EVEN DECOMPOSITION OF GRAPHS

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Abstract

A decomposition (G_1, G_2, G_3 , ..., G_n) of G is said to be a Linear decomposition or Arithmetic Decomposition if each G_i is connected and $|E(G_i)| = a+(i-1)d$, for all i = 1, 2, 3, ...,n and a, d ∈ Z+. E.Ebin Raja Merly introduce the concept of even decomposition of a connected graph and investigate their variations. The arithmetic Decomposition with a = 2 and d = 2 is known as Even Decomposition (ED) since the number of edges of each subgraph of G is even, we denote ED as (G_2, G_4, G_6, ..., G_{2n}). In this paper, we study the Even Decomposition (ED) of special class of graph namely $C_n \wedge K_2$, , W_{n+1} and G_n.

Keywords

Continuous Monotonic Decomposition, Arithmetic Decomposition, Arithmetic Odd Decomposition and Even Decomposition.

AMS Subject Classification: 05C70

1.Introduction

In this paper, we consider simple undirected graph without loops or multiple edges.The concept of Continuous Monotonic Decomposition was introduced by N.Gnana Dhas and J.Paulraj Joseph. The concept of Arithmetic Odd Decomposition was introduced by E.Ebin Raja Merly and N.Gnanadhas in [2] .We introduce the concept of even decomposition of a connected graph and investigate their variations.

Definition 1.1

Let G = (V, E) be a simple graph of order p and size q. If G_1, G_2, G_3, ..., G_n are connected subgraphs of G with $E(G) = E(G_1) \cup E(G_2) \cup E(G_3) \ldots \cup E(G_n)$, then ($G_1$, G_2, G_3, ..., G_n) is said to be a decomposition of G.

Definition 1.2

A decomposition (G_1, G_2, G_3, ..., G_n) of G is said to be continuous monotonic decomposition (CMD) if each G_i is connected and $|E(G_i)| = i$, for each i = 1, 2, 3, ..., n.

Definition 1.3

A decomposition (G_1, G_2, G_3 , ..., G_n) of G is said to be a Linear decomposition or Arithmetic Decomposition if each G_i is connected and $|E(G_i)| = a+(i-1)d$, for all i = 1, 2, 3, ...,n and a, d ∈ Z+..

Definition 1.4

The Arithmetic Decomposition of G is said to be Arithmetic Odd Decomposition(AOD) of G only when a = 1 and d = 2.

Remark 1.5

If a = 1 and d= 2, then the number of edges of G is n^2. Since the number of edges of G is n^2, q is the sum first n odd numbers 1, 3, 5, ...,2n-1. Since the number of edges of each subgraph of G is odd, we denote the AOD as (G_1, G_3, G_5, ..., G_{2n-1}).

2.EVEN DECOMPOSITION

Definition 2.1

The Arithmetic Decomposition of G is said to be Even Decomposition (ED) of G only when a = 2 and d = 2.

Remark 2.2

If a = 2 and d= 2, then the number of edges of G is n(n+1). Since the number of edges of G is n(n+1), q is the sum first n even numbers 2, 4, 6, ...,2n. Since the number of edges of each subgraph of G is even, we denote the ED as (G_2, G_4, G_6, ..., G_{2n}).

Lemma 2.3

Let $m \equiv 0(mod 2)$.The set $\{2,4,6, \dots .2m\}$ can be partitioned into two sets S_1 and S_2 such that

$$\sum_{x \in S_1} x = \sum_{y \in S_2} y = n$$

Here $m(m+1) = 2n$.

Proof

Let $m = 2k, k \geq 1, k \in \mathbb{Z}$.

Case 1: k is odd , $k = 2\lambda + 1$, $\lambda \geq 1$

Then $m = 2k = 2(2\lambda + 1)$ and $m + 1 = 4\lambda + 3$. Hence n is odd.

Case 2: k is even , $k = 2\lambda$, $\lambda \geq 1$

Proof is by induction on λ.

If $\lambda = 1$ then $k = 2, m = 4$ and $n = 10$.

