INTRODUCTION TO MEAN LABELING

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Abstract

Graph labelling is an essential and interesting topic in graph theory. There are nearly 200 graph labeling techniques. Graph theory was introduced in the year 1960.In this paper, the graph is taken as simple, finite and undirected. V(G) represents vertex set and E(G) represents Edge set. A graph labeling is the assignment of labels, that is represented by integers, to the edges or vertices, or both, of a graph .we shall study some of the types of labelling with illustration which gives a clear idea about it. The topics are explained with examples and supporting results and diagrams for clear idea about the concept. The study is limited with certain types.

Keywords: Mean Graph – Even Mean labelling – Odd Mean Labelling – Strongly Multiplicative – Harmonious Multiplicative – Geometric mean labelling – Skolem difference

Introduction

In this paper, the graph are taken as simple, finite and undirected. V(G) represents vertex set and E(G) represents Edge set. A graph labeling is the assignment of labels, that is represented by integers, to the edges or vertices, or both, of a graph. A vertex labelling is a function of V to a set of labels. A graph with such a vertex labelling function is defined as Vertex – labeled graph.

An edge labeling is a function of E to a set of labels and a graph with such a function is called as an edge-labelled graph. We shall discuss about labelling concept in this paper.

Definition – 1

A graph is said to be *mean graph* if a graph with p vertices and q edges has an injective function f from the vertices of G to { 0,1,2…….q} such that when each edge uv is labeled with (f(u) + f(v))/2 if f(u) + f(v) is even and (F(u) + f(v)+1)/2 if f(u) + f(v) is odd , then the resulting edge labels are distinct.

Definition -2

A graph with p vertices and q edges is said to be a *relaxed mean graph* if there exists a function f from the vertex set of G to {0, 1, 2, · · · , q − 1, q + 1} such that the induced map f ∗ from the edge set of G to {1, 2, · · · , q} defined by

$$f * (e = uv) = \begin{cases} \frac{f(u)+f(v)}{2} & if\ f(u)+f(v)\ is\ even \\ \frac{f(u)+f(v)+1}{2} & if\ f(u)+f(v) is\ odd \end{cases}$$

then the edges get distinct from 1,2,3,…..q.

Definition – 3

A function f is called an *Even Mean Labeling* of a graph G with p vertices and q edges. If f is an injection from the vertices of G to the set {2,4,6,….2q} such that when each edge uv is assigned the label [f(u)+f(v)/2],then the resulting edge labels are distinct. A graph, which admits an Even Mean labeling, is said to be Even Mean Graph.

Definition – 4

A function f is called an *Odd mean labeling* of a graph G with p vertices and q edges. If f is an injection from the vertices of G to the set {1,3,5,…..2q-1} such that when each edge uv is assigned the label [f(u)+f(v)/2], then the resulting edge labels are distinct. A graph, which admits an odd mean labeling, is said to be odd mean graph.

Definition-5

A graph with p vertices is *Strongly Multiplicative* if the vertices of G can be labeled with distinct integers 1,2,3,…..p such that the labels induced on the edges by the product of the end vertices are distinct.

Definition – 6

A graph of order n is said to be a *Strongly graph* if its vertices can be assigned the values 1,2,3,4…..n in such a way that, when an edge whose vertices are labeled i and j is labeled with the value i+j+ij, all edges have different labels

Definition – 7

A *harmonious labeling* on a graph *G* is an injection from the vertices of *G* to the group of integers modulo *k*, where *k* is the number of edges of *G*, that induces a bijection between the edges of *G* and the numbers modulo *k* by taking the edge label for an edge (*x*, *y*) to be the sum of the labels of the two vertices *x*, *y* (mod *k*).

Definition – 8

A *harmonious graph* is one that has a harmonious labeling. Odd cycles are harmonious, as is the Petersen graph. It is conjectured that trees are all harmonious if one vertex label is allowed to be reused.[8] The seven-page book graph $K_{1,7} \times K_2$ provides an example of a graph that is not harmonious.

Definition – 9

A Graph G=(V,E) with p vertices and q edges is said to be a *Geometric mean* if it is possible to label the vertices $x \in v$ with distinct labels f(x) from 1,2,….q+1 in such a way that when each edge e=uv is labeled

with f(e=uv) = $\lceil\sqrt{f(u)f(v)}\rceil$ or $\lfloor\sqrt{f(u)\ f(v)}\ \rfloor$, then the resulting edge labels are distinct. In this case f is called Geometric mean labeling of G.

Definition – 10

Let G = (V,E) be a skolem difference mean graph with p vertices and q edges. If one of the *Skolem difference mean labeling* of G satisfies the condition that all the labels of the vertices are odd, then we call this skolem difference mean labeling an extra skolem difference mean labeling and call the graph G an extra skolem difference mean graph.

