# THE REVAN WEIGHTED SZEGED INDEX OF GRAPHS

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**ABSTRACT**

In Chemical Graph Theory, the topological indices are applied to measure the chemical characteristics of chemical compounds. To the continuity of Kulli's[21] work on Revan index and by Illic´ and Milosavljevic[9] on weighted vertex szeged index, we introduce the Revan weighted szeged index of graph, which is an another weighted version of szeged index. We present the exact formula of Revan weighted szeged index of corona product of two connected graphs in terms of other graph invariants including the szeged index, first Zagreb index and second Zagreb index. Finally, we apply this result to compute the exact value of Revan weighted szeged indices for some molecular graphs.

**Keywords: Corona product, Revan index, Szeged index, Weighted szeged index.**
**MSC:** 05C12, 05C76

## 1 Introduction

All the graphs considered in this paper are simple. A vertex $x \in V(G)$ is said to be *equidistant* from the edge $e = uv$ of $G$ if $d_G(u, x) = d_G(v, x)$, where $d_G(u, x)$ denotes the distance between $u$ and $x$ in $G$; otherwise, $x$ is a nonequidistant vertex. The degree of a vertex $x \in V(G)$ is denoted by $d_G(x)$.

For an edge $uv = e \in E(G)$, the number of vertices of $G$ whose distance to the vertex $u$ is less than the distance to the vertex $v$ in $G$ is denoted by $n_u^G(e) = n_u(e, G)$; analogously, $n_v^G(e) = n_v(e, G)$ is the number of vertices of $G$ whose distance to the vertex $v$ in $G$ is less thanthe distance to the vertex $u$; the vertices equidistant from both the ends of the edge $e = uv$ are not counted.

The two topological indices, namely, the *szeged index* and *weighted szeged index* of $G$, denoted by $Sz(G)$ and $Sz_W(G)$, respectively, are defined as follows:

$$Sz(G) = \sum_{e=uv\in E(G)} n_u(e,G)n_v(e,G),$$
$$Sz_w(G) = \sum_{e=uv\in E(G)} \left(d_G(u)+d_G(v)\right)n_u(e,G)n_v(e,G).$$

Graph operations play an important role in the study of graph decompositions into isomorphic subgraphs. It is well known that many graphs arise from simpler graphs via various graph operations and one can study the properties of smaller graphs and deriving with it some information about larger graphs. Hence it is important to understand how certain invariants of such product graphs are related to the corresponding invariants of the original graphs.The corona of two graphs was first introduced by Frucht and Harary in[8]. Let G and H be two simple graphs. The *corona product* $G \circ H$, see Fig. 1, is obtained by taking one copy of $G$ and $|V(G)|$ copies of $H$; and by joining each vertex of the $i$ -th copy of $H$ to the $i$ -th vertex of $G$, where $1 \le i \le |V(G)|$ .

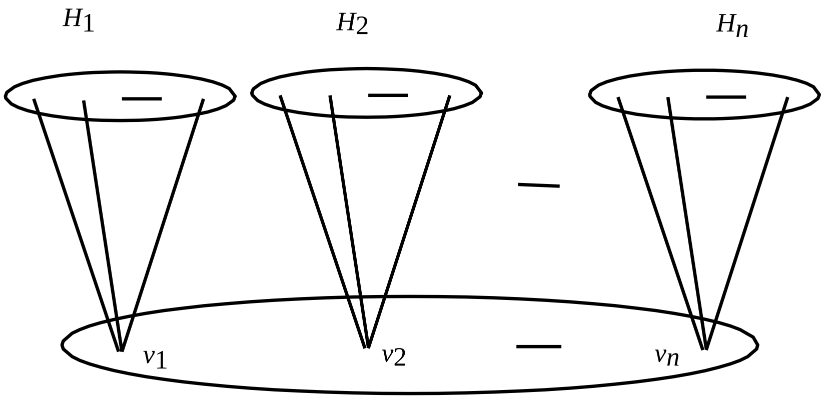


Fig.2 The corona product of two graphs

Corona product of graphs appears in chemical literature as plerographs of the hydrogen suppressed molecular graphs known as kenographs see [24] more information. Different topological indices such as Wiener - type indices[4] , Szeged, vertex PI, first and second Zagreb indices [33], weighted PI index [26], weighted szeged index [27], Rewan weighted PI index [16] etc. of the corona product of two graphs have already been studied. For more application refer [33, 1, 32, 4].