Let $S_1 = \{2, 8\}$ and $S_2 = \{4, 6\}$. Now,

$$\sum_{x \in S_1} x = 2 + 8 = 10 = n$$

$$and \sum_{y \in S_2} y = 4 + 6 = 10 = n$$

Hence the result is true for $\lambda = 1$.

Assume that the result is true for $\lambda - 1$. Hence the set $\{2,4,6, \dots ,2(4\lambda - 4)\}$ can be partitioned into two sets S_1 and S_2 such that

$$\sum_{x \in S_1} x = \sum_{y \in S_2} y = n = (2\lambda - 2)(4\lambda - 3)$$

To prove the result is true for λ. The set $\{2,4,6, \dots .2(4\lambda)\}$ can be partitioned into two sets S_1' and S_2' where $S_1' = S_1 \cup \{2(4\lambda - 3), 2(4\lambda)\}$ and $S_2' = S_2 \cup \{2(4\lambda - 2), 2(4\lambda - 1)\}$.

Now,

$$\sum_{x \in S_1'} x = \sum_{x \in S_1} x + 2(4\lambda - 3) + 2(4\lambda)$$
$$= (2\lambda - 2)(4\lambda - 3) + 2(4\lambda - 3) + 2(4\lambda)$$
$$= 8\lambda^2 + 2\lambda$$
$$= 2\lambda(4\lambda + 1)$$
$$= n$$

Similarly,

$$\sum_{y \in S_2'} y = \sum_{y \in S_2} y + 2(4\lambda - 2) + 2(4\lambda - 1)$$
$$= (2\lambda - 2)(4\lambda - 3) + 2(4\lambda - 2) + 2(4\lambda - 1)$$
$$= 8\lambda^2 + 2\lambda$$
$$= 2\lambda(4\lambda + 1)$$
$$= n$$

Hence by induction the lemma is true for all λ.

Lemma 2.4

Let $m + 1 \equiv 0(mod 2)$. The set $\{2,4,6, \dots .2m\}$ can be partitioned into two sets S_1 and S_2 such that

$$\sum_{x \in S_1} x = \sum_{y \in S_2} y = n$$

Here $m(m+1) = 2n$.

Proof

Let $m+1=2k,\ k\geq 1, k\in \mathrm{Z}$.

Case 1: k is odd , $k=2\lambda+1\ ,\lambda\geq 1$

Then $m=2k-1$

$$=2(2\lambda+1)-1$$
$$=4\lambda+1$$

and $m+1=4\lambda+2$.

Hence n is odd.

Case 2: k is even , $k=2\lambda\ ,\lambda\geq 1$

Proof is by induction on λ.

If $\lambda=1$then $k=2, m=3$ and $n=6$.

Let S_1 = {2, 4} anS_2 = {6}. Now,

$$\sum_{x\in S_1} x=2+4=6=n \ and \ \sum_{y\in S_2} y=6=n$$

Hence the result is true if $\lambda=1$. Assume that the result is true for $\lambda-1$. Hence the set $\{2,4,6,\dots,2(4\lambda-5)\}$ can be partitioned into two sets S_1 and S_2 such that

$$\sum_{x\in S_1} x=\sum_{y\in S_2} y=n=(2\lambda-2)(4\lambda-5)$$

To prove the result is true for λ. The set $\{2,4,6,\dots,2(4\lambda-1)\}$ can be partitioned into two sets $S_1^{'}$ and $S_2^{'}$ where $S_1^{'}=S_1\cup\{2(4\lambda-3), 2(4\lambda-2)\}$ and $S_2^{'}=S_2\cup\{2(4\lambda-4), 2(4\lambda-1)\}$.

Now,

$$\sum_{x\in S_1^{'}} x=\sum_{x\in S_1} x+2(4\lambda-3)+2(4\lambda-2)=(2\lambda-2)(4\lambda-5)+2(4\lambda-3)+2(4\lambda-2)$$
$$=8\lambda^2-2\lambda$$
$$=2\lambda(4\lambda-1)$$
$$=n.$$

Similarly,

$$\sum_{y\in S_2^{'}} y=\sum_{y\in S_2} y+2(4\lambda-4)+2(4\lambda-1)$$
$$=(2\lambda-2)(4\lambda-5)+2(4\lambda-4)+2(4\lambda-1)$$
$$=8\lambda^2-2\lambda$$
$$=2\lambda(4\lambda-1)$$
$$=n.$$

Hence by induction the lemma is true for all λ.