Results

In this section, it is proved that comb $P_n \Theta K_1$, $\forall_n C_{2n+1}$, $\forall_n K_{1,n}$, $\forall_n$ and P_n, $\forall$ n are even and odd mean graphs.

Result – 1 Every Comb graph is odd and even mean graph.

Proof: Let G = $P_n \Theta K_1$, $\forall$ n be a comb graph with 2n vertices and 2n-1 edges. The comb graph $P_n \Theta K_1$ The even mean labeling for vertices of comb $P_n \Theta K_1$ is defined by

$u_i = 4i - 2$, i = 1,2,… n

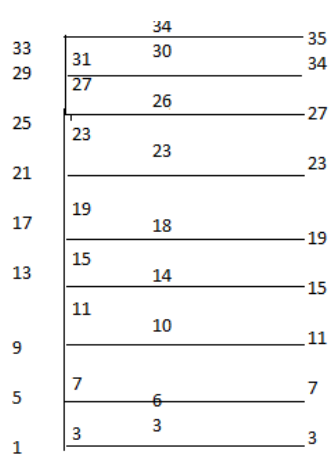
$v_i = 4i$, i = 1,2,… n

Edge labelings are defined by ei = 4i, i = 1,2,… n – 1 ei′ = 4i – 1, i = 1,2,… n .

The odd Mean labeling for vertices of comb $P_n \Theta K_1$ is defined by

$u_i = 4i - 3$, i = 1,2,… n

$v_i = 4i - 1$, i = 1,2,… n



Edge labelings are defined by

$e_i = 4i – 1$, $i = 1,2,\ldots n – 1$

$ei' = 4i – 2$, $i = 1,2,\ldots n$

Therefore, the labeling of vertices and edges are distinct. Hence the graph is even and odd mean graph.

Result – 2

Every cycle of odd length is even and odd mean graph

Proof: Let G = Cn , ∀ n ≥ 3 and n is odd

The even mean labeling for vertices of Cn is defined by

ui = 2i , i =1,2, ….n

Edge labelings are defined by

ei = 2i +1, i = 1,2, ….n-1,

en = n+1.

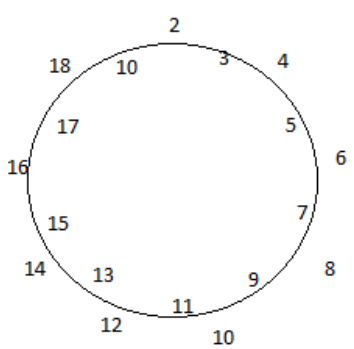
The odd mean labeling for vertices of cycle Cn is defined by ui = 2i-1,i = 1, 2, 3, 4……..n.

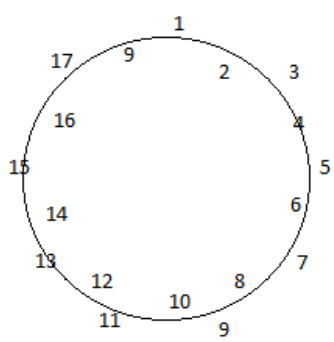
Edge labelings are defined by ei = 2i, i = 1,2,3….n-1. en = n.

From above t, the labeling of vertices and edges are distinct.

Hence the graph is even and odd mean graph.

Example Every cycle of odd length is an even and odd mean graph.





Result – 3

Every star graph is odd and even mean graph

Proof: Let G = K 1,n ∀ n be a star graph with n + 1 vertices and n edges. 100 .

The odd mean labeling for vertices of star graph K 1,n is defined by

v0 = 1, ui = 2i+1, i = 1,2,3…..n

Edge labelings are defined by ei = i + 1, i = 1,2,3…n.

The even mean labeling for vertices of star graph K1,n , ∀ n is defined by

v0 = 2, ui = 2i+2, i = 1,2,3….n

Edge labeling are defined by ei = i +2, i = 1,2,3….n

From the above, the vertex and edge labeling are distinct.

Hence the star graph is odd and even mean graph.

Example - Star graph K1,12 is odd and even mean graph

The Graph K 1, 12 Figure 6.9 The Graph K 1,12 Theorem 6.4: Every path graph Pn ,∀ n is odd and even mean graph. Proof: Let G = Pn , ∀ n be a path graph with n vertices and n-1 edges.

The odd mean labeling for vertices of path graph Pn is defined by

ui = 2i-1, i = 1,2,3..n

Edge labelings are defined by

ei = 2i, i = 1,2,…n - 1

The even mean labeling for vertices of path graph Pn ∀ n is defined by

ui =2i, i =1,2….n.

ei = 2i+1, i =1,2,3……n - 1 .

From the above assignment, the vertex and edge labelings are distinct.