The Szeged index studied by Gutman [10], Gutman and Dobrynin [11] and Khadikar et. al. [15] is closely related to the Wiener index of a graph. Basic properties of Szeged index and its analogy to the Wiener index are discussed by Klavzˇar et al.[14] . It is proved that for a tree $T$ the Wiener index of $T$ is equal to its Szeged index. Ashrafi et. al. [25] have explained the differences between Szeged and Wiener indices of graphs. The mathematical properties and chemical applications of Szeged index are well studied by Dobrynin et. al. [7], Gutman et. al. [12] and Randic et. al. [29]. Recently Pisanski and Randic [28] studied the measuring network bipartivity using Szeged index

Weighted PI index and weighted szeged index of graph $G$ has been introduced by Ilic´ and Milosavljevic [9] and obtained the upper and lower bounds for weighted vertex PI index of graph, for more results see[2, 21, 3, 5, 17, 18, 34, 27]. Kandan et al.[16] recently introduced the Revan weighted PI index of graph and obtained the exact formula for corona product of graphs, also derived the exact value for some standard graphs. In this paper, the exact formula for the Revan weighted szeged index of corona product of two connected graphs is obtained and using this results we deduced exact value of some important classes of graphs.

Let $\Delta(G)$ $(\delta(G))$ denote the maximum (minimum) degree among the vertices of $G$. The revan vertex degree of a vertex $u$ in $G$ is defined as $r_G(u) = \Delta(G) + \delta(G) - d_G(u)$. In[21] Kulli introduced first and second Revan indices of a graph G are respectively defined as

$$R_1 = \sum_{uv\in E(G)}(r_G(u) + r_G(v)) \quad \text{and} \quad R_2 = \sum_{uv\in E(G)}(r_G(u) . r_G(v))$$

and derived the exact value for the various molecular structure. For more of its applications refer [22, 23]. Motivated by the invariants like weighted szeged indices and Revan indices,we define here the Revan Weighted Szeged index of a graph G as follows:

$$Sz_r(G) = \sum_{uv\in E(G)}(r_G(u) + r_G(v))n_u(e, G)n_v(e, G) \qquad (1.1)$$

To connect the Revan weighted szeged index with the well known indices called *first Zagreb index* and *second Zagreb index* which are defined by $M_1(G) = \sum_{e=uv\in G}(d_G(u) + d_G(v))$

and $M_2(G) = \sum_{e=uv\in G} d_G(u)\, d_G(v)$ . The *edge a - Zagreb index* of $G$ is defined as

$Z_a(G) = \sum_{e=uv\in G}(d_G(u)^a + d_G(v)^a)$ It is not hard to see that $Z_1(G) = M_1(G)$, where $M_1(G)$ is the first Zagreb index of $G$. The *edge* $(a, b)$ *- Zagreb index* of $G$ is defined as

$Z'_{a,b}(G) = \sum_{e=uv\in G}(d_G(u)^a + d_G(v)^b + d_G(u)^b + d_G(v)^a)$ . The Zagreb indices are found to have apilications in QSPR and QSAR studies as well, see [6].

## 2 Revan weighted Szeged index of the Corona product of graphs

In this section,we compute the Revan weighted szeged index of the corona product of two graphs. For our convenience,we partition the edge set of $G \circ H$ into three sets. $E_1 = \{e \in E(G \circ H)|e \in E(H_i), 1 \le i \le n\}$, $E_2 = \{e \in E(G \circ H)|e \in E(G)\}$ and $E_3 = \{e \in E(G \circ H)|e = uv, u \in V(H_i), 1 \le i \le n, v \in V(G)\}$. It is easy to see that $E_1$, $E_2$ and $E_3$ are partition of the edge set of $G \circ H$ and also $|E_1| = |V(G)||E(G)|$, $|E_2| = |E(G)|$ and $|E_3| = |V(G)||V(H)|$. Let $t_e(G)$ denote the number of triangles containing an edge $e$ in $G$. The following lemma is used in the proof of the main theorem of this section.

**Lemma 2.1.** *[26] Let $G$ and $H$ be graphs, then*

*1. $|V(G \circ H)| = |V(G)|(1 + |V(H)|)$ and $|E(G \circ H)| = |E(G)| + |V(G)|(|V(H)| + |E(H)|)$.*

*2. (a) If $e = uv \in E_1$, then $d_{G\circ H}(u) = d_H(u) + 1$ and $d_{G\circ H}(v) = d_H(v) + 1$.*

*(b) If $e = uv \in E_2$, then $d_{G\circ H}(u) = d_G(u) + |V(H)|$ and $d_{G\circ H}(v) = d_G(v) + |V(H)|$.*