Theorem 2.5

For any integer n, $C_n\wedge K_2$ has an ED $\{H_2,H_4,\dots,H_{2m}\}$ if and only if there exists an integer m satisfying the following properties:

(i) $m=2k \ or\ 2k-1\ (k\geq 1, k\in\mathbb{Z})$

(ii) $m(m+1)=2n$

Proof

Let $G=C_n\wedge K_2$. Then, $|E(G)|=2n$. Assume $C_n\wedge K_2$ has a ED $\{H_2,H_4,\dots,H_{2m}\}$

Then

$$|E(H_2)|+|E(H_4)|+\cdots|E(H_{2m})|=|E(G)|\Rightarrow 2+4+\cdots+2m=2n$$
$$2(1+2+\cdots+m)=2n$$
$$2\left(\frac{m(m+1)}{2}\right)=2n$$
$$m(m+1)=2n$$
$$m(m+1)\equiv 0(mod2)$$

$$m(m+1) = 2k, k \geq 1, k \in \mathbb{Z}$$

$$m = 2k \; or \; m = 2k - 1 \; (k \geq 1, k \in \mathbb{Z})$$

Conversely, assume $m(m+1) \equiv 0(mod 2)$. Let $G = C_n \wedge K_2$.
Let $V(C_n) = \{u_1, u_2, u_3, \dots, u_n\}$ and $V(K_2) = (v_1, v_2)$.
Define $w_{ij} = (u_i, v_j)$where $1 \leq i \leq n, 1 \leq j \leq 2$.
Now
$V(G) = \{w_{ij}: 1 \leq i \leq n, 1 \leq j \leq 2\}$ and $|E(G)| = 2n$.
Define
$T_1 = \{(w_{i1}, w_{(i+1)2}): 1 \leq i \leq n-1, i - odd\} \cup \{(w_{i2}, w_{(i+1)1}): 1 \leq i \leq n-2, i - even\} \cup \{(w_{11}, w_{n2})\}$ and
$T_2 = \{(w_{i2}, w_{(i+1)1}): 1 \leq i \leq n-1, i - odd\} \cup \{(w_{i1}, w_{(i+1)2}): 1 \leq i \leq n-2, i - even\} \cup \{(w_{12}, w_{n1})\}$.
Here $|T_1| = n$ and $|T_2| = n$. Also, $|T_1| + |T_2| = 2 + 4 + \cdots + 2m = m(m+1)$.By lemma 2.3 and lemma 2.4,
$\{2,4,6, \dots .2m\} = S_1 \cup S_2$ where

$$\sum_{x \in S_1} x = n \; and \; \sum_{y \in S_2} y = n$$

Decompose T_1 and T_2 into trees $\{H_i\}$ as follows

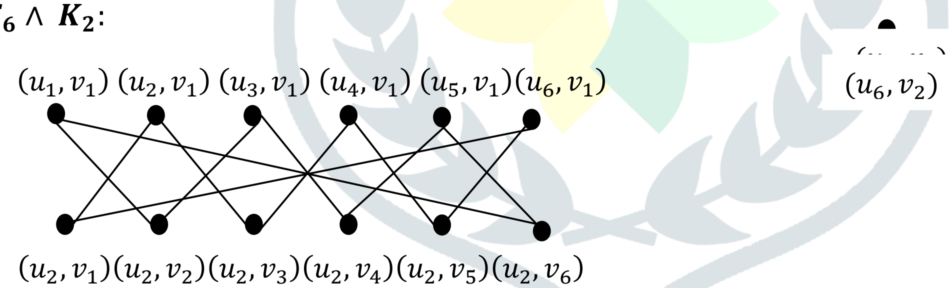
$$T_1 = \bigcup_{i \in S_1} H_i \; and \; T_2 = \bigcup_{i \in S_2} H_i$$

Also,$|E(H_i)| = i, 1 \leq i \leq 2m$.
Clearly $\{H_2, H_4, \dots, H_{2m}\}$ form an ED of $C_n \wedge K_2$.

