuzzy multifunctions and studied their lower and upper intuitionistic fuzzy semi continuity from a topological space to an intuitionistic fuzzy topological space. Recently the author Sobana.et.al [11] of this paper introduced the concepts of intuitionistic fuzzy e-open sets and maps in intuitionistic fuzzy topological space. In the present paper we study the concepts of lower and upper e-continuous intuitionistic fuzzy multifunctions.

Preliminaries Throughout this paper (X,T) and (Y,S) represents a topological space and an intuitionistic fuzzy topological space respectively.

Definition 2.1 1 *[12] A subset A of a topological space (X,T) is called*
- regular open if $A = intcl(A)$,
- regular closed if $A = clint(A)$,

Definition 2.2 2 *[12] Let (X,T) be a topological space and A be a subset of X. Then the δ-interior and δ-closure of A are defined by:*

$$int_\delta(A) = \cup \{K: K \text{isregularopenset} K \subseteq A\},$$
$$cl_\delta(A) = \cap \{K: K \text{isregularclosedset} A \subseteq K\}.$$

Definition 2.3 3 *A subset A of a topological space (X,T) is called*
- δ-semi open [9] if $A \subseteq clint_\delta(A)$,
- δ-semi closed [9] if $A \supseteq intcl_\delta(A)$,
- δ-pre open [10] if $A \subseteq intcl_\delta(A)$,
- δ-pre closed [10] if $A \supseteq clint_\delta(A)$,
- e-open [7] if $A \subseteq clint_\delta(A) \cup intcl_\delta(A)$,
- e-closed [7] if $A \supseteq intcl_\delta(A) \cap clint_\delta(A)$.

Remark 2.1 4 *Every open (resp. closed) set is e-open (resp. e-closed) and every δ-semi open, δ-pre open (resp. δ-semi closed, δ-pre closed) set is e-open (resp. e-closed) but the separate converses may not be true.*

The family of all e-open (resp. e-closed, δ-semi open, δ-pre open, δ-semi closed, δ-pre closed) subsets of topological space (X,T) is denoted by $eO(X)$ (resp. $eC(X)$, $\delta SO(X)$, $\delta PO(X)$, $\delta SC(X)$, $\delta PC(X)$). The intersection of all e-closed (resp. δ-semi closed, δ-pre closed) sets of X containing a set A of X is called the e-closure (resp. δ-semi closure, δ-pre closure) of A. It is denoted by $ecl(A)$ (resp. $\delta scl(A)$, $\delta pcl(A)$). The union of all e-open (resp. δ-semi open, δ-pre open) sub sets of A of X is called the e-interior (resp. δ-semi interior, δ-pre interior) of A. It is denoted by $eint(A)$ (resp. $\delta sint(A)$, $\delta pint(A)$). A subset N of a topological space (X,T) is called e-neighborhood of a point x of X if there exist a e-open set O of X such that $x \in O \subset N$. A is a e-open in X if and only if it is a e-neighborhood of each of its points. A mapping f from a topological space (X,T) to another topological space (X^*,T^*) is said to be e-continuous if the inverse image of every open set of X^* is e-open in X. Every continuous mapping is e-continuous but the converse may not be true. A multifunction F from a topological space (X,T) to another topological space (X^*,T^*) is said to be lower e-continuous (resp. upper e-continuous) at a point $x_0 \in X$ if for every e-neighborhood U of x_0 and for any open set W of X^* such that $F(x_0) \cap W \neq \phi$ (resp. $F(x_0 \subset W)$ there is a e-neighborhood U of x_0 such that $F(x) \cap W \neq \phi$ (resp. $F(x) \subset W$) every $x \in U$.

Definition 2.4 5 *[1,2,3] Let Y be a nonempty fixed set. An IFS $\tilde{A}$ in Y is an object having the form $\tilde{A} = \{\langle \mu_A(y), \nu_A(y) \rangle : y \in Y\}$ here the functions $\mu_A(y): Y \to I$ and $\nu_A(y): Y \to I$ denotes the degree of membership (namely $\mu_A(y)$) and the degree of non membership (namely $\nu_A(y)$) of each element $y \in Y$ to the set $\tilde{A}$ respectively, and $0 \le \mu_A(y) + \nu_A(y) \le 1$ for each $y \in Y$.*