*(c) If $e = uv \in E_3$, and $u \in V(H)$, $v \in V(G)$, then $d_{G\circ H}(u) = d_H(u) + 1$ and $d_{G\circ H}(v) = d_G(v) + |V(H)|$.*

*(d) $\Delta(G \circ H) = \Delta(G) + |V(H)|$ and $\delta(G \circ H) = \delta(H) + 1$*

**Theorem 2.2.** *Let $G$ be connected graph of order $n$ and size $p$. If $H$ is a graph of order $m$ and size $q$, then*

$$Sz_r(G\circ H) = 2n(\Delta(G)+m+\delta(H))\,M_2(H)$$
$$+\left[n\sum_{e=uv\in E_1}(d_H(u)+d_H(v))^2 - 2n(\Delta(G)+m+\delta(H))\sum_{e=uv\in E_1}(d_H(u)+d_H(v))\right]t_e(H)$$
$$+\left[2n\sum_{e=uv\in E_1}\Delta(G)+m+\delta(H)) - n\sum_{e=uv\in E_1}(d_H(u)+d_H(v))\right]t_e{}^2(H)$$
$$-\ n\ Z'_{2,1}(H)+(m+1)^2[2(\Delta(G)+m+\delta(H))Sz(G)-Sz_w(G)]$$
$$+(\mathrm{nm(n(m+1)-1)-2qn)(2(\Delta(G)+\delta(H))+m+1)}$$
$$-\mathrm{2(n(m+1)-1)(qn+pm)+nM_1(H)+4pq}$$

*Proof.* As in the beginning of this section, we partition the edges of $G\circ H$ into three sets $E_1$, $E_2$ and $E_3$. First we compute for the set $E_1$. Let $e = uv \in E_1$ and let $x$ $V(H)$ is not adjacent to both $u$ and $v$ in $H$. Then $x$ is equidistant to $e$ in $G\circ H$, that is, $d_{G\circ H}(x, u) = d_{G\circ H}(x, v) = 2$. If $x$ is adjacent to both $u$ and $v$ in $H$, then $x$ is an equidistant to $e$ in $G\circ H$.

Hence $n_u^{G\circ H}(e) = d_H(u) - t_e(H)$ and $n_v^{G\circ H}(e) = d_H(v) - t_e(H)$

$$\sum_{e=uv\in E_1}\left(r_{G\circ H}(u)+r_{G\circ H}(v)\right)n_u^{G\circ H}(e)n_v^{G\circ H}(e)$$

$$= n\sum_{uv\in E_1}(\Delta(G\circ H)+\delta(G\circ H)-d_{G\circ H}(u)+\Delta(G\circ H)+\delta(G\circ H)$$
$$-d_{G\circ H}(v))(d_H(u)-t_e(H))(d_H(v)-t_e(H))$$

$$= n\sum_{e=uv\in E_1}\Delta(G)+m+\delta(H))+1-d_H(u)-1+\Delta(G)+m+\delta(H)$$
$$+1-d_H(v)-1)(d_H(u)-t_e(H))(d_H(v)-t_e(H)),by\ \ lemma\ \ 2.1$$

$$= n\sum_{e=uv\in E_1}(2(\Delta(G)+m+\delta(H))-(d_H(u)+d_H(v))(d_H(u)-t_e(H))(d_H(v)-t_e(H))$$

$$= n\ \{\sum_{e=uv\in E}2(\Delta(G)+m+\delta(H)d_H(u)d_H(v)$$
$$-\sum_{e=uv\in E_1}2t_e(H)(\Delta(G)+m+\delta(H)(d_H(u)+d_H(v))$$
$$+\sum_{e=uv\in E_1}2t_e^2(H)(\Delta(G)+m+\delta(H))-\sum_{e=uv\in E_1}\left(d_H(u)+d_H(v))d_H(u)d_H(v)\right)$$
$$+\sum_{e=uv\in E_1}t_e(H)\left(d_H(u)+d_H(v)\right)^2-\sum_{e=uv\in E_1}t_e^2(H)(d_H(u)+d_H(v))\}$$

$$= 2n(\Delta(G)+m+\delta(H)\ M_2(H)-2n(\Delta(G)+m+\delta(H)\sum_{e=uv\in E_1}\left(d_H(u)+d_H(v)\right)t_e(H)$$
$$+2n(\Delta(G)+m+\delta(H)\sum_{e=uv\in E_1}t_e{}^2(H)-n\sum_{e=uv\in E_1}\left(d_H^2(u)d_H(v)+d_H^2(v)d_H(u)\right)$$
$$+n\sum_{e=uv\in E_1}(d_H(u)+d_H(v))^2t_e(H)-n\sum_{e=uv\in E_1}(d_H(u)+d_H(v))t_e{}^2(H)$$