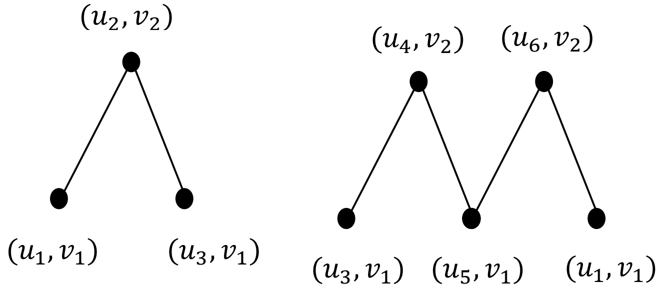
Illustration 2.6

As an illustration, let us decompose $C_6 \wedge K_2$.
Let$V(C_6) = \{u_1, u_2, \dots u_6\}$.Let $V(K_2) = \{v_1, v_2\}$

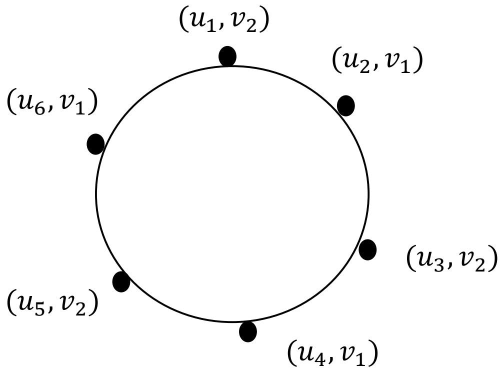
$\boldsymbol{C_6 \wedge K_2}$:



ED of $\boldsymbol{C_6 \wedge K_2}$:



$\boldsymbol{H_2}$ $\boldsymbol{H_4}$



$\boldsymbol{H_6}$

Theorem 2.7

For any integer n, W_{n+1}has an ED $\{H_2, H_4, \dots, H_{2m}\}$ if and only if there exists an integer m satisfying the following properties:

(i) $m = 2k \text{ or } 2k - 1\ (k \geq 1, k \in \mathbb{Z})$

(ii) $m(m+1) = 2n$

Proof

Let $G = W_{n+1}$. Then, $|E(G)| = 2n$. Assume W_{n+1} has a ED $\{H_2, H_4, \dots, H_{2m}\}$

Then

$$|E(H_2)| + |E(H_4)| + \cdots |E(H_{2m})| = |E(G)|$$
$$\Rightarrow 2 + 4 + \cdots + 2m = 2n$$
$$2(1 + 2 + \cdots + m) = 2n$$
$$2\left(\frac{m(m+1)}{2}\right) = 2n$$
$$m(m+1) = 2n$$
$$m(m+1) \equiv 0 (mod 2)$$
$$m(m+1) = 2k, k \geq 1, k \in \mathbb{Z}$$
$$m = 2k \text{ or } m = 2k - 1\ (k \geq 1, k \in \mathbb{Z})$$

Conversely,assume $m(m+1) \equiv 0 (mod 2)$.

Let $G = W_{n+1}$.

Let $V(W_{n+1}) = \{u, u_1, u_2, u_3, \dots, u_n\}$ and $|E(G)| = 2n$.

Define

$T_1 = \{(u, u_i) : 1 \leq i \leq n\}$and $T_2 = \{(u_i, u_{i+1}) : 1 \leq i \leq n-1\} \cup \{(u_n, u_1)\}$.

Here$|T_1| = n$ and $|T_2| = n$. Also, $|T_1| + |T_2| = 2 + 4 + \cdots + 2m = m(m+1)$.By lemma 2.3 and lemma 2.4, $\{2,4,6, \dots .2m\} = S_1 \cup S_2$ where

$$\sum_{x \in S_1} x = n \text{ and } \sum_{y \in S_2} y = n$$

Decompose T_1 and T_2 into trees $\{H_i\}$ as follows

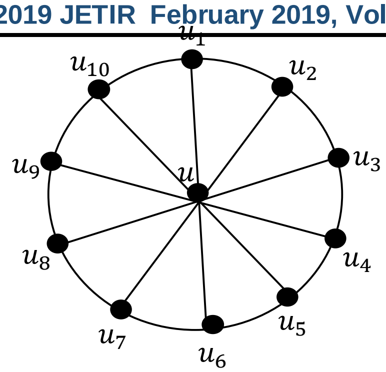
$$T_1 = \bigcup_{i \in S_1} H_i \text{ and } T_2 = \bigcup_{i \in S_2} H_i$$

Also,$|E(H_i)| = i, 1 \leq i \leq 2m$. Clearly $\{H_2, H_4, \dots, H_{2m}\}$ form an ED of W_{n+1}.