Definition 2.56 *[1,2,3] Let Y be a nonempty set and the IFS $\tilde{A}$ and $\tilde{B}$ be in the form $\tilde{A} = \{\langle \mu_A(y), \nu_A(y) \rangle : y \in Y\}, \tilde{B} = \{\langle \mu_B(y), \nu_B(y) \rangle : y \in Y\}$ and let $\{\tilde{A}_\alpha : \alpha \in \wedge\}$ be an arbitrary family of intuitionistic fuzzy sets in Y. Then:*

- $\tilde{A} \subseteq \tilde{B}$ if $\forall y \in Y[\mu_{\tilde{A}}(y) \le \mu_{\tilde{B}} \text{and} \nu_{\tilde{A}}(y) \ge \nu_{\tilde{B}}(y)]$,
- $\tilde{A} = \tilde{B}$ if $\tilde{A} \subseteq \tilde{B}$ and $\tilde{B} \subseteq \tilde{A}$,
- $\tilde{A}^c = \{\langle y, \nu_{\tilde{A}}(y), \mu_{\tilde{A}}(y) : y \in Y\}$,
- $\tilde{0} = \{\langle y, 0,1 \rangle : y \in Y\}$ and $\tilde{1} = \{\langle y, 1,0 \rangle : y \in Y\}$,
- $\cap \tilde{A}_\alpha = \{\langle y, \wedge \mu_{\tilde{A}}(y), \vee \nu_{\tilde{A}}(y) \rangle : y \in Y\}$,
- $\cup \tilde{A}_\alpha = \{\langle y, \vee \mu_{\tilde{A}}(y), \wedge \nu_{\tilde{A}}(y) \rangle : y \in Y\}$.

Definition 2.6 7 *[6] Two IFS`s $\tilde{A}$ and $\tilde{B}$ of Y are said to be quasi coincident ($\tilde{A}q\tilde{B}$ for short) if $\exists y \in Y$ such that $\mu_{\tilde{A}}(y) > \nu_{\tilde{B}}(y)$ or $\nu_{\tilde{A}}(y) < \nu_{\tilde{B}}(y)$.*

Definition 2.7 8 *[6] For any two IFS`s $\tilde{A}$ and $\tilde{B}$ of Y, $(\tilde{A}q\tilde{B}) \Leftrightarrow \tilde{A} \subset \tilde{B}^c$.*

Definition 2.8 9 *[5] An intuitionistic fuzzy topology on a non empty set Y is a family T of IFS in Y which satisfy the following axioms:*

- $\tilde{0}, \tilde{1} \in S$,
- $\tilde{A}_1 \cap \tilde{A}_2 \in S$ for any $\tilde{A}_1, \tilde{A}_2 \in S$,
- $\cup \tilde{A}_i$ for any arbitrary family $\{\tilde{A}_i : i \in \wedge\} \in S$.

In this case the pair (Y,S) is called an IFTS and each IFS in S is known as an intuitionistic fuzzy open set in Y. The complement $\tilde{B}^c$ of an intuitionistic fuzzy open set $\tilde{B}$ is called an intuitionistic fuzzy closed set in Y.

Definition 2.9 10 *[5] Let (Y,S) be an IFTS and $\tilde{A}$ be an IFS in Y. Then the interior and closure of $\tilde{A}$ are defined by*
$cl(\tilde{A}) = \cap \{\tilde{K} : \tilde{K} \text{isanintuitionisticfuzzyclosedsetin} Y \text{and} \tilde{A} \subseteq \tilde{K}\}$,
$int(\tilde{A}) = \cup \{\tilde{K} : \tilde{K} \text{isanintuitionisticfuzzyopensetin} Y \text{and} \tilde{K} \subseteq \tilde{A}\}$.

Definition 2.10 11 [8] *Let X and Y are two non empty sets. A function $F : (X,T) \to (Y,S)$ is called intuitionistic fuzzy multifunction if $F(x)$ is an intuitionistic fuzzy set in $Y, \forall x \in X$.*

Definition 2.11 12 [13] *Let $F : (X,T) \to (Y,S)$ is an intuitionistic fuzzy multifunction and A be a subset of X. Then $F(A) = \cup_{x \in A} F(x)$.*

Definition 2.12 13 [8] *Let $F : (X,T) \to (Y,S)$ be an intuitionistic fuzzy multifunction. Then the upper inverse $F^+(\tilde{A})$ and lower inverse $F^-(\tilde{A})$ A of an intuitionistic fuzzy set $\tilde{A}$ in Y are defined as follows:*
$F^+(\tilde{A}) = \{x \in X : F(x) \subseteq \tilde{A}\}$ and
$F^-(\tilde{A}) = \{x \in X : F(x) q \tilde{A}\}$