$$= 2n(\Delta(G)+m+\delta(H)M_2(H)$$
$$+\left[n\sum_{e=uv\in E_1}(d_H(u)+d_H(v))^2-2n(\Delta(G)+m+\delta(H)\sum_{e=uv\in E_1}\left(d_H(u)+d_H(v)\right)\right]t_e(H)$$
$$+\left[2n\sum_{e=uv\in E_1}(\Delta(G)+m+\delta(H))-n\sum_{e=uv\in E_1}\left(d_H(u)+d_H(v)\right)\right]t_e{}^2(H)-\ n\ Z'_{2,1}(H)$$

Next we compute for the set $E_2$. Let $e = uv \in E_2$ and let $x \in T_{G\circ H}(e; u)$. Then all the vertices of the copy of $H$ attached to $x$ are in $T_{G\circ H}(e; u)$. Since $|v(H)| = m$, $n_u^{G\circ H}(e) = (m + 1)(n_u^G)(e)$ and $n_v^{G\circ H}(e) = (m + 1)(n_v^G)(e)$ we have

$$\sum_{e=uv\in E_2}\left(r_{G\circ H}(u)+r_{G\circ H}(v)\right)n_u^{G\circ H}(e)n_v^{G\circ H}(e)$$
$$= \sum_{e=uv\in E_2}(\Delta(G\circ H)+\delta(G\circ H)-d_{G\circ H}(u)+\Delta(G\circ H)+\delta(G\circ H)$$
$$-d_{G\circ H}(v))(m+1)n_u^G(e)(m+1)n_v^G(e)$$

$$= \sum_{e=uv\in E_2}(\Delta(G)+m+\delta(H))+1-d_G(u)-m+\Delta(G)+m+\delta(H)+1-d_G(v)-m)$$
$$(m+1)n_u^G(e)(m+1)n_v^G(e)$$

$$= (m+1)^2\sum_{e=uv\in E_2}(2(\Delta(G)+\delta(H))+1)-(d_G(u)+d_G(v)))\ n_u^G(e)n_v^G(e)$$

$$= (m+1)^2\left[2(\Delta(G)+\delta(H))+1)\sum_{e=uv\in E_2} n_u^G(e)n_v^G(e) - \sum_{e=uv\in E_2}(d_G(u)+d_G(v))\, n_u^G(e)n_v^G(e)\right]$$

$$= (m+1)^2[2(\Delta(G)+m+\delta(H)+1)Sz(G) - Sz_w(G)]$$

Finally, for the set $E_3$. Let $u_1, u_2, \ldots, u_r$ be the vertices adjacent to $u$ in $H$. Then $u_j$ is equidistant to $e$ in $G \circ H$, for $j = 1, 2, \ldots, r$. On the other hand every vertex of $G \circ H$ other than $u_1, u_2, \ldots, u_r$ are in $T_{G\circ H}(e;v)$. Hence $n_u^{G\circ H}(e) = 1$ and $n_v^{G\circ H}(e) = |v(G\circ H)| - (d_H(u)+1)$, we have

$$\sum_{e=uv\in E_3}\left(r_{G\circ H}(u)+r_{G\circ H}(v)\right)n_u^{G\circ H}(e)n_v^{G\circ H}(e)$$

$$= \sum_{u\in V(H)}\sum_{v\in V(G)}(\Delta(G\circ H)+\delta(G\circ H)-d_{G\circ H}(u)+\Delta(G\circ H)$$
$$+\delta(G\circ H)-d_{G\circ H}(v))|(V(G\circ H)| - (d_H(u)+1)) \quad by\ lemma\ 2.1$$

$$= \sum_{u\in V(H)}\sum_{v\in V(G)}(2(\Delta(G)+\delta(H)+m+1-d_H(u)-d_G(v))[n(m+1)-d_H(u)+1]$$

$$= \sum_{u\in V(H)}\sum_{v\in V(G)}(2(\Delta(G)+\delta(H)+m+1)[n(m+1)-1]$$
$$-\sum_{u\in V(H)}\sum_{v\in V(G)}(2(\Delta(G)+\delta(H)+m+1)(d_H(u))$$
$$-[n(m+1)-1]\sum_{u\in V(H)}\sum_{v\in V(G)}(d_H(u)+d_G(v))$$
$$+\sum_{u\in V(H)}\sum_{v\in V(G)}d_H^2(u)+\sum_{u\in V(H)}\sum_{v\in V(G)}d_H(u)d_G(v)$$
$$= nm(n(m+1)-1)(2(\Delta(G)+\delta(H))+m+1) - 2qn(2(\Delta(G)+\delta(H)+m+1)$$
$$-(n(m+1)-1)(2qn+2pm)+nM_1(H)+(2p)(2q)$$
$$= (nm(n(m+1)-1)-2qn)(2(\Delta(G)+\delta(H))+m+1) - 2(n(m+1)-1)$$
$$(qn+pm)+nM_1(H)+4pq$$