Illustration 2.8

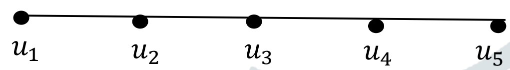
As an illustration, let us decompose W_{10+1}.

Let $V(W_{10+1}) = \{u_1, u_2, \dots u_{10}, u\}$. W_{10+1} is given

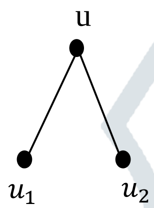


W_{10+1}

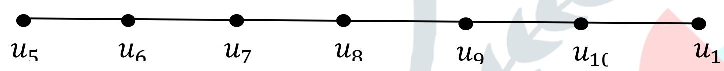
ED of W_{10+1}



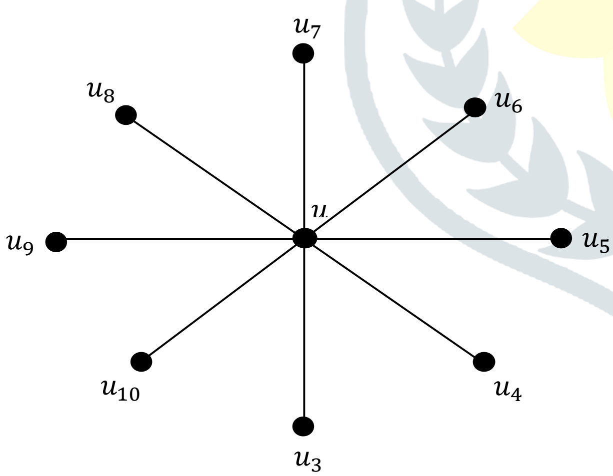
H_4



H_2



H_6



H_8

Theorem 2.9

For any integer n, Globe $G_n$has an ED $\{H_2, H_4, \ldots, H_{2m}\}$ if and only if there exists an integer m satisfying the following properties:

(i) $m = 2k \ or \ 2k - 1 \ (k \geq 1, k \in \mathbb{Z})$

(ii) $m(m + 1) = 2n$

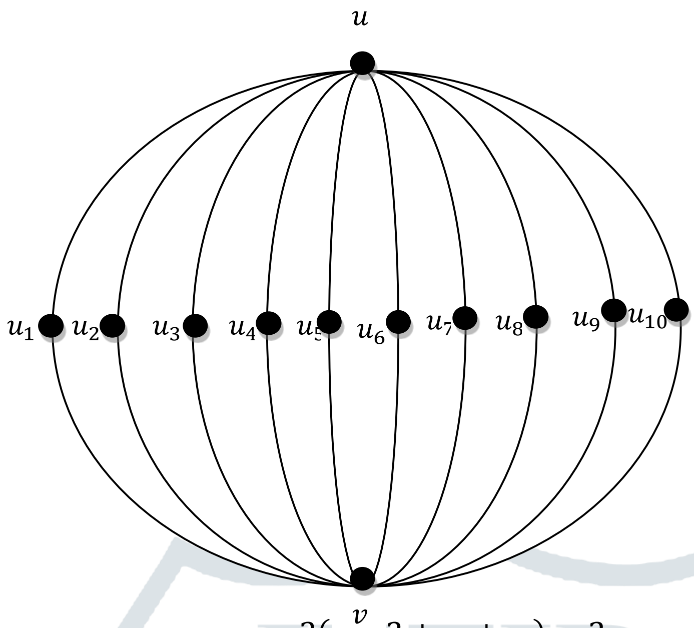
Proof

Let $G = G_n$. Then, $|E(G)| = 2n$. Assume G_n has a ED$\{H_2, H_4, \ldots, H_{2m}\}$