Definition 2.13 14 [14] *Let A be IFS (Y,S). A is called an intuitionistic fuzzy regular open if $A = intcl(A)$ and intuitionistic fuzzy regular closed if $A = clint(A)$*

Definition 2.14 15 *[14]Let (Y,S) be an IFTS and A be an IFS in Y, then the fuzzy δ-closure and δ-interior of A are denoted and defined by*

$$cl_\delta(A) = \wedge \{K : K \text{isanintuitionisticfuzzyregularclosedsetin} Y \text{and} A \le K\},$$
$$int_\delta(A) = \vee \{K : K \text{isanintuitionisticfuzzyregularopensetin} Y \text{and} K \le A\}.$$

Definition 2.15 16 [11] *Let A be an IFS in an IFTS (Y,S). A is called an intuitionistic fuzzy*

- δ-semi open if $A \leq clint_{\delta}(A)$,
- δ-semi closed if $A \geq intcl_{\delta}(A)$,
- δ-pre open if $A \leq intcl_{\delta}(A)$,
- δ-pre closed if $A \geq clint_{\delta}(A)$,
- e-open if $A \leq clint_{\delta}(A) \vee intcl_{\delta}(A)$,
- e-closed if $A \geq clint_{\delta}(A) \wedge intcl_{\delta}(A)$,

3. Upper and lower e-continuous intuitionistic fuzzy multifunctions

Definition 3.1 17 *An intuitionistic fuzzy multifunction $F:(X,T) \to (Y,S)$ is said to be*

• intuitionistic fuzzy upper e-continuous (resp. δ-semi continuous, δ-pre continuous) at a point $x_0 \in X$, if for any intuitionistic fuzzy open set $\widetilde{W} \subset Y$ such that $F(x_0) \subset \widetilde{W}$ there exists a set $U \subset eO(X)$ (resp. $U \in \delta SO(X), U \in \delta PO(X)$) containing x_0 such that $F(U) \subset \widetilde{W}$,

• intuitionistic fuzzy lower e-continuous (resp. δ-semi continuous, δ-pre continuous) at a point $x_0 \in X$, if for any intuitionistic fuzzy open set $\widetilde{W} \subset Y$ such that $F(x_0)q\widetilde{W}$ there exists a set $U \subset eO(X)$ (resp. $U \in \delta SO(X), U \in \delta PO(X)$) containing x_0 such that $F(U)q\widetilde{W}$.

Intuitionistic fuzzy upper e-continuous (resp. δ-semi continuous, δ-pre continuous) and intuitionistic fuzzy lower e-continuous (resp. δ-semi continuous, δ-pre continuous) if it is intuitionistic fuzzy e-continuous (resp. δ-semi continuous, δ-pre continuous) and intuitionistic fuzzy lower e-continuous (resp. δ-semi continuous, δ-pre continuous) at each point of X.

Lemma 3.1 18 *Let A be a subset of a space (X,T). Then A is e-open in (X,T) if and only if A is δ-semi open and δ-pre open*

Theorem 3.119 *An intuitionistic fuzzy multifunction $F:(X,T) \to (Y,S)$ is intuitionistic fuzzy upper e-continuous if and only if it is intuitionstic fuzzy upper δsemi-continuous and intuitionistic fuzzy upper δpre-continuous.*

Proof. It follows from Lemma (3.1) width 0.22 true cm height 0.22 true cm depth 0pt

Corollary 3.120 *A fuzzy multifunction $F:(X,T) \to (Y,S)$ is fuzzy upper e-continuous if and only if it is fuzzy upper δsemi-continuous and fuzzy upper δpre-continuous.*

Corollary 3.221 *A multifunction $F:(X,T) \to (Y,S)$ is upper e-continuous if and only if it is upper δsemi-continuous and upper δpre-continuous.*

Theorem 3.222 *An intuitionistic fuzzy multifunction $F:(X,T) \to (Y,S)$ is intuitionistic fuzzy lower e-continuous if and only if it is intuitionistic fuzzy lower δsemi-continuous and intuitionistic fuzzy lower δpre-continuous.*

Proof. It follows from Lemma (3.1) width 0.22 true cm height 0.22 true cm depth 0pt

Corollary 3.323 *A fuzzy multifunction $F:(X,T) \to (Y,S)$ is fuzzy lower e-continuous if and only if it is fuzzy lower δsemi-continuous and fuzzy lower δpre-continuous.*