Now we shall obtain $Sz_r(G\circ H)$. By the definition of $Sz_r(G\circ H)$

$$Sz_r(G\circ H) = \sum_{e\in E(G\circ H)}\left(r_{G\circ H}(u)+r_{G\circ H}(v)\right)n_u^{G\circ H}(e)n_v^{G\circ H}(e)$$
$$= \sum_{e=uv\in E_1}\left(r_{G\circ H}(u)+r_{G\circ H}(v)\right)n_u^{G\circ H}(e)n_v^{G\circ H}(e)$$
$$+\sum_{e=uv\in E_2}\left(r_{G\circ H}(u)+r_{G\circ H}(v)\right)n_u^{G\circ H}(e)n_v^{G\circ H}(e)$$
$$+\sum_{e=uv\in E_3}\left(r_{G\circ H}(u)+r_{G\circ H}(v)\right)n_u^{G\circ H}(e)n_v^{G\circ H}(e)$$

$$Sz_r(G\circ H) = 2n(\Delta(G)+m+\delta(H)\ M_2(H)$$
$$+\left[n\sum_{e=uv\in E_1}(d_H(u)+d_H(v))^2 - 2n(\Delta(G)+m+\delta(H))\sum_{e=uv\in E_1}(d_H(u)+d_H(v))\right]t_e(H)$$
$$+\left[2n\sum_{e=uv\in E_1}\Delta(G)+m+\delta(H)) - n\sum_{e=uv\in E_1}(d_H(u)+d_H(v))\right]t_e^{\,2}(H)$$
$$-\ n\ Z'_{2,1}(H)+(m+1)^2[2(\Delta(G)+m+\delta(H))Sz(G)-Sz_w(G)]$$
$$+(nm(n(m+1)-1)-2qn)(2(\Delta(G)+\delta(H))+m+1)$$
$$-2(n(m+1)-1)(qn+pm)+nM_1(H)+4pq$$

Using the theorem 2.2, we have the following corollaries

**Corollary 2.3.** *If $G$ is a connected graph of order $n$ and $H$ is a $r$ regular graph with order $m$ then*

$$Sz_r(G\circ H) = 2n(\Delta(G)+m+r)(qr^2)$$
$$+\left[n\sum_{e=uv\in E_1}(2r)^2 - 2n(\Delta(G)+m+r)\sum(2r)\right]t_e(H)$$

$$+(2n\sum(\Delta(G)+m+r)-n\sum(2r))\, t_e^2(H)$$

$$- nq(2r^3) + (m+1)^2\,[2(\Delta(G)+r+1)Sz(G) - Sz_w(G)]$$
$$+ (nm(n(m+1)-1) - 2qn)(2(\Delta(G)+r)+m+1)$$
$$- 2(n(m+1)-1)(qn+pm) + n(2rq) + 4pq.$$

*For a triangle free graph H,* $t_e(H) = 0$.

**Corollary 2.4.** *Let* $G$ *be connected graph of order* $n$ *and size* $p$*. If* $H$ *is a triangle free graph of order* $m$ *and size* $q$ *then*

$$Sz_r(G\circ H) = 2n(\Delta(G)+m+\delta(H))M_2(H) - n Z'_{2,1}(H)$$
$$+ (m+1)^2[2(\Delta(G)+\delta(H)+1)Sz(G) - Sz_w(G)]$$
$$+ (nm(n(m+1)-1) - 2qn)(2(\Delta(G)+\delta(H)) + m + 1)$$
$$- 2(n(m+1)-1)(qn+pm) + nM_1(H) + 4pq.$$

**Corollary 2.5.** *Let* $G$ *be connected graph of order* $n$ *and size* $p$*. If* $H$ *is a triangle free and* $r$ *regular graph of order* $m$ *and size* $q$*, then*

$$Sz_r(G\circ H) = 2n(\Delta(G)+m+r)(qr^2) - 2nqr^3$$
$$+ (m+1)^2[2(\Delta(G)+r+1)Sz(G) - Sz_w(G)]$$
$$+ (nm(n(m+1)-1) - 2qn)(2(\Delta(G)+r)+m+1)$$
$$- 2(n(m+1)-1)(qn+pm) + 2nrq + 4pq$$