Hence the graph G = Pn , ∀ n is odd and even mean graph.

Results

Theorem 2.1 Ln A $\bar{K}2$ is a Geometric mean graph.

Proof: Let u_1 u_2 ……u_n and v_1v_2….vn be two paths of length n. Join u_i and v_i, 1≤i≤n.

The resultant graph is Ln. For 1≤i≤n, join u_i with two vertices s_i, t_i and v_i with two vertices x_i, y_i.

The resultant graph is LnAK2 whose edge set is E = $\{u_iu_{i+1}, v_iv_{i+1}, | 1 \le i \le n-1\} \cup \{s_iu_i , t_iu_i ,u_iv_i ,v_ix_i ,v_iy_i ,/1 \le i \le n\}$

Define a function f:V(LnAK2) On geometric mean graphs

f(s1) =1,

f(si) = 7(i-1), 2≤i≤n

f(ti) = 7i-5, 1≤i≤n

f(ui) = 7i-4, 1≤i≤n

f(xi) = 7i-2, 1≤i≤n

f(yi) = 7i-1, 1≤i≤n

Edges are labeled with f(siui) = 7i-6, 1≤i≤n

f(tiui) = 7i-5, 1≤i≤n

f(uivi) = 7i-4, 1≤i≤n

f(vixi) = 7i-3, 1≤i≤n

f(viyi) = 7i-2, 1≤i≤n

f(uiui+1) = 7i-1, 1≤i≤n-1

f(vivi+1) = 7i, 1≤i≤n-1

Then we get distinct edge labels. Thus f provides a Geometric mean labeling for LnAK2.

Example:

Theorem 2.3: Triangular ladder TLn is a Geometric mean graph.

Proof: Let u1, u2…..un and v1, v2…,vn be the two path of length n.

Join ui and vi, 1≤i≤n.

The resultant graph is Ln. For 1≤i≤n-1, Join ui and vi+1, and the resultant graph is TLn and whose edge set is E= {ui ui+1, vivi+1, uivi+1 /1≤i≤n-1}∪{uivi/1≤i≤n-1}

Triangular LadderTLn has 4n-3 edges.

Define a function f: V (TLn) → {1,2,…q+1} by

f(u1) = 1,

f(ui) = 4(i-1), 2≤i≤n-1.

f(v1) =3 ,

f(vi) = 4i-2, 2≤i≤n

Edges are labeled with f(uivi) = 4i-3, 1≤i≤n

f(uiui+1) = 4i-2, 1≤i≤n-1

f(uivi+1) = 4i-1, 1≤i≤n-1

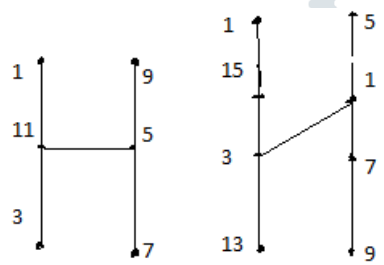
f(vivi+1) = 4i, 1≤i≤n-1

Hence TLn is a Geometric mean graph.

Theorem: TLnAK1 is a Geometric mean graph.

Proof: Let u1, u2….un and v1 v2….vn two paths of length n.

Join ui and vi, (1≤i≤n).



The resultant graph is Ln. For 1≤i≤n-1, join ui and vi+1the resultant graph is TLn For 1≤i≤n, add two new vertices xi and yi and join ui with xi and vi with yi.

The resultant graph is TLnAK1, Whose edge set is E = {ui ui+1, vivi+1, uivi+1 | 1≤i≤n-1}∪ {ui vi, xiyi, yivi 1≤i≤n} On geometric mean graphs .

Define a function f: V (LnAK1) → {1,2,…q+1} by

f(x1) =1,

f(xi) = 6i-2, 2≤i≤n

f(y1) =2,

f(yi) = 6i-5, 2≤i≤n

f(u1) =3,

f(ui) = 6i-3, 2≤i≤n

f(v1)=4,

f(vi) = 6i-4, 2≤i≤n

Edges are labeled with

f(u1v1)=3,

f(uivi) = 6i-4, 2≤i≤n

f(x1u1) = 1,

f(xiui) = 6i-3, 2≤i≤n

f(y1v1) =2, f(yivi) = 6i-5, 2≤i≤n

f(u1v2) = 4,

f(uivi+1) = 6i-2, 2≤i≤n-1

f(uiui+1) = 6i, 1≤i≤n-1

f(v1v2) = 5, f(vivi+1) = 6i-1, 2≤i≤n-1

Hence f provides a Geometric mean labeling of TLnAK1.

Conclusion: We have discussed about various types of Graph labelling and its application with certain results. There are many graph labelling types apart from this study, Only selective study has been made.

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