Corollary 3.424 *A multifunction $F:(X,T) \to (Y,S)$ is lower e-continuous if and only if it is lower δsemi-continuous and lower δpre-continuous.*

Lemma 3.225 *Let $F:(X,T) \to (Y,S)$ be an intuitionistic fuzzy multifunction. Then $[ecl(F)]^{-}(\tilde{V}) = F^{-}(\tilde{V})$ for each intuitionnistic fuzzy open set $\tilde{V}$ of Y.*

Proof. Suppose that $\tilde{V}$ be any intuitionistic fuzzy open set of Y. Let $x \in [ecl(F)]^{-}(\tilde{V})$. If $x \notin F^{-}(\tilde{V})$. Then, $(F(x)q\tilde{V})$. Which implies $F(x) \subset \tilde{V}^{c}$. Since $\tilde{V}^{c}$ is intuitionistic fuzzy closed and hence intuitionistic fuzzy e-closed, $ecl(F(x)) \subset \tilde{V}^{c}$. Consequently $x \notin [ecl(F)]^{-}(\tilde{V})$, which is a contradiction. Hence, $x \in F^{-}(\tilde{V})$. This shows that $[ecl(F)]^{-}(\tilde{V}) \subset F^{-}(\tilde{V})$. Conversely, let $x \in F^{-}(\tilde{V})$. Then $F(x)q\tilde{V}$. Suppose that $x \notin [ecl(F)]^{-}(\tilde{V})$. Then $(ecl(F(x)))q\tilde{V}$. And so $ecl(F(x)) \subset \tilde{V}^{c}$, which implies that $F(x) \subset \tilde{V}^{c}$. Hence, $(F(x)q\tilde{V})$. Which is a contradiction. Hence $x \in [ecl(F)]^{-}(\tilde{V})$. This shows that $[ecl(F)]^{-}(\tilde{V}) \supset F^{-}(\tilde{V})$. Consequently, we obtain, $[ecl(F)]^{-}(\tilde{V}) = F^{-}(\tilde{V})$. width 0.22 true cm height 0.22 true cm depth 0pt

Theorem 3.326 *An intuitionistic fuzzy multifunction $F:(X,T) \to (Y,S)$ is intuitionistic fuzzy lower e-continuous if and only if $ecl(F):(X,T) \to (Y,S)$ is intuitionistic fuzzy lower e-continuous.*

Proof. Necessity. Suppose that F is intuitionistic fuzzy lower e-continuous. Let $x \in X$ and $\tilde{V}$ be any intuitionistic fuzzy open set of Y such that $ecl(F(x))q\tilde{V}$. By Lemma (3.2), we have $x \in [ecl(F)]^{-}(\tilde{V}) = F^{-}(\tilde{V})$. Since F is intuitionistic fuzzy lower e-continuous, there exists $U \in eO(X)$ containing x such that $F(u)q\tilde{V}, \forall u \in U$. Now $\tilde{V}$ be an intuitionistic fuzzy open set of Y. $F(u)q\tilde{V} \Rightarrow ecl(F(x))q\tilde{V}$. This shows that $ecl(F)$ is intuitionistic fuzzy lower e-continuous.

Sufficiency. Suppose that $ecl(F):(X,T) \to (Y,S)$ is intuitionistic fuzzy lower e-continuous. Let $x \in X$ and V be any e-open set of Y such that $F(x)q\tilde{V}$. By lemma (3.2), we have $x \in [ecl(F)]^{-}(\tilde{V}) = F^{-}(\tilde{V})$ and hence $ecl(F(x))q\tilde{V}$. Since, $ecl(F)$ is intuitionistic fuzzy lower e-continuous, there exists $U \in eO(X)$ containing x such that hence, $ecl(F(x))q\tilde{V}, \forall u \in U$. Hence, $F(u)q\tilde{V}, \forall u \in U$ and F is intuitionistic fuzzy lower e-continuous. width 0.22 true cm height 0.22 true cm depth 0pt

Corollary 3.527 *An fuzzy multifunction $F:(X,T) \to (Y,S)$ is fuzzy lower e-continuous if and only if $ecl(F):(X,T) \to (Y,S)$ is fuzzy lower e-continuous.*

Corollary 3.628 *An multifunction $F:(X,T) \to (Y,S)$ is lower e-continuous if and only if $ecl(F):(X,T) \to (Y,S)$ is lower e-continuous.*

Lemma 3.329 *Let A and B be subsets of a topological space (X,T),*

- If $A \in \delta SO(X) \cup \delta PO(X)$ and $B \in eO(X)$, then $A \cap B \in eO(X)$,
- If $A \subset B \subset X, A \in eO(X)$ and $B \in eO(X)$ then $A \in eO(X)$.