**3. Revan weighted szeged indices of some special classes of graphs**

For a cycle $C_n$, path $P_n$ and complete graph $K_n$ on $n$ vertices, it is known that $Sz(C_n)=\frac{n^3}{4}$ When n is even, and $\frac{n(n-1)^2}{4}$ otherwise and $Sz(P_n)=\binom{n+1}{3}$; see[19].Also it can be easily seen that $Sz_w(C_n) = n^3$ when $n$ is even, and $n(n-1)^2$ otherwise, $Sz_w(P_n)=\frac{2(n-1)(n^2+n+3)}{3}$, $Sz(K_n) = \frac{n(n-1)}{2}$ and $Sz_w(K_n) = n(n-1)^2$.

Further it is known that the Zagreb indices on path and cycles are $M_1(C_n) = 4n$, $n \geq 3$, $M_1(P_1) = 0$, $M_1(P_n) = 4n-6$, $n > 1$, $M_1(K_n) = n(n-1)^2$ and $M_2(P_n) = 4(n-2)$, $M_2(C_n) = 4n$, $M_2(K_n) = \frac{n(n-1)}{2}$.

Its a direct consequence from the definition of Revan weighted szeged index for the cycle, path and complete graph is

$Sz_r(C_n)=\frac{n^3}{2}$ when n is even, $\frac{n(n-1)^2}{2}$ otherwise, $Sz_r(P_n)=\frac{(n-1)(n^2+n+6)}{3}$ and

$$Sz_r(K_n)=\frac{n(n-1)^2}{2}$$

For a given graph $G$, its $t-$ *fold bristled* graph $Brs_t(G)$ is obtained by attaching $t$ vertices of degree 1 to each vertex of $G$. This graph can be represented as the corona product of $G$ and complement of a complete graph on $t$ vertices. The $t-$ fold bristled graph of a given graph is also known as its $t-$ thorny graph.

**Example 1.** Let $G$ be a graph with $n$ vertices. Then

$$Sz_r(G\circ\overline{K_t}) = (t+1)^2\,[2\,\Delta(G)+m)Sz(G) - Sz_w(G)] + (\,nt\,(n(t+1)-1))(2\,\Delta(G)+t+1)$$
$$- 2(n(t+1)-1)(pt)$$

The $t-$ fold bristled graph of $P_n$ and $C_n$ are shown in Fig.2. From the above formula, the Revan weighted szeged indices of these graphs can easily be computed.

$$Sz_r(P_n \circ \overline{K}_t) = (t+1)^2(n-1)(n(n+1) - \frac{2(n^2+n-3)}{3} + t(n(t+1)-1)(nt+3n+2), \quad for\ n \geq 2$$

$$Sz_r(C_n \circ \overline{k}_1) = \begin{cases} \frac{n^3(t+1)^2}{2} + nt(t+3)(n(t+1)-1) & if\ n\ is\ even \\ \frac{n(n-1)^2(t+1)^2}{2} + nt(t+3)(n(t+1)-1) & if\ n\ is\ odd \end{cases}$$

A special corona graph $C_n \circ K_1$ is called a *sunlet* graph on $2n$ vertices.

$$Sz_r(C_n \circ k_1) = \begin{cases} 2n(n^2+4n-2n) & if\ n\ is\ even \\ 2n(n-1)^2 & if\ n\ is\ odd \end{cases}$$

**Example 2.** Let $H$ be a graph with $m$ vertices. Then

$$\begin{aligned} Sz_r(K_n \circ H) &= 2n(n+m+\delta(H)-1)\, M_2(H) \\ &+ \left[ n \sum_{e=uv\in E_1} (d_H(u)+d_H(v))^2 - 2n(n+m+\delta(H)-1) \sum_{e=uv\in E_1} (d_H(u)+d_H(v)) \right] t_e(H) \\ &+ \left[ 2n \sum_{e=uv\in E_1} n+m+\delta(H)-1) - n \sum_{e=uv\in E_1} (d_H(u)+d_H(v)) \right] t_e^{\,2}(H) \\ &- n\, Z'_{2,1}(H) + (m+1)^2[n(n-1)(\delta(H)+1)] \\ &+ (nm(n(m+1)-1) - 2qn)(2(n+\delta(H)) + m - 1) \\ &- (n(m+1)-1)(2qn + n(n-1)m) + nM_1(H) + 2n(n-1)q \end{aligned}$$