Then

$|E(H_2)| + |E(H_4)| + \cdots |E(H_{2m})| = |E(G)|$

$$\Rightarrow 2 + 4 + \cdots + 2m = 2n$$



G_{10}

$$2(1+2+\cdots+m)=2n$$
$$2\left(\frac{m(m+1)}{2}\right)=2n$$
$$m(m+1)=2n$$
$$m(m+1)\equiv 0(mod2)$$
$$m(m+1)=2k, k\geq 1, k\in\mathbb{Z}$$
$$m=2k\ or\ m=2k-1\ (k\geq 1, k\in\mathbb{Z})$$

Conversely, assume $m(m+1)\equiv 0(mod2)$.
Let $G=G_n$. Let $V(G_n)=\{u,v,u_1,u_2,u_3,\dots,u_n\}$ and $|E(G)|=2n$.
Define
$T_1=\{(u,u_i):1\leq i\leq n\}$and $T_2=\{(v,u_i):1\leq i\leq n\}$.
Here$|T_1|=n$ and $|T_2|=n$. Also, $|T_1|+|T_2|=2+4+\cdots+2m=m(m+1)$.By lemma 2.3 and lemma 2.4, $\{2,4,6,\dots.2m\}=S_1\cup S_2$ where

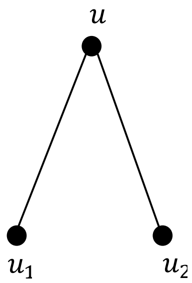
$$\sum_{x\in S_1}x=n\ and\ \sum_{y\in S_2}y=n$$

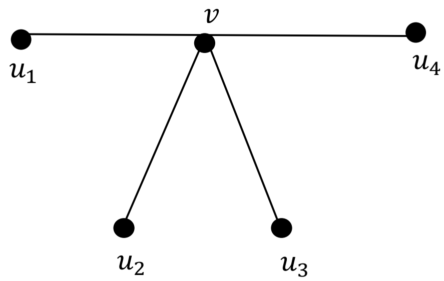
Decompose T_1 and T_2 into trees $\{H_i\}$ as follows
$$T_1=\bigcup_{i\in S_1}H_i\ and\ T_2=\bigcup_{i\in S_2}H_i$$
Also,$|E(H_i)|=i, 1\leq i\leq 2m$. Clearly $\{H_2,H_4,\dots.,H_{2m}\}$ form an ED of G_n.

Illustration2.10

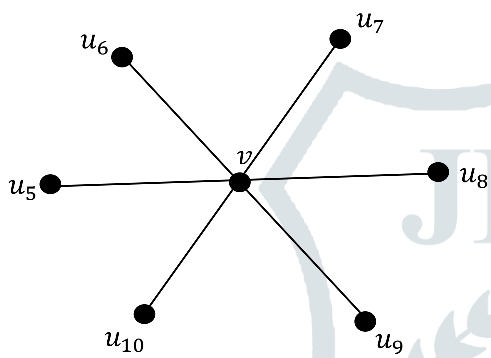
As an illustration, let us decompose G_{10}. Let $V(G_{10})=\{u,v,u_1,u_2,u_3,\dots,u_{10}\}$. G_{10}. is given below.

ED of G_{10}

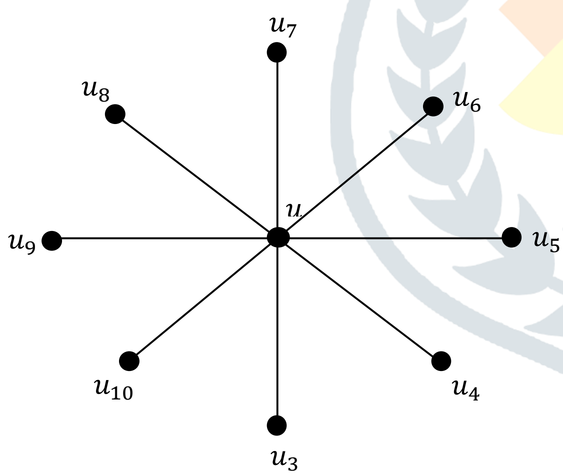




H_2 H_4



H_6



H_8

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