Theorem 3.430 *If an intuitionistic fuzzy multifunction $F:(X,T) \to (Y,S)$ is intuitionistic fuzzy upper e-continuous (resp. intuitionistic fuzzy lower e-continuous) and $X_0 \in \delta PO(X) \cup \delta SO(X)$ then the restriction $F|X_0: X_0 \to Y$ is intuitionistic fuzzy upper e-continuous (resp. intuitionistic fuzzy lower e-continuous).*

Proof. We prove only the assertion for F intuitionistic fuzzy upper e-continuous, the proof for F intuitionistic fuzzy lower e-continuous being analogous. Let $x \in X$ and $\tilde{V}$ be any intuitionistic fuzzy open set of Y such that $F|X_0(x) \subset \tilde{V}$. Since F intuitionistic fuzzy upper e-continuous and $F|X_0(x) = \tilde{V}$, there exists $U \in eO(x)$ containing x such that $F(U) \subset \tilde{V}$. Set $U_0 = U \cap X_0$, then By Lemma 3.225 we have $x \in U_0 \in eO(X_0)$ and $(F|X_0)(U_0) \subset \tilde{V}$. This shows that $F|X_0: X_0 \to Y$ is intuitionistic fuzzy upper e-continuous. width 0.22 true cm height 0.22 true cm depth 0pt

Corollary 3.731 *If a fuzzy multifunction $F:(X,T) \to (Y,S)$ is fuzzy upper e-continuous (resp. fuzzy lower e-continuous) and $X_0 \in \delta PO(X) \cup \delta SO(X)$ then the restriction $F|X_0: X_0 \to Y$ is fuzzy upper e-continuous (resp. fuzzy lower e-continuous).*

Corollary 3.832 *If a multifunction $F:(X,T) \to (Y,S)$ is upper e-continuous (resp. lower e-continuous) and $X_0 \in \delta PO(X) \cup \delta SO(X)$ then the restriction $F|X_0: X_0 \to Y$ is upper e-continuous (resp. lower e-continuous).*

Theorem 3.533 *An intuitionistic fuzzy multifunction $F:(X,T) \to (Y,S)$ is intuitionistic fuzzy upper e-continuous (resp. intuitionistic fuzzy lower e-continuous) if for each $x \in X$ there exists $X_0 \in eO(X)$ containing x such that the restriction $F|X_0: X_0 \to Y$ is intuitionistic fuzzy upper e-continuous (resp. intuitionistic fuzzy lower e-continuous).*

Proof. We prove only the assertion for F intuitionistic fuzzy upper e-continuous, the proof for F intuitionistic fuzzy lower e-continuous being analogous. Let $x \in X$ and $\tilde{V}$ be any intuitionistic fuzzy open set of Y such that $F(x) \subset \tilde{V}$. There exists $X_0 \in eO(X)$ containing x such that $F|X_0(x) = F(x)$, is intuitionistic fuzzy upper e-continuous, there exists $U \in eO(x)$ containing x such that $F|X_0(U_0) \subset \tilde{V}$. Then By Lemma (3.3) we have $x \in U_0 \in eO(X)$ and $F(u) = F|X_0(u)$ for $u \in U_0$. This shows that $F:(X,T) \to (Y,S)$ is intuitionistic fuzzy upper e-continuous. width 0.22 true cm height 0.22 true cm depth 0pt

Corollary 3.934 *A fuzzy multifunction $F:(X,T) \to (Y,S)$ is fuzzy upper e-continuous (resp. fuzzy lower e-continuous) if for each $x \in X$ there exists $X_0 \in eO(X)$ containing x such that the restriction $F|X_0: X_0 \to Y$ is fuzzy upper e-continuous (resp. fuzzy lower e-continuous).*

Corollary 3.1035 *A multifunction $F:(X,T) \to (Y,S)$ is upper e-continuous (resp. lower e-continuous) if for each $x \in X$ there exists $X_0 \in eO(X)$ containing x such that the restriction $F|X_0: X_0 \to Y$ is upper e-continuous (resp. fuzzy lower e-continuous).*

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