The *star* graph $S_{m+1}$ on $m+1$ vertices is the corona product of $K_1$ and $K_m$. The $\overline{fan}$ graph $F_{m+1}$ and the *wheel* graph $W_{m+1}$ on $m+1$ vertices are also corona product of $K_1$ and $P_m$ and $C_m$, seeFig.3.From the above formula the Revan weighted szeged indices of these graphs are obtained.

$$Sz_r(K_1 \circ \overline{K}_t) = t^2(t+1)$$

$$Sz_r(K_1 \circ P_t) = \begin{cases} 12 & \text{if } t=2 \\ t^3+7t^2-14t+4 & \text{if } t \geq 3 \end{cases}$$

$$Sz_r(K_1 \circ C_t) = \begin{cases} 36 & \text{if } m=3 \\ t(t^2+9t+2) & \text{if } t > 2. \end{cases}$$

For a given graph $H$ the graph $K_2 \circ H$ is called the *bottleneck* graph of $H$. The Revan weighted Szeged index of this graph can easily be obtained from Example 2

**Example 3.** Let $H$ be a graph with $m$ vertices. Then

$$\begin{aligned} Sz_r(K_2 \circ H) &= 4(m+\delta(H)+1)M_2(H) \\ &+ \left[ 2 \sum_{e=uv\in E_1} (d_H(u)+d_H(v))^2 - 4(m+\delta(H)+1) \sum_{uv\in E_1} (d_H(u)+d_H(v)) \right] t_e(H) \\ &+ \left[ 4 \sum_{uv\in E_1} (m+\delta(H)+1) - 2 \sum_{uv\in E_1} d_H(u)+d_H(v) \right] t_e^2(H) - 2Z'_{2,1}(H) \\ &+ (m+1)^2[2(\delta(H)+1)] + (2m(2m+1)-4q)(2\delta(H)+m+1) \\ &- 2(2m+1)(2q+m) + 2M_1(H) + 4q. \end{aligned}$$

.

In particular, the Revan weighted Szeged index of the bottleneck graph of $P_m$ is equal to $Sz_r(K_2 \circ P_t) = 2(t^3 + 7t^2 - 14t + 2)$.

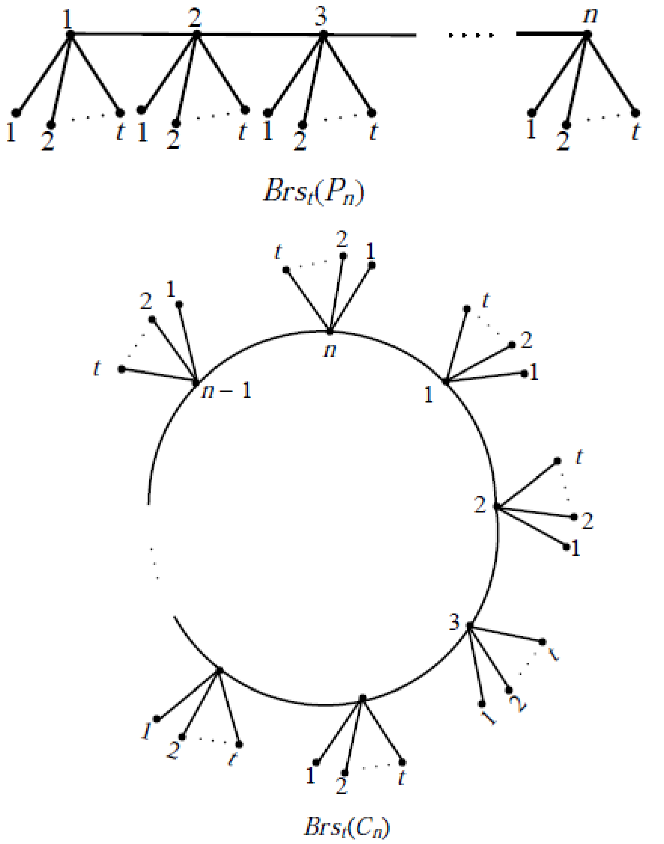


Fig.2 The $t-$ fold bristled graph of $Pn$ and $Cn$

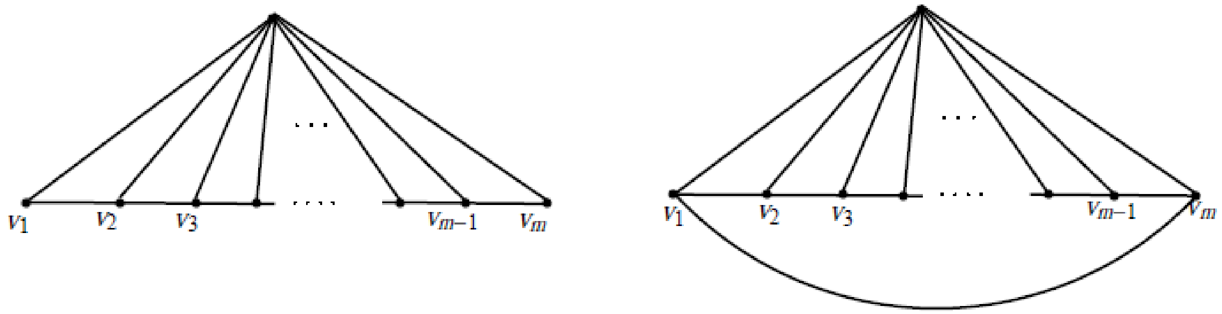


Fig. 3 Fan graph and wheel graph

Let $\{G_i\}_{i=1}^{n}$ be a set of finite pairwise disjoint graphs with $v_i \in V(G_i)$. The *bridge graph* $\{G_i\}_{i=1}^{n}$ with respect to the $\{v_i\}_{i=1}^{n}$ obtained from the graphs $G_1, G_2, \ldots, G_n$ by connecting the vertices $v_i$ and $v_{i+1}$ by an edge for all $i = 1, 2, \ldots, n-1$, see Fig.4.

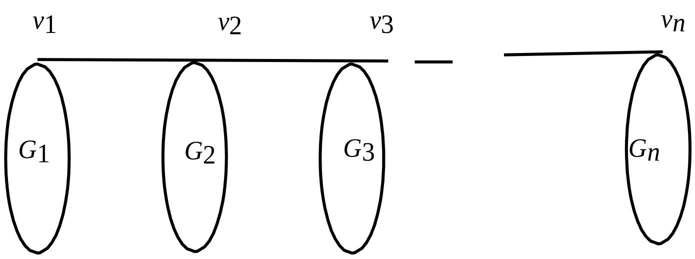


Fig.4 The bridge graph

We define $G_n(H,V) = B(H,H,\ldots,H;v,v,\ldots,v)$ (n times) which is the special case of the bridge graph. For example, let $P_n$ be the path on $n$ vertices $v_1, v_2, \ldots, v_n$, define $B_n = G_n(P_3, v_2)$, see Fig.5(Polyethene when $n = 4$ ). As another example, let $C_k$ be the cycle with $k$ vertices and define $T_n = G_n(C_k, v_1)$, see Fig.6 (when $k = 3$ and $n = 5$ ). As a final example , define the bridge graph $J_{n,m+1} = G_n(W_{m+1}, v_1)$, where $W_{m+1}$ is the wheel graph on $m+1$ vertices $v_1, v_2, \ldots, v_{m+1}$ such that $deg(v_1) = m$ and $deg(v_i) = 3$, $i = 1, 2, \ldots, m+1$. By the definition of corona product, $B_n = P_n \circ \overline{K_2}$, $T_{n,3} = P_n \circ K_2$ and $J_{n,m+1} = P_n \circ C_m$.

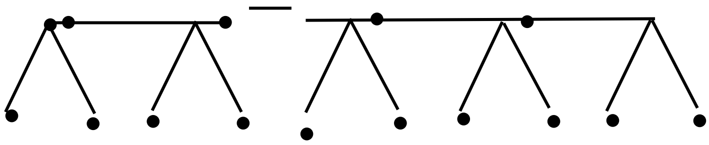

Fig.5 The graph $B_n$

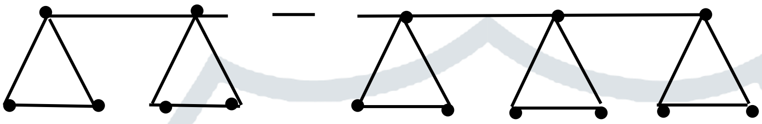

Fig.6 The graph $T_{n,3}$

**Example 4.** Using Theorem 2.2, we obtain the Revan szeged indices of the following graphs.

1. $Szr(B_n) = 3n^3 + 30n^2 + 17n - 22$.
2. $Szr(T_{n,3}) = 9n^3 + 36n^2 - 4n - 17$.
3. $Szr(J_{n,m+1}) = m(n + 5nm - 6) + (m + 1)^2(n - 1)(n^2 + n + 2) + nm(m + 1)(nm + 5n + 2)$, for $m > 3$